This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

University of Wisconsin
LIBRARY
Class Tb
Book .W43
.E
3
2

20896

MECHANICS OF ENGINEERING

AND OF

MACHINERY.

BY DR. JULIUS WEISBACH.

VOLUME III.

PART I., SECTION II.

THE MECHANICS OF THE MACHINERY OF TRANSMISSION.

SECOND EDITION, THOROUGHLY REVISED AND GREATLY ENLARGED,

BY

GUSTAV HERRMANN,

PROFESSOR AT THE ROYAL POLYTECHNIC SCHOOL, AACHEN, GERMANY.

DESIGNED AS A TEXT-BOOK FOR TECHNICAL SCHOOLS AND COLLEGES, AND FOR
THE USE OF ENGINEERS, DRAUGHTSMEN, ETC.

TRANSLATED BY

J. F. KLEIN, D.E.,

(Joseph)

PROFESSOR MECHANICAL ENGINEERING, LEHIGH UNIVERSITY, BETHLEHEM, PA.

AUTHORIZED TRANSLATION.

WITH MORE THAN FOUR HUNDRED FINE ILLUSTRATIONS

NEW YORK:
JOHN WILEY & SONS,
53 EAST TENTH STREET.
1890.

COPYRIGHT, 1890.
BY
JOSEPH F. KLEIN.

ROBERT DRUMMOND,
Electrotyper,
444 & 446 Pearl Street,
New York.

FERRIS BROS.,
Printers,
326 Pearl Street,
New York.

6730384

TABLE OF CONTENTS.

CHAPTER IV.

ROPES AND CHAINS.

ARTICLE PAGE

114.—Flexible Transmitters 535
115.—Hempen Rope 536
116.—Strength of Hempen Rope 545
117.—Wire Rope 548
118.—Strength of Wire Rope 556
119.—Chains 560
120.—Comparison of Ropes and Chains 563
121.—The Uniting of Ropes and Chains 566
122.—Sheaves and Guide Pulleys 570
123.—Rope-rods 577

CHAPTER V.

SCREWS.

124.—Screws in General 584
125.—Helical Motion 591
126.—The Relations of the Forces Acting on Screws 594
127.—Screws used as Fastenings 600
128.—Check Nuts 605
129.—The Differential Screw 609
130.—Screws with Differential Motion 612
131.—Right- and Left-handed Screws 617
132.—The Endless Screw 622
133.—Screw Wheels 627
134.—The Dimensions of Screws 634

CHAPTER VI.

CRANK TRAINS.

135.—The Quadric Crank Train 639
136.—The Drag-link Coupling 643
137.—Parallel Cranks 646
138.—The Ordinary Crank Train 651
139.—The Motions in the Crank Train 657

ARTICLE PAGE
140.—The Oscillating Crank Train 669
141.—Crank with Variable Velocity 673
142.—Roemer's Mechanism 678
143.—Influence of Mass 683
144.—The Crank with Constant Force on the Piston 689
145.—Influence of the Length of the Connecting Rod 696
146.—The Double Crank 702
147.—Influence of the Length of the Connecting Rod 708
148.—The Triple Crank 715
149.—The Crank and the Beam 720
150.—The Crank with Variable Force on the Piston 727
151.—Accelerating Force 734
152.—Piston Pressure 738
153.—Graphical Investigation of the Crank Motion 742
154.—Friction of the Crank Train 747
155.—Crank Constructions 754
156.—Eccentrics 757
157.—Dimensions of the Crank 763
158.—The Connecting Rod 771

CHAPTER VII.

CAM TRAINS.

159.—Cam Surfaces 784
160.—Wipers 788
161.—Flat Cams 799
162.—Inversions 802
163.—Eccentrics 805
164.—Triangular Cams 812
165.—Cylindrical Cams 815
166.—Cam Rails 826

CHAPTER VIII.

ENGAGING AND DISENGAGING GEAR.

167.—Disengaging in General 832
168.—Clutches 833
169.—Toothed Wheels as Disengaging Gear 840
170.—Belts as Disengaging Gear 854
171.—Click Gear 860
172.—Ratchet Gear 868
173.—Escapements 882

CHAPTER IX.

REGULATORS.

174.—Regulating in General 895
175.—Brakes 898

ARTICLE PAGE
176.—Block-brakes........ 900
177.—Wagon-brakes........ 905
178.—Band-brakes........ 912
179.—Other Brakes........ 919
180.—The Fly........ 924
181-182.—Counterweights........ 926
183.—Hydraulic Counterweight........ 937
184.—Force Regenerator........ 941
185.—Draw-bridges........ 945
186.—Counterweighting Cranks........ 953
187.—Inertia of the Counterweight........ 958
188-189.—Fly Wheels........ 962
190.—Action of Fly Wheels........ 967
191-192.—Weight of the Rotating Mass........ 973
193.—Distribution of the Mass of the Fly Wheel........ 983
194.—Strength of Fly Wheels........ 985
195.—Governors........ 990
196.—Sensitiveness of Governors........ 1000
197-198.—Static and Astatic Governors........ 1006
199.—Pseudo-astatic Governors........ 1018
200 —Proell's Governor........ 1027
201.—The Cosine Governor........ 1031
202.—Differential Governors........ 1040
203.—Hydraulic and Pneumatic Governors........ 1042
204.—Poncelet's Governor........ 1045

APPENDIX.

Graphical Statics of Mechanisms........ 1049

CHAPTER IV.

ROPES AND CHAINS.

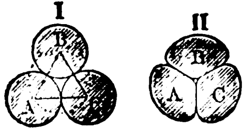
§ 114. **Flexible Transmitters.**—Ropes and chains are often used to transmit rectilinear motion when they are subjected to *tensile* stress only, for these flexible pieces cannot transmit *compressive forces.* They are generally employed when considerable motion is to be transmitted, for in such cases the flexibility of the transmitter permits its being wound on drums or pulleys, thus greatly simplifying the construction, and avoiding the difficulties which the use of rigid pieces, such as rods, would involve. For instance, it is much easier to make deep borings by means of ropes than by means of rigid boring rods. Ropes find their most natural and extensive use in lifting loads to great heights, as in windlasses, cranes, hoists, hauling apparatus, and on inclined planes. These same means of transmission are also employed in transportation, and in illustration we need only mention rope and chain towage. Finally, we may employ ropes as a substitute for rods which have a reciprocating motion, and are alternately subjected to tension and compression, as in pump rods attached to bobs, but the number of ropes must then be doubled, and they must be so attached to the extremities of equal levers that the ropes will act alternately and thus produce the required oscillating motion of the bob. Here also the whole arrangement is greatly simplified by the possibility of making any change of direction by passing the ropes over guide pulleys, and thus avoiding the use of the clumsy bobs. The employment of ropes and chains in the transmission of rotary motions has already been discussed in the second chapter under the head of rope and chain wheels.

For the purposes enumerated ropes are generally used; it is only in certain cases—for example, in powerful cranes, where very large forces are to be transmitted—that chains are used. Ropes are made of either hemp or wire, wire ropes in particular having recently come into extensive use.

§ 115. Hempen Rope.—Before wire ropes were made, hemp was the only material used in rope-making; but now hempen rope is seldom employed, except for the smaller forces and where drums and pulleys can have but a small diameter, as in tackle; wire ropes, on account of their stiffness, cannot be bent into sharp curves without a considerable loss of strength. Hempen rope is made from yarn spun from the best of hemp; occasionally Manilla hemp is used for ship cordage, because it is lighter, and stands alternate wetting and drying better than hemp. In order that the rope may be as strong as possible, only well *hackled* hemp should be used, from which the hackling (or combing) has removed the weak as well as the short and coarse fibres that add little or nothing to the strength of the rope. The weaker cordage can be made by twisting together some of the yarns, which are generally spun fine—for example, store cord, which needs but two or three of these yarns. In stronger cordage, composed of many yarns (often several hundred), these yarns are first twisted into *strands*, a few of which (usually three or four) are also twisted together and constitute the rope; while the strongest cordage, for example a cable having a girth[*] of 0.2 to 0.5 meters [8 to 20 ins.], is formed by twisting together three or four of these ropes.

That these steps are necessary in rope-making may be easily shown. Let A, B, C, Fig. 449, I, be the circular cross-sections of three parallel and adjacent yarns of equal length: then if they are twisted the axis of each yarn will arrange itself in helical coils, which may be regarded as lying upon the surface of a cylinder passing through the centres of gravity A, B, C of the three cross-sections. The pitch of such a helix is given by $\frac{L}{n} = s$, where L is the length of the piece after turning, and n the number of turns given to the piece. The length of such a helical coil will be given by

FIG. 449.



---

[*] The size of rope is often referred to its circumference instead of its diameter.

$$l = \sqrt{(\pi d)^2 + s^2},$$

where d is the diameter of the circle $A\,B\,C$. Now if we suppose the two ends of the portion of cord under consideration to be kept at a constant distance from each other, so that the pieces of yarn before turning have the same length L as the cord after turning, then it is clear that during the turning each yarn will be stretched so that the original length of the yarn $\frac{L}{n} = s$ will by the turning be made equal to a length of the helical coil

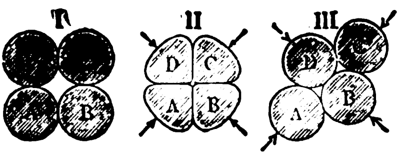
$$l = \sqrt{(\pi d)^2 + s^2};$$

that is, it will be stretched an amount equal to

$$l - s = \sqrt{(\pi d)^2 + s^2} - s.$$

As a consequence, a certain tension will exist in each yarn, from which follows that the individual yarns will all press *inward*. The result is that the soft yarns in the twisted cord will assume a cross-section represented by Fig. 449, II. The pressing inward of the individual yarns can be explained in the same manner as the tendency of a stretched string to resume its position of equilibrium when it has been deflected from its straight direction. Now, even if the ends do not, as was here assumed, remain at a constant distance apart while the yarns are being twisted into a strand, but approach each other to a certain extent, thus shortening the strand, nevertheless a certain tension will still remain in the yarns, giving rise to their pressure against each other. On this depends the contracting action which accompanies the twisting in all spinning processes. If instead of three yarns we suppose four to be placed, as in Fig. 450, I, these will, on account of the yielding character of the material, assume the cross-sections of II by reason of the pressures which they exert upon each other when twisted. But it is necessary that the intensity of this pressure

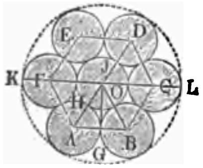
FIG. 450.



be the same for all yarns; for if we suppose, for example, that the yarns A and C are subject to less tension than B and D the cross-section will take a form like III, the less taut threads, being pressed outward by the others. Such inequality of tension occurs when the individual yarns do not exactly agree, and even when the lengths are equal the differences in the constitution of the individual yarns might cause this defect. Aside from the non-circular form of the cord, the material would in such a case be strained very unequally, that is, very disadvantageously.

It is now easily seen that with a greater number of yarns the defect mentioned will be still more liable to appear; for example, if we suppose that there are six yarns $ABCDEF$, as in Fig. 451, then, when the ideal condition obtains, *i.e.*, when there is exact equality of length and otherwise exact likeness of constitution, the six yarns can readily assume, relatively to each other, the positions given by the regular hexagon $ABCDEF$, but the slightest difference would cause the yarn with the greatest tension to slip inwards, and the others would then twist about it.

Fig. 451.



On this account it is customary to employ only three or at most four *strands* with hempen rope, and with large cables only three or four *ropes*. In making *strands*, however, a greater number of yarns (up to 100 and more) are generally taken, the defect above mentioned being avoided by filling the middle portion of the strand with a special cord or strand. This middle portion is called the *core*. In hempen rope, cores are only used in the strands, for the three or four strands constituting a rope are so few in number that they do not need a core. In wire rope, however, where the strands usually number six, not only each strand, but the rope itself, is provided with a core, which principal core is often only a special strand placed within the others.

The size of the core depends on the number, n, of yarns or threads and their thickness, d. The core must not be too thick, else the yarns will be separated; nor can it be too thin, for then the defect above mentioned as due to inequalities in the strand will still exist, though to a less degree. If d_0 represents the diameter HJ of the circle which touches the cross-sections of all the yarns on the inside, and $\alpha = AOB = \frac{360^\circ}{n}$ the angle at the centre subtended by each yarn, then will

$$d_0 = 2\,OH = 2(OA - AH) = \left(\frac{1}{\sin\frac{\alpha}{2}} - 1\right)d.$$

The diameter $AD = d_1$ of the circle on which lie the centres of the cross-sections of the yarns can be found from

$$d_1 = d + d_0 = \frac{d}{\sin\frac{\alpha}{2}},$$

and the diameter $KL = d_2$ of the external circle of contact from

$$d_2 = d + d_1 = \left(\frac{1}{\sin\frac{\alpha}{2}} + 1\right)d.$$

From this we get for the different values of n the following table:

$n =$	4	5	6	8	12
$\alpha =$	90°	72°	60°	45°	30°
$d_0 =$	$0.414\,d$	$0.702\,d$	d	$1.613\,d$	$2.863\,d$

The necessity for the twisting in hempen rope is easily explained. If we imagine the yarn to be formed by placing the hempen fibres parallel to each other and side by side, they will not have any strength, for with their short length (at most 1 meter [3.28 ft.]) they will slide along each other when their longer thread is subjected to a pull, without calling forth their resistance to tearing. During the twisting of the fibres in process of spinning, they arrange themselves in helices, and exert, as shown above, certain pressures upon each other. These pressures cause frictional resistances between the fibres, which prevent their sliding on each other. Now when the sum of these frictional resistances exceeds the tensile strength of the fibres, they will tear before they will slide. Consequently in spinning it is of the first importance to twist the fibres until this friction exceeds the strength. It is clear that the amount of twist, that is, the angle of inclination of the helical lines to the axis of the thread, can be smaller the longer the individual fibres and the rougher the material. The number of turns n in a thread of the length L, whose outermost helical fibre has a given angle of inclination δ to the axis of the thread, evidently depends on the diameter d of the thread, for if $s = \frac{L}{n}$ is the pitch of this helix, we have the equation

$$tan\ \delta = \frac{\pi d}{s}.$$

The size of the threads or yarns commonly used for ropes is such that one kilogram of fibre will make 200 or 250 meters [656 to 770 ft.] of thread, which corresponds to a diameter of 2.2 millimeters [0.088 in.]. If with *Karmarsch* we assume for such yarns 40 turns per meter [13 turns per ft.], and consequently a pitch $s = 0.025$ meter [$s = 1$ in.], we shall have, for the outermost layer of fibres, an angle of inclination of the helices to the axis given by $tan\ \delta = \frac{3.14 \times 2.2}{25} = 0.276$, or, about 15°.

The considerations which hold for the twisting of fibres into yarns apply also to the *laying up* or uniting of yarns into strands and of strands into ropes, for these operations are essentially alike. The proper amount of twist to be given to the yarns and strands is of great importance to the strength and durability of the ropes. With too little twist the rope is not sufficiently dense, round, and smooth, and moisture can too easily penetrate it, thus greatly lessening its durability. On this account it is customary to impregnate ropes with a sticky, insoluble substance like tar. The substance must be sticky in order that the friction between the fibres may not be diminished. The yarns or strands are tarred separately or in the finished rope. The yarns may be twisted immediately after they are drawn through the hot tar or after they have become cold. In the first case they are said to have been made by the *warm register* and in the second case by the *cold register.* Laying up a rope while hot renders it closer and less pliable than when laid up cold. The tarring itself diminishes the strength of the rope by rendering the fibres somewhat brittle, but prevents a still greater loss from the effects of moisture. Thus, according to *Muschinbroek's* experiments, an untarred rope when wet through will sustain only seven tenths of the load which it can sustain when dry. Tarring increases the weight of the rope from 10 to 20 per cent, according to the manner of tarring.

On the other hand, the strands and rope must not be twisted too much, because they will lose in flexibility, and because the twisting itself is always accompanied by a diminution of strength. Experience shows that a strand or rope never sustains as great a load as could be sustained by all the separate untwisted fibres when placed side by side. This can be explained as follows: If a strand composed of a certain number of yarns is so twisted that there is one complete twist for every length s of the strands, all the threads will lie in helices having the pitch s. The angle of inclination δ of these helices to the straight axis depends on their distance from the axis, and is determined from $\tan \delta = \frac{\pi d}{s}$, where d represents the diameter of the cylindrical layer in which the yarns are located. The length l of one such coil of yarn is evidently given by

$$s = l \cos \delta.$$

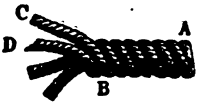
If we suppose the twisting to be unaccompanied by any tensile stress and consequent elongation in the yarns, the effect of twisting on the strand will be to diminish its length from l to s, shortening it by the amount $l - s = l(1 - \cos \delta)$. But as the separate yarns do subject each other to tension and elongation in twisting, the actual shortening will be less than that given by the above expression. Thus, for example, the strands of a rope 142 fathoms long,* after being twisted through an angle $\delta = 40°$, had a length of 118 fathoms, *i.e.*, it was shortened $\frac{24}{142} = 16.9$ per cent, while the expression

$$l\,(1 - cos\,40°) = 0.234\,l$$

gives more than 23 per cent. This difference is due to the elongation of the fibres during twisting. It follows, of course, that the strength of the rope is diminished, for the fibres are subjected to stress before the load comes on the rope, this twisting stress increasing with the angle of twist δ. For these reasons the angle of twist in strands and ropes is usually taken between 30° and 40°.

This also explains the rule universally observed in rope-making, according to which, during the successive steps of twisting fibres into yarns, yarns into strands, and strands into ropes, each of these twistings must be opposite in direction to the preceding one. For instance, the fibres are usually laid up *right*-handed into yarns, the yarns *left*-handed into strands *B C*, Fig. 452, and the strands *right*-handed into ropes *A B*. If the strands were turned in the same direction as the yarns, the effect of the two operations would be summed up in the yarns, not only making the latter very tight and stiff, and the strands less flexible, but also considerably diminishing the strength of the material. In using these opposite directions of twisting, the effect of each step of the operation is to partly undo the work of the preceding one. For instance, if a certain yarn be given n_1 right-handed turns during spinning, and if it receives afterward during the formation of the strand n_2 left-handed turns, there will remain in the yarn only $n_2 - n_1$ right-handed twists or coils. Frequently this result is left unchanged, for the number of twists on a given length diminishes at each succeeding step, n_2 being less than n_1. Occasionally, however, when the loosening of the yarns, which always accompanies a twisting in the opposite direction, is in-

FIG. 452.



---

* See *Prechtl*, Technolog. Encyklopaedie, vol. 14, p. 480.

compatible with the desired closeness of the rope, this untwisting action during the formation of the strands can be wholly neutralized by giving to the yarns an additional right-handed rotation at the same time that the left-handed rotation is laying up the yarns into strands. A similar supplementary rotation takes place in making wire rope, though for different reasons. These opposite directions of twisting are needed to effect a lasting union between the parts of a rope, for if the twisting were all in one direction, they would have a strong tendency to untwist.

There is still another result of the twisting which diminishes the strength of the rope. The stress on the rope due to twisting is not, of course, the same in all yarns, for it depends upon the angle of inclination of the helical coils of the yarns. While the yarns lying on the surface of a strand form helical lines having a large angle of inclination, and are therefore subject to considerable stress, the angles of inclination of the lines toward the centre of the strand become smaller, as is evident from the equation $tan\ \delta = \frac{\pi d}{s}$. As a consequence, the yarns are stretched less the nearer they are to the centre of the strand. The middle yarns are even slack to a certain extent, as is evident from the fact that before twisting all the yarns have for each coil the same length l, which length after twisting is drawn down to the shorter length s of the strand. Now if we suppose the rope to be subjected to a certain load, the middle, slack yarns will not at first take any part in sustaining it; the load is therefore held only by the yarns already stretched by twisting, and it may happen that the outer yarns will break before the inner ones are stretched at all. This defect is common to all the older hand-made ropes, in which all the yarns were of the same length. If to remedy this we start with the inner yarns enough shorter so that all the yarns will be equally taut after twisting, and therefore capable at first of helping to sustain the load, we shall fall into the opposite error of breaking the inner yarns before the outer ones reach their full working stress. For it is clear that the elongation of the rope caused by the load can be followed by the inner, nearly straight, yarns only by an actual stretching of those yarns. On the other hand, the outer, helical yarns, can, by virtue of their form, follow this elongation without stretching, for in consequence of the pull the rope becomes a little smaller and the helical lines steeper, a coil requiring less length than before. The consequence of such an arrangement is therefore that the inner yarns will break first—a more serious defect than the other, because the tearing of the inner fibres would be completely hidden from observation.

Now that cordage is made by machinery, all these circumstances are carefully considered, and the difficulties obviated by obedience to the following principles.* The yarns of a strand are arranged in *concentric layers*, each layer having the exact number of yarns corresponding to its circumference. The *lengths* of the yarns in the *separate layers* are *different*, *decreasing* as they *approach the centre*. The lengths of the yarns in the individual layers are not, however, determined so that the yarns will all have the same tension at first, but so that the tension of the yarns due to twisting shall gradually decrease toward the centre. Now when a rope is stretched by a load, the tension in its inner yarns increases more rapidly in proportion as their steeper helices lose the power of following the elongation through diminution of the angle of inclination. In this way it is possible to strain all the fibres equally when the rope is subjected to its maximum working stress. This is accomplished by calculating the length of the yarn of all the separate layers from a certain angle of twist δ_1 of the outer layer, which is smaller than the actual angle of twist δ in the finished rope. Now if the twisting be continued till this angle δ_1, which may be taken about 27°, is reached, all the yarns will have the same tension, for each has been strained the amount due to its special helix. Now if a supplementary rotation is imparted to the rope so that the outer yarns will form a final angle of inclination δ of about 36° with the axis, the yarns will all receive an increase of tension which will be greater the farther they are from the centre. The experiments made with these ropes show a considerably greater strength than those made by hand according to the older methods.

---

* See *Prechtl*, Technolog. Encyklopaedie, vol. 14, p. 582.

§ **116. Strength of Hempen Rope.**—The strength of hempen rope does not depend merely on the material and the cross-section, but, according to what has preceded, on the method of manufacture and on the amount of twisting. The loss of strength in a rope when greatly twisted is shown by the following experiment by Muschenbroek: a rope twisted until it was one fifth shorter, broke with a load of 6834 lbs.; when it was twisted until it was one fourth shorter, it broke with a load of 5335 lbs.; and when the twisting was increased so as to shorten it by one third, the ultimate strength was 4519 lbs.

Moreover, it is plain that the larger sizes, consisting of a great number of yarns, are comparatively weaker than the smaller ones, for in the former there not only exists a greater inequality in the tension of the yarns, but this inequality is increased by bending around pulleys and drums. The strength is of course diminished by this inequality, for the more severely stretched yarns tear sooner than if the tension were uniform.

The size of a hempen rope for a given load P can be calculated from the breaking strength given in Vol. I., §218, a factor of safety of 3 being assumed for *stationary* ropes used only for fastening, and a factor of safety of 5 for *running* ropes, that is, such as are wound around drums and pulleys.

Consequently, if we assume the ultimate strength to be 5 kilograms per square millimeter [7110 lbs. per sq. in.] and take 5 for the factor of safety, we shall obtain the diameter of a running rope from

$$P = 1 \frac{\pi d^2}{4} = 0.785 d^2 \quad [P = 1120 d^2]$$

or

$$d = 1.13 \sqrt{P} \text{ millimeters.} \quad [d = 0.03 \sqrt{P} \text{ in.}]$$

For wet and tarred rope the diameter may be assumed 10 per cent greater, $d = 1.24 \sqrt{P}$ $[d = 0.033 \sqrt{P}]$; for running rope the diameter may be taken 20 per cent smaller, $d = 0.9 \sqrt{P}$ $[d = 0.024 \sqrt{P}]$. According to the experiments made at the rope manufactory of *Felten & Guillaume*, of Cologne, the average ultimate strength of their ropes was

$$K = 18{,}675 \text{ lbs. per square inch,}$$

or

$$K = 13.13 \text{ kilograms per square millimeter;}$$

that is, more than twice as large as is usually assumed.

The weight of round rope per running meter [per running foot] when it has a diameter of d millimeters [d inches] is

$$G = 0.0009\, d^2, \quad [G = 1.28\, d^2,]$$

or when we substitute $d = 1.13 \sqrt{P}$ [$d = 0.03 \sqrt{P}$] we have $G = 0.00115\, P$ and $P = 870\, G$ [$G = 0.00035\, P$ and $P = 2855\, G$].

Tarred rope on an average weighs 20 per cent more: for this therefore

$$G = 0.00108\, d^2; \quad [G = 1.54\, d^2;]$$

or taking $d = 1.24 \sqrt{P}$ [$d = 0.033 \sqrt{P}$], we get

$$G = 0.00166\, P \text{ and } P = 602 G. \quad [G = 0.0005 P \text{ and } P = 1975 G.]$$

Consequently the weight of a tarred rope 870 meters [2850 ft.] long is sufficient to subject it to its working tension. For tarred running rope, 602 meters [1975 ft.] represents the working tension. From what was said above, we see that for standing rigging these lengths can be taken greater in the ratio of 5 to 3.

According to the data furnished by *Felten & Guillaume*, we must distinguish between closely laid ropes to be used as ship cordage and for hauling purposes, and the more loosely laid ropes suitable for tackle and windlasses. When both are of the same size, the former are said to be 1.8 times heavier than the latter, but only 1.5 times as strong. Experiments made by the above-mentioned firm with ropes of Rhine hemp having 3 strands which contain from 25 to 80 yarns, were found to have an average ultimate strength of 13.13 kilograms per square millimeter [18,675 lbs. per sq. in.], the results being tabulated on the following page.

Ropes which are subject to great loads and are to be wound on drums are often made flat in the form of bands, several (4 to 6) round ropes about 30 millimeters [1.2 ins.] diameter being placed side by side and woven or sewed together by zigzag lines of hempen cord or brass wire. In this way the large diameters are avoided which would be necessary in single round ropes of the same strength, and an easier winding about the drums is obtained. Moreover, the coils of this flat rope can also wind themselves *upon* instead of *alongside* of each other, thus gradually increasing the radius of the drum. It is well to make this flat band of ropes half of which have a right-handed twist, while the other half have a left-handed twist, and place them so that they will alternate, for with this arrangement the rope keeps its flat form better, and its tendency to turn about its axis when under tension is considerably diminished. This last circumstance and the flexibility of the flat rope make it particularly suitable for well-boring and hoisting in shafts.

If such a rope contains m cords of diameter d_1, we have

$$P = 0.785\, m\, d_1^2, \text{ and consequently } m = 1.27 \frac{P}{d_1^2}.$$

Given by *Felten & Guillaume* for

TENSILE STRENGTH OF HEMPEN ROPE OF AVERAGE CLOSENESS.

No. of Strands = 3.

No.	Girth		Area of rope.	No. of yarns in strand.	Load of 11600 lbs. on rope 24.72 ins. long.		Weight of rope 1 foot long. lbs.	Tearing Load		Remarks.
	of rope. ins.	of strand. ins.	sq. ins.		Elongation. ins.	Girth of rope. ins.		of rope. lbs.	per sq. in. lbs.	
1	3.26	1.67	0.657	25			0.384	12125	18420	Tore at beginning of hitch.
2	3.77	1.89	0.848	34	1.41		0.501	16303	19200	
3	4.12	2.19	1.146	34			0.501	19275	16830	
4	4.12	2.14	1.104	40	2.83	3.77	0.652	17582	15930	
5	4.12	2.14	1.104	40	2.57	3.77	0.652	19579	17750	Wet outside, dry inside, tore at end of hitch.
6	4.80	2.44	1.432	51	3.60	4.29	0.786	29184	20400	Tore at beginning of hitch.
7	5.16	2.57	1.590	63	2.57	4.72	0.947	33212	20850	
8	6.18	3.09	2.289	82	2.83	5.49	1.176	45688	19980	

N. B.—The average ultimate stress is therefore 18675 lbs. per sq. in.

Given by *Felten & Guillaume* for

TENSILE STRENGTH OF HEMPEN ROPE OF AVERAGE CLOSENESS.*

No. of Strands = 3.

No.	Girth		Area of rope.	No. of yarns in strand.	Load of 5264 kg. on rope 0.628 meters long.		Weight of rope 1 meter long.	Tearing Load		Remarks.
	of rope. mm.	of strand. mm.	mm².		Elongation. mm.	Girth of rope. mm.	kg.	of rope. kg.	per mm². kg.	
1	82.7	42.5	424	25	....	....	0.571	5500	12.95	Tore at beginning of hitch.
2	95.8	47.9	547	34	35.9	....	0.746	7395	13.50	
3	104.6	55.6	739	34	....	....	0.746	8743	11.83	
4	104.6	54.4	712	40	71.8	95.8	0.970	7975	11.20	
5	104.6	54.4	712	40	65.4	95.8	0.970	8881	12.48	Wet outside, dry inside, tore at end of hitch.
6	122	62.1	924	51	91.5	109	1.17	13238	14.34	Tore at beginning of hitch.
7	131	65.4	1026	63	65.4	120	1.41	15065	14.66	
8	157	78.5	1477	82	71.8	139.4	1.75	20724	14.05	

N. B.—The average ultimate strength is therefore 13.13 kilograms per sq. millimeter.

This may also be expressed in terms of the diameter d of a round rope having the same cross-section $m = \left(\frac{d}{d^1}\right)^2$; but it should be noticed that greater strength may be expected of a flat rope than of a single round rope of the same cross-section, for in the latter, as in all thick ropes, the inequalities of tension in the different fibres are greater. The weight of the flat rope is somewhat greater than the sum of the weights of the round ropes forming it, on account of the wires or cords used in sewing it together.

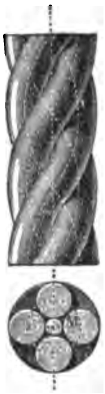
§ **117. Wire Ropes.**—Wire rope has been in use about fifty years.† The advantage consists in the greater strength of the material, which is usually iron or steel wire. In consequence of their greater strength, wire ropes are smaller than hempen ropes for the same load; but having less flexibility they require larger pulleys and drums, for which reason wire rope cannot be used for tackle and the smaller windlasses.

---

* See Kunst und Gewerbeblatt des polytechn. Vereins fuer das Koenigreich Bayern, 1856; also, Polytechn. Centralblatt, 1856.

† Wire ropes composed of single, parallel, straight wires secured by fine wire wound spirally around the whole, were used earlier in suspension bridges, but *twisted* wire rope was first used by Oberbergrath *Albert* in Clausthal, in 1834. See *Karsten's*, Archiv fuer Huettenkunde, 1837, p. 235.

Fig. 453.



Wire rope is made by twisting together fine metal wires, in a manner similar to that employed in making hemp rope from the yarns. The wires, of which there are often several hundred in a rope, are first laid up into strands, Fig. 453, and these strands are laid up into rope. The rope can never be formed by a direct twisting together of all the wires. From what was said under the head of hempen rope, we know that strands of more than four wires require a core; this is usually of hemp. Without such a central piece it would be impossible to prevent some of the wires from entering the inner portion of the strand, and therefore the inner threads with their steeper helices and consequent shorter length would be especially strained by the load, while the outer threads on account of their flatter inclination would yield more easily. For the same reason, when the strands are laid up into rope a core is placed between them, for the number of strands is always greater than 4, usually 6, and sometimes more. In hempen rope such a principal core between the strands is never used, as there are usually only three strands, rarely four. Instead of increasing the number of strands in a hempen rope when greater strength is needed, the ropes themselves are laid up into cables, but this is never done with wire rope. From the discussion of hempen rope we know that the thickness of the core depends on the number and size of the wires in the strand and of the strands in the rope. In wire as in hempen rope, the rule is to lay up the strands into rope in a direction opposite to that in which the wires (or yarns) were twisted into strands.

Sections of several ropes consisting of three to four strands are shown in Figs. 454, 455, 456, and 457, Fig. 454 representing the cross-section of a rope containing three strands of six wires each, and Fig. 455 one having six strands of three wires

Fig. 454.

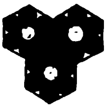


Fig. 455.

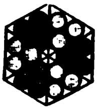


Fig. 456.

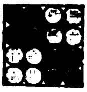
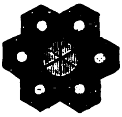


Fig. 457.



each. From what has preceded, we know that the former is without the principal core, while the latter is without cores in the strands. In like manner the cores in the four strands of four wires each composing the rope, Fig. 456, may be left out, while in all ropes containing more wires, for example, in the rope of thirty-six wires, Fig. 457, the use of cores is indispensable. For the size of the cores in the strands we have the relation already established,

$$d_0 = \left(\frac{1}{sin \frac{\alpha}{2}} - 1 \right) d,$$

where α is the angle at the centre, and d the diameter of the wire. The greater the number of strands the more nearly does the rope approach a cylindrical form after twisting. This twisting causes the axes of the strands to lie on the surface of a cylinder, whose diameter D_1 is in each case given by simple geometrical relations. Thus for the cross-sections, Fig. 454 and Fig. 455, we have

$$D_1 = 2\,d + \frac{d}{sin\,60^\circ} = 3.155\,d;$$

for the cross-section Fig. 456,

$$D_1 = \frac{2\,d}{sin\,45^\circ} = 2.828\,d:$$

and for the cross-section Fig. 457,

$$D_1 = 2\,d\,sin\,60^\circ + d\,cot\,15^\circ = 5.464\,d.$$

The exterior diameter D of the rope depends also upon the closeness of the twisting; *Reuleaux* gives for the ratio $\nu = \frac{D}{d}$ of diameter of rope to diameter of wire, when i represents the number of wires, this table:*

$i =$	36	48	54	60	66	72
$\nu = \frac{D}{d} =$	8	10.25	11.33	12.80	13.25	14.20

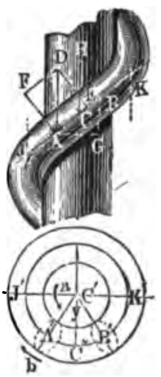
---

* *Kuehn* deduces from comparisons of samples of wire rope $\nu = \frac{i}{4}$ for numbers under 48, and for greater numbers $\nu = 9.4 + \frac{i}{19}$ See Zeitschrift fuer Berg-, Huetten- und Salinenwesen im preussischen Staats, 1873.

In the manufacture of wire rope, although the process is in many respects like that of hempen rope, the difference of material causes such a difference in stress that it must be specially taken into account. In particular, the stiffness of the wire, which has recently come to be used in its hard unannealed condition, prevents the strands being as easily twisted as the yarns, for in this process each wire would be subject to a considerable twisting, which would greatly tax its torsional strength. To avoid this straining of the wire, we must prevent the twisting which would accompany the laying up of the wires during the formation of the rope, by giving an opposite rotation to each wire.

The kind of stress developed in the spirally-wound wires of a strand during its formation can be ascertained as follows:

Fig. 458.



Let *A B*, Fig. 458, be a piece of helically-wound wire short enough to be regarded as straight, whose centre line is at the distance r from the axis of the strand, making with this axis an angle of inclination δ. We may regard the piece *A B* as being generated during the formation of the strand, by supposing that the straight wire starting from *A* keeps its distance $C'A' = r$ from the axis, turning about the latter through the angle $A'\,C'\,B' = \gamma$, while the strand itself moves in the direction of its length a distance $B\,G = l \cos \delta$, where l represents the length *A B*. If $L = \dfrac{2\pi r}{\sin \delta}$ be taken for the length of a helical coil, it is evident that the angle γ is given by

$$\gamma = 2\pi \frac{l}{L} = \frac{l}{r} \sin \delta.$$

This rotation may be laid off in amount and direction along the axis *H C* in the well-known manner. Now transfer this axis of rotation from *C* to *A* parallel to itself, adding, according to § 4 of the Introduction, a corresponding translation $A'B' = r\gamma$, and resolve this axis of rotation *D A* into its two components, *E A* in the direction of the piece *A B* and *F A* at right angles to it, these components lying in the tangent plane to the cylinder which contains the helical line AB. This at once shows us that during the formation of the strand the piece of wire is subjected to two different actions:

(1) A *twisting* action due to the rotation whose axis is $EA = DA \cos\delta = \gamma \cos\delta$, and which subjects the piece of wire to torsion as if it were a shaft; and

(2) A *bending* action by a rotation whose axis is $FA = \gamma \sin\delta$.

If we represent by γ_t and γ_b the angles of rotation corresponding to a unit of length of wire, we shall have for the angle of torsion

$$\gamma_t = \frac{\gamma \cos\delta}{l} = \frac{1}{r} \sin\delta \cos\delta = \frac{1}{2r} \sin 2\delta,$$

and for the angle which corresponds to the bending,

$$\gamma_b = \frac{\gamma \sin\delta}{l} = \frac{1}{r} \sin^2\delta.$$

First considering the bending, we see that $\gamma_b = \frac{1}{r} \sin^2\delta$ represents the angle included between the consecutive radii of the helix forming the axis of the wire, consequently the radius of curvature of the helix will be $\rho = \frac{1}{\gamma_b} = \frac{r}{\sin^2\delta}$; this may be proved in another way by considering that the radius of curvature of the helix must coincide with that of the ellipse JK obtained by intersecting its cylinder by a plane passed through the element AB perpendicular to the axis FA. The semi-axes of this ellipse are $C'C'' = r$ and $CK = \frac{r}{\sin\delta}$; consequently the radius of curvature at the vertex C'' will be

$$\rho = \frac{CK^2}{C'C''} = \frac{r}{\sin^2\delta}.$$

This bending causes a tensile or compressive stress k in the outer fibres of the wire, which stress may be found from $\frac{E}{\rho} = \frac{k}{\frac{1}{2}d}$ (see Vol. I. § 115), for $\frac{k}{\frac{1}{2}d}$ represents the stress occurring at a distance unity from the neutral axis. This formula can also be written

$$k = \frac{d}{2\rho} E = \frac{d}{2r} E \sin^2 \delta ;$$

and we see that for a given diameter of wire d and diameter of rope $(2r)$, the bending stress increases with the square of the sine of the angle δ. This shows how necessary it is to assume the angle δ as small as possible. It is often taken equal to 10° or more, which is not too great when soft annealed wires are used. When the wire is not annealed calculation shows that for most cases an assumption of an angle $\delta = 10°$ will cause the tension of the outer fibres to exceed the elastic limit.

The second rotation γ_t about an axis EA coinciding with the direction of the wire subjects the wire to torsion. The amount of the angle of torsion, *i.e.*, the twist per unit of length of wire, was found above to be

$$\gamma_t = \frac{\sin 2\delta}{2r}.$$

This value reaches its maximum $\gamma_t = \frac{1}{2r}$ when $\delta = 45°$, and then diminishes both with the diminution and with the increase of δ, becoming equal to zero when $\delta = 0$ and when $\delta = 90°$, *i.e.*, when the helix becomes a *flat ring*, and when it *coincides with the straight axis.* The tension caused by this twisting in the outer fibres may be calculated, according to Vol. I. § 262 and § 264, from

$$\gamma_t = \frac{\sin 2\delta}{2r} = \frac{S^*}{\frac{1}{2} d C} \quad \text{or} \quad S = \frac{d \sin 2\delta}{4r} C;$$

---

* According to Vol. I. § 262, the moment of torsion is

$$P a = \frac{\alpha}{l} C W,$$

and according to § 264

$$P a = \frac{S W}{e},$$

consequently

$$\frac{\alpha}{l} = \frac{S}{e C};$$

or, with the notation here used,

$$\gamma_t = \frac{S}{\frac{1}{2} d C}.$$

where C is the modulus of transverse elasticity and S the greatest torsional stress, *i.e.*, the stress occurring at the distance $\frac{1}{2}d$ from the axis of the wire. So great a twist can only be resisted by the very finest wires, and it is therefore necessary to lay up the wires into strands in such a way that the wires themselves will not receive any rotation whatever. To accomplish this it is only necessary to give to each wire, *i.e.*, to the bobbins on which they are wound, an additional rotation about its own axis equal and opposite to that which it receives when twisted about the axis of the strand. If we suppose the bobbin holding the wire shown in Fig. 458 to be carried by an arm $C'A'$ turning about C', and give to this arm a left-handed rotation γ in the direction of the arrow a, moving the arm from the position $C'A'$ to $C'B'$, and at the same time give the bobbin itself an equal rotation in the direction of the arrow b about the axis A' of the wire, then we shall have a rotation pair whose effect, as we know, is equivalent to a simple translation $r\gamma$, *i.e.*, in the present case a shifting from A' to B'. This is accomplished, in machinery for manufacturing wire ropes, by placing the bobbins holding the wires on a disk whose axis coincides with the axis of the strand; the bobbins are so placed on the disk that they can turn around pins which are parallel to the axis. Now if this disk be carried about the core of the strand, suitable mechanism must be provided for communicating to every bobbin, during every revolution of the disk, one rotation in the opposite direction. For smaller sizes of wire used in fringe work, the mechanism shown in Fig. 150, § 47, suffices; but in forming wire rope of larger size use is made of an arrangement like that employed in *Buchanan's* steering wheel for keeping the floats parallel, an arrangement described in the chapter treating of crank trains. According to *Albert* *), the original wire rope made by hand did not need any such arrangement: the wires being stretched full length, were not subjected to torsion where the laying up was taking place, because the other end was perfectly free.

Two wire ropes or strands, in one of which the wires have a twist of their own, while the wires of the other are free from twist, do not differ in appearance so long as the wires are circular in cross-section. The existing difference may be made clear, however, by the help of Figs. 459 and 460. Fig. 459 shows a wire *a b c* without any twist of its own wound about the core *A B*; and here all the originally straight longitudinal fibres of the wire are equal helices of the same radius *r* and the same pitch, as is shown by *a a a*, *b b b*, *d d d*, . . . On the other hand, the fibres of a wire, Fig. 460, which has not been freed from twist by an opposite rotation of the bobbin, form helices, all of which have the same pitch but different radii. For example, the innermost fibre *b b b* lies on a cylinder whose radius is $r - \frac{1}{2}d$, and the outermost fibre *a a a* on a cylinder whose radius is $r + \frac{d}{2}$. If the individual wires had, say, a square cross-section, the sharp edges would render the differences of form visible to the eye.

---

* *Karsten's* Archiv fuer Mineralogie, Geognosie, Bergbau- und Huettenkunde, 1837, vol. x. p. 215.

FIG. 459.

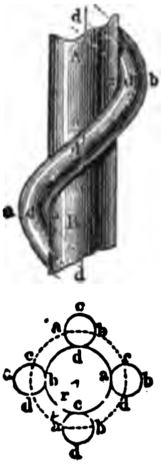
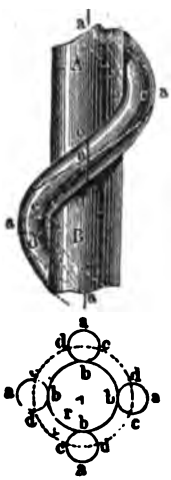


FIG. 460.



What has here been said concerning the necessity of avoiding a twist in the individual wires by giving them a suitable rotation backward while they are being laid up into a strand, applies in a similar manner to the laying up of the strands into rope; for unless such precaution is taken during the laying up of the strands, the latter will partially or wholly unwind on account of the opposite directions of twist in strand and rope. In forming the strands into a rope the angle of inclination of the helices to the axis is usually taken between 10° and 30°, but here also it seems advisable to take the angle as small as possible.

Occasionally ropes have been made of wires laid together parallel, but it is easy to see that such rope can only be used for standing rigging,—for example, ship shrouds,—and are not at all suitable when the ropes are to be bent around pulleys and drums. In the latter case only the outer parallel wires would be stretched, the consequent inequalities of stress diminishing the strength of the rope. On the other hand, if the rope has been laid up, *i.e.*, twisted, the same wire alternately appears on the outside and inside, thus greatly increasing the probability of obtaining a uniform tension in all the wires.

The shortening of the wire rope by the twisting may be determined from the angle of twist or pitch s of the individual helical coils. If r is the distance of the centre of a wire from the axis of the strand, or of a strand from the axis of the rope, the corresponding length of wire or strand for a complete coil is given by

$$l = \sqrt{s^2 + (2\pi r)^2};$$

consequently the shortening will be

$$\lambda = l - s = \sqrt{s^2 + (2\pi r)^2} - s = \text{approximately } \frac{(2\pi r)^2}{2s}.$$

In long ropes the joining of the ends of separate wires is accomplished by winding the ends together for a distance of 1 meter [about 3 ft.] or by soldering. It is of course desirable to keep the number of such joints small by having the pieces forming the rope as long as possible, and the joints of the separate wires so distributed that no two joinings will occur at the same place.

To keep wire ropes from rusting, they are greased or tarred; galvanized wires have for some time past been used for this purpose. Recently wire rope has been made of phosphor bronze, but thus far little is known concerning its behavior. It does not seem as if the greater strength or durability of these ropes would outweigh their greater cost.

§ **118. Strength of Wire Rope.**—The working stress of wire rope may be found from the ultimate strength of iron wire, $k = 62.1$ kilograms [137 lbs.], by assuming a factor of safety of 3 for standing rope and a factor 5 for running rope. This gives

$k = 20$ [28,500 lbs. per sq. in.] for standing rope;
$k = 12$ [17,000 lbs. " " "] for running rope.

A rope having n wires of diameter d can therefore sustain a load

$$P = \frac{\pi d^2}{4} n k = 0.785\, n d^2 k,$$

the size of the wire may be found from

$$d = 1.13 \sqrt{\frac{P}{nk}},$$

and the number of wires from

$$n = 1.27 \frac{P}{d^2 k}.$$

If for iron wire we assume $k = 12$ [17,000 lbs. per sq. in.], we have

$$P = 9.42\, n d^2, \quad d = 0.326 \sqrt{\frac{P}{n}}, \quad n = 0.106 \frac{P}{d^2}.$$

$$\left[P = 13350\, n d^2, \quad d = 0.0087 \sqrt{\frac{P}{n}}, \quad n = 0.000075 \frac{P}{d^2}.\right]$$

The external diameter D of the wire rope may then be determined from data given in the preceding article.

The same formulas hold for flat rope having m ropes of n wires each, but we must then substitute $m \times n$ for n.

To determine the weight of wire rope we must take into account the shortening caused by the twisting. If we assume the average shortening to be 10 per cent,* and the specific gravity of iron $\gamma = 7.7$, we have for the weight of wire rope per running meter [per running foot]

$$G = \frac{10}{9} 7.7 \times 0.785 \frac{n d^2}{1000} = 0.00672\, n d^2; \quad [G = 2.91\, n d^2;]$$

---

* If the angle of inclination of the wires in the strands is $\delta_1 = 10°$, and the angle of inclination of the strands to the axis of the rope is $\delta_2 = 20°$, the shortening will be $\lambda = 1 - \cos 10° \cos 20° = 0.074 = 7.4$ per cent; and if $\delta_2 = 25°$, we have $\lambda = 1 - \cos 10° \cos 25° = 0.102 = 10.2$ per cent.

or when we substitute $P = 9.42\,nd^2$ (corresponding to $k = 12$ [$k = 17000$]) [$P = 13350\,nd^2$]

$$G = \frac{0.00672}{9.42}P = 0.000714\,P, \quad [G = 0.000218\,P,]$$

or

$$P = 1400\,G. \quad [P = 4590\,G.]$$

Since $P = 870\,G$ for untarred rope and $P = 602\,G$ for tarred rope, we see that for the same load wire ropes are $\frac{870}{1400} = 0.623$ lighter than untarred and $\frac{602}{1400} = 0.429$ lighter than tarred hempen rope. This constitutes a special advantage of wire rope when used for hoisting in very deep shafts, where the weight of the rope is a considerable part of its load. The maximum permissible depth of shaft when hempen rope is to be employed is 870 and 602 meters [2850 and 1975 ft.] respectively, while its limit for wire rope is 1400 meters [4590 ft.].

The character of wire rope suitable for transmitting rotation between distant shafts has already been discussed in § 59.

For the weight, working, and ultimate loads of wire ropes, reference can be made to the tables given by *Felton & Guilleaume* in Polytechn. Centralblatt, 1856, and copied into manuals like "Ingenieurs Taschenbuch" Verein Huette. [See also American pocket-books by *Nystrom* and *Haswell.*] The following table contains English data on the strength of steel, iron, and hempen rope taken from the periodical, Der Berggeist, 1860, No. 97, and transformed into French measures:

TABLE OF STRENGTH OF ROPES.

(*A. S. Newell & Co., Birkenhead.*)

a. ROUND ROPES.

Steel.		Iron.		Hemp.		Working Load.	Ultimate Strength.
Girth. mm.	Weight of 1 Meter. kg.	Girth. mm.	Weight of 1 Meter. kg.	Girth. mm.	Weight of 1 Meter. kg.	kg.	kg.
88.7	2.48	114.3	4.47	....	....	5486	36376
82.4	1.98	98.3	3.23	241	5.96	3962	27432
63.4	1.37	85.5	2.23	203	3.97	2743	18288
50.8	0.852	66.6	1.49	165	2.48	1829	12192
44.4	0.620	53.9	0.982	140	1.74	1219	8128
38.1	0.372	44.4	0.620	....	....	762	5080
25.4	0.248	38.1	0.372	....	...	457	3048
....	...	25.4	0.248	69.8	0.496	305	2032

b. Flat Ropes.

Steel.			Iron.			Hemp.			Working Load.	Ultimate Strength.
Width.	Thick-ness.	Weight of 1 Meter.	Width.	Thick-ness.	Weight of 1 Meter.	Width.	Thick-ness.	Weight of 1 Meter.		
mm.	mm.	kg.	mm.	mm.	kg.	mm.	mm.	kg.	kg.	kg.
82.4	9.52	4.47	114	19	7.95	241	60.2	13.67	6502	56896
76.2	9.52	3.97	108	19	6.96	229	63.4	12.45	5690	50800
69.7	9.52	3.72	102	17.5	6.22	216	57.0	11.20	5080	45720
63.4	12.7	3.23	95.2	17.5	5.96	210	53.9	9.94	4470	40640
57.0	12.7	2.73	82.5	15.9	4.47	152	38.1	7.45	3658	32512
50.8	12.7	2.48	76.2	15.9	3.97	146	38.1	6.95	3251	28448

TABLE OF STRENGTH OF ROPES.

(*A. S. Newell & Co., Birkenhead.*)

a. Round Ropes.—For Inclined Planes, Mines, Collieries, Ship's Standing Rigging, etc.

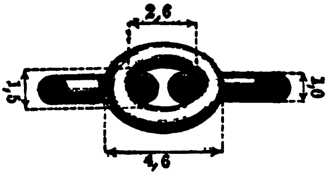
Hemp.		Iron.		Steel.		Tensile Strength.	
Girth.	Weight per fathom.	Girth.	Weight per fathom.	Girth.	Weight per fathom.	Working load.	Ultimate Strength.
inches.	pounds.	inches.	pounds.	inches.	pounds.	cwts.	tons.
2¾	2	1	1			6	2
		1½	1½	1	1	9	3
3¾	4	1⅝	2			12	4
		1¾	2½	1½	1½	15	5
4½	5	1⅞	3			18	6
		2	3½	1⅝	2	21	7
5½	7	2⅛	4	1¾	2½	24	8
		2¼	4½			27	9
6	9	2⅜	5	1⅞	3	30	10
		2½	5½			33	11
6½	10	2⅝	6	2	3½	36	12
		2¾	6½	2⅛	4	39	13
7	12	2⅞	7	2¼	4½	42	14
		3	7½			45	15
7½	14	3⅛	8	2⅜	5	48	16
		3¼	8½			51	17
8	16	3⅜	9	2½	5½	54	18
		3½	10	2⅝	6	60	20
8½	18	3⅝	11	2¾	6½	66	22
		3¾	12			72	24
9½	22	3⅞	13	3¼	8	78	26
10	26	4	14			84	28
		4¼	15	3⅜	9	90	30
11	30	4⅜	16			96	32
		4½	18	3½	10	108	36
12	34	4⅝	20	3¾	12	120	40

b. FLAT ROPES.—For Pits, Hoists, etc.

Hemp.		Iron.		Steel.		Tensile Strength.	
Girth.	Weight per fathom.	Girth.	Weight per fathom.	Girth.	Weight per fathom.	Working Load.	Ultimate Strength.
inches.	pounds.	inches.	pounds.	inches.	pounds.	cwts.	tons.
4 × 1⅛	20	2¼ × ⅛	11			44	20
5 × 1¼	24	2⅛ × ⅛	13			52	23
5⅛ × 1⅜	26	2¾ × ⅜	15			60	27
5¾ × 1⅛	28	3 × ⅜	16	2 × ⅛	10	64	28
6 × 1⅛	30	3¼ × ⅜	18	2¼ × ⅛	11	72	32
7 × 1⅞	36	3⅛ × ⅜	20	2¼ × ⅛	12	80	36
8½ × 2⅛	40	3¾ × 7/16	22	2⅛ × ⅛	13	88	40
8⅛ × 2¼	45	4 × 7/16	25	2¾ × ⅛	15	100	45
9 × 2⅛	50	4¼ × ½	28	3 × ⅜	16	112	50
9⅛ × 2⅜	55	4⅛ × ½	32	3¼ × ⅜	18	128	56
10 × 2⅛	60	4⅜ × ½	34	3⅛ × ⅜	20	136	60

§ 119. Chains.—Chains used for the transmission of rotations between distant shafts have already been discussed in the section devoted to chain wheels, but those here treated are chains used in windlasses and hoists for lifting loads. They are generally open-link chains like that shown in Fig. 461, though occasionally flat-link chains (Figs. 316 to 318) have been used for the same purpose. The oval links of the chain are usually 1.5 *d* and 2.6 *d* wide on the inside, *d* representing diameter of iron used. Since the tearing of a link of a chain is accompanied by fracture at two places, we have for the load on the chain

FIG. 461.



$$P = 2 \frac{\pi d^2}{4} k,$$

where the permissible stress *k*, experience shows, may be taken equal to 6 kilograms (8500 lbs. per square in.) when the chain has open links as in Fig. 461, *i.e.*, links without studs.

Accordingly, for $k = 6$ [$k = 8500$]

$$P = 9.42 d^2 \quad \text{and} \quad d = 0.326 \sqrt{P}.$$

$$[P = 13350 d^2 \quad \text{and} \quad d = 0.0087 \sqrt{P}.]$$

To determine the weight of a link of the chain we first find the length of the middle fibre, regarding it as an ellipse whose semi-axes are $a = 1.8\,d$ and $b = 1.25\,d$, from the known formula

$$l = \pi(a+b)\left[1+\frac{1}{4}\left(\frac{a-b}{a+b}\right)^2+\dots\right]$$
$$= 3.05 \times 1.008\,\pi d = 9.66\,d.$$

From this we find the weight of a single link of the chain to be

$$G_0 = \frac{\pi d^2 l \gamma}{4} = 0.785 \times 9.66 \times 0.0000077\,d^3$$
$$= 0.0000584\,d^3 \text{ kg.} \quad [G_0 = 2.10\,d^3 \text{ lbs.}]$$

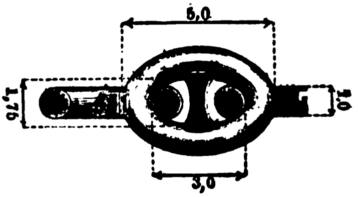
Since each link is a piece of chain $2.6\,d$ long, we get for the weight of the chain per running meter [per running ft.],

$$G = 1000\,\frac{G_0}{2.6\,d} = 0.0225\,d^2 \text{ kilograms}; \quad [G = 9.73\,d^2 \text{ lbs.};]$$

or, substituting $d = 0.326\,\sqrt{P}$, $[d = 0.0087\,\sqrt{P},]$

$$G = 0.0225 \times 0.106\,P = 0.00239\,P \text{ kilograms and } P = 418\,G.$$
$$[G = 0.000737\,P \quad\text{and}\quad P = 1357\,G.]$$

Fig. 462.



Large chain-cables, such as are used on ships, are usually provided with stud-links like those shown in Fig. 462. The stud has a square cross-section at the middle whose side is $\frac{5}{8}\,d$, and a rectangular shape at the ends $\frac{5}{8}\,d$ in width and d in depth. One advantage of this chain is that it will not entangle, and another that it is $\frac{4}{3}$ times as strong as the open-link variety; we can consequently assume $k = 8$ [$k = 11400$ lbs. per sq. in.], which gives

$$P = 12.56\,d^2 \text{ and } d = 0.282\,\sqrt{P}. \quad [P = 17800\,d^2;\ d = 0.0076\,\sqrt{P}.]$$

The length l of iron needed for a link whose semi-axes are $a = 2\,d$ and $b = 1.375\,d$ is

$$l = 3.14 \times 3.375\left[1+\frac{1}{4}\left(\frac{0.625}{3.375}\right)^2+\dots\right]d = 10.694\,d.$$

From this we obtain, for the weight of the link without the stud,

$$0.785 \times 10.694 \times 0.0000077\,d^3$$
$$= 0.0000647\,d^3 \text{ kilograms.} \quad [= 2.33\,d^3 \text{ lbs.}]$$

Adding to this the weight of the stud, which is approximately equal to

$$2.25 \times \tfrac{3}{8} \times \tfrac{5}{8} \times 0.0000073\,d^3 = 0.0000091\,d^3, \quad [= 0.328\,d^3,]$$

we get for the weight of a whole link $G_0 = 0.0000738\,d^3$ [$= 2.66d^3$ lbs.], and for the weight per running meter [per running foot],

$$G = 1000\frac{G_0}{3d} = 0.0246\,d^2 = 0.00195\,P \text{ and } P = 512\,G.$$

$$[G = 10.65\,d^2 = 0.0006\,P \text{ and } P = 1660\,G.]$$

For equal loads, therefore, this chain is somewhat lighter than the open-link chain.

TABLE I.

WORKING LOAD, DIAMETER, AND WEIGHT OF ROPES AND CHAINS.

(Formulas expressed in French Units.)

	Hempen Rope.		Wire Rope.	Open-link Chain.	Stud-link Chain.
	Dry and Untarred.	Wet or Tarred.			
k (kg.) =	1	0.82	12	6	8
d (mm.) =	$1.13\sqrt{P}$	$1.24\sqrt{P}$	$0.326\sqrt{\frac{P}{n}}$	$0.326\sqrt{P}$	$0.282\sqrt{P}$
P (kg.) =	$0.785\,d^2 = 870\,G$	$0.652\,d^2 = 600\,G$	$9.42\,n\,d^2 = 1400\,G$	$9.42\,d^2 = 420\,G$	$12.56\,d^2 = 512\,G$
G (kg.) =	$0.0009\,d^2 = 0.00115\,P$	$0.00108\,d^2 = 0.00166\,P$	$0.00672\,n\,d^2 = 0.000714\,P$	$0.0225\,d^2 = 0.00239\,P$	$0.0246\,d^2 = 0.00195\,P$

(English Units.)

	Dry and Untarred.	Wet or Tarred.	Wire Rope.	Open-link Chain.	Stud-link Chain.
k (lbs.) =	1420	1160	17000	8500	11400
d (ins.) =	$0.03\sqrt{P}$	$0.033\sqrt{P}$	$0.0087\sqrt{\frac{P}{n}}$	$0.0087\sqrt{P}$	$0.0076\sqrt{P}$
P (lbs.) =	$1120\,d^2 = 2855\,G$	$916\,d^2 = 1973\,G$	$13350\,n\,d^2 = 4590\,G$	$13350\,d^2 = 1360\,G$	$17800\,d^2 = 1660\,G$
G (lbs.) =	$1.28\,d^2 = 0.00035\,P$	$1.54\,d^2 = 0.0005\,P$	$2.91\,n\,d^2 = 0.000218\,P$	$9.73\,d^2 = 0.000737\,P$	$10.65\,d^2 = 0.0006\,P$

REMARK. Chain cables and the construction of chains are treated in *Prechtl's* Encyklopaedie, vol. viii., article "Ketten;" also in *Karmarsch* and *Heeren's* Technisches Woerterbuch, and in the Verhandlungen zur Befoerderung des Gewerbfleisses in Preussen, 1824 and 1835. Theoretical determinations of the size of chain-links, etc., are contained in Dr. F. Grashof's Festigkeitslehre, Berlin, 1866, and in Dr. Winkler's Lehre von der Elasticitaet und Festigkeit, part i., Prag., 1867. Tables giving the strength and weight of chains may be found in most of the engineering pocket-books, and in works on Machine Design.

TABLE II.

STRENGTH, WEIGHT, AND COST OF WIRE ROPE, HEMPEN ROPE AND CHAIN CABLES.

Breaking Load in Tons, Pounds, and Kilograms.	Kind of Cable.	Girth of Wire Rope and of Hempen Rope; Diameter of Iron of Chain.		Weight of		Cost per Fathom.
		inches.	millimeters.	one Foot. pounds.	one Meter. kilograms.	dollars.
1 ton	Wire Rope	1.0	25.4	0.125	0.186	0.025
2240 lbs.	Hempen Rope	2.0	50.8	0.177	0.264	0.028
1016 kg.	Chain	$\frac{1}{4}$	6.35	0.500	0.744	0.09
8 tons	Wire Rope	2.0	50.8	0.438	0.652	0.09
17920 lbs.	Hempen Rope	5.0	127.0	0.978	1.49	0.158
8128 kg.	Chain	$\frac{1}{2}$	12.7	2.667	3.97	0.24
12 tons	Wire Rope	2.5	63.5	0.753	1.12	0.148
26880 lbs.	Hempen Rope	7.0	178.0	2.036	3.03	0.32
12192 kg.	Chain	$\frac{11}{16}$	17.5	4.502	6.70	0.36
16 tons	Wire Rope	3.0	76.2	1.136	1.69	0.218
35840 lbs.	Hempen Rope	8.0	203.2	2.365	3.52	0.373
16256 kg.	Chain	$\frac{13}{16}$	20.6	6.169	9.18	0.48
20 tons	Wire Rope	3.5	88.9	1.546	2.30	0.30
44800 lbs.	Hempen Rope	9.0	228.6	3.225	4.80	0.50
20320 kg.	Chain	$\frac{29}{32}$	23.0	7.674	11.42	0.575
24 tons	Wire Rope	4.0	101.6	2.043	3.04	0.396
53760 lbs.	Hempen Rope	10.0	254.0	4.166	6.20	0.656
24384 kg.	Chain	$\frac{31}{32}$	24.6	8.836	13.15	0.65
30 tons	Wire Rope	4.5	114.0	2.721	4.05	0.53
67200 lbs.	Hempen Rope	11.0	279.0	5.000	7.44	0.786
30480 kg.	Chain	$1\frac{1}{16}$	27.0	10.335	15.38	0.775
36 tons	Wire Rope	5.0	127.0	3.723	5.54	0.725
80640 lbs.	Hempen Rope	12.5	318.0	5.940	8.84	0.988
36576 kg.	Chain	$1\frac{3}{16}$	30.2	13.01	19.36	0.975
44 tons	Wire Rope	5.5	140.0	4.50	6.70	0.895
98560 lbs.	Hempen Rope	14.0	356.0	6.94	10.32	1.097
44704 kg.	Chain	$1\frac{5}{16}$	33.4	16.00	23.81	1.20
54 tons	Wire Rope	6.0	152.0	5.67	8.43	1.11
120960 lbs.	Hempen Rope	15.0	381.0	7.92	11.78	1.247
54864 kg.	Chain	$1\frac{7}{16}$	36.6	19.16	28.52	1.44

§ **120. Comparison of Ropes and Chains.**—The formulas for the loads, weights, and dimensions of ropes and chains determined in the preceding articles are collected in Table I, where d represents respectively the diameter of the hempen rope, the diameter of wire, and the diameter of the chain iron. A comparison of these flexible transmitters with respect to strength, weight, and cost, deduced from experiments made by order of the British Admiralty, is contained in Table II. Although these (English) prices vary greatly at different times, yet the data may be of service, giving some idea of the comparative cost of ropes and chains. We see from this table that for equal loads the wire rope is always the lighter and cheaper, and the chain cable heavier and usually more costly, means of transmission. On an average we may assume that, for equal loads, the weight of wire ropes is only half, and that of chain cable from two to two and a half times, as great as that of hempen rope; and also, that the diameter of wire rope is only about 0.4, and that of round iron forming the links of the chain about 0.3, of the diameter of the hempen rope.

The length sufficient to produce the maximum working stress k, which we will call the permissible length, is found from Table I. to be for hempen rope 870 and 600 meters [2855 and 1975 ft.] respectively, for chains 420 and 512 meters [1360 and 1660 ft.] respectively, while for wire rope it is 1400 meters [4590 ft.]. This shows us the advantage of wire rope for very deep shafts, where chains and hempen ropes would be useless.

Sometimes, when the depths are very great, ropes are given approximately the form of a body of uniform strength (see Vol. I. § 208), by making them of separate pieces whose diameters diminish toward the lower end. It is evident that by this means the tension in the fibres caused by the rope's own weight can be considerably diminished. We may also note in passing, that entire immersion in water diminishes the tension in the material caused by the chain's or rope's own weight in the ratio $\frac{\gamma - 1}{\gamma}$, γ denoting the specific gravity of the material. This is particularly important for sounding lines, the buoyant effort of the water doubling the permissible length of line of the rope, while the permissible length of wire rope used in borings filled with water, or in telegraph cables, is increased only 12 to 15 per cent.

Another special advantage of all ropes over chains is that ropes are less liable to have defective places than chains. In the first place, defects of material in a link of a chain cannot usually be detected, while the method of making hemp rope gives a better guarantee for the excellence of the material used, the fibres during the combing or hackling of the hemp not readily escaping the observation of the workman. In wire rope the fineness of the wire is sufficient proof of the excellence of the iron that can be so drawn out without breaking. In the second place, chains, subject as they are to unavoidable small shocks, easily change their fibrous texture for a crystalline one; consequently it is well to avoid their use wherever breakage would endanger human life, and where this is not practicable, to frequently test the strength of the chains.

The resistance experienced by chains when bent around pulleys or drums consists simply of the friction of the links of the chain on each other; it is estimated in the same way as journal or bolt friction (see § 90), and is proportional to the tension of the advancing portion of the chain. On the other hand, the resistance in ropes is due to two causes—the natural stiffness of the materials and the friction of the separate fibres or wires on each other. In unwinding ropes or chains from drums there is also *frictional resistance*, but here the tension of the receding portion must be taken into calculation. Now as regards the resistance due to stiffness, we find that there is no loss from this source in wire rope during the unwinding, for its elasticity is sufficient to straighten it without any additional help, provided the limit of elasticity of the material has not been exceeded by winding it on a pulley of too small diameter. The elasticity of hempen rope, however, is so slight that it has hardly any tendency to straighten itself, and consequently the stiffness causes some loss of work during the unwinding. Fuller data concerning the magnitude of these resistances may be found in Vol. I. § 196. In order to diminish as much as possible that portion of the resistance to bending which in chains and wire ropes is due to friction, an occasional oiling is customary and desirable. On the other hand, tarring the hempen rope and puttying the wire rope make them stiffer, but must be done to increase their durability by protecting them from moisture.

EXAMPLES: 1) What should be the diameter of a hempen rope for a windlass if the rope is to lift 1000 kilograms [2200 lbs.]?

We have here for untarred rope

$$d = 1.13 \sqrt{1000} = 35.7 \text{ millimetres}, \quad [1.43 \text{ ins.,}]$$

and for tarred rope

$$d = 1.24 \sqrt{1000} = 39.2 \text{ millimetres}. \quad [1.57 \text{ ins.}]$$

2) The load on the chain of a crane is 5000 kilograms [11,000 lbs.]; what must be the diameter of the iron of the chain?

For an open-link chain we get

$$d = 0.326 \cdot \sqrt{5000} = 23.05 \text{ millimeters}, \quad [1.12 \text{ ins.,}]$$

while for a stud-link chain we have

$$d = 0.282 \sqrt{5000} = 19.9 \text{ millimeters}. \quad [0.8 \text{ in.}]$$

The weight per running meter [per running ft.] of chain is respectively

$$G = 0.0225 \times 24^2 = 12.96 \text{ kilograms}. \quad [8.7 \text{ lbs. per ft.}]$$

and

$$G = 0.0246 \times 20^2 = 9.84 \text{ kilograms}. \quad [6.6 \text{ lbs. per ft.}]$$

3) A wire rope containing 72 wires is to lift a load of 3000 kilograms [6600 lbs.] up a shaft 400 meters [1312 ft.] deep; what diameter must be given to the wire? The weight of the wire rope is $G = 0.00672 \times 72 \times 400 d^2 = 193.6 d^2$; consequently the diameter of the wire must be

$$d = 0.326 \sqrt{\frac{3000 + 193.6 d^2}{72}}$$

or

$$d = \sqrt{\frac{4.420}{0.715}} = 2.49. \quad [6.1 \text{ ins.}]$$

The diameter of the wire rope will then, according to *Reuleaux*, be (see § 116)

$$D = 14.2 \times 2.5 = 35.5 \text{ millimeters}, \quad [1.42 \text{ ins.,}]$$

and according to *Riehn*

$$D = \left(9.4 + \frac{72}{19}\right) 2.5 = 33.0 \text{ millimeters}. \quad [1.32 \text{ ins.}]$$

§ 121. The Uniting of Ropes and Chains. To unite the ends of two wire ropes, a muff $A D$, Fig. 463, may be used. The muff is composed of two separate parts $A B$ and $C D$, whose ends have a slight taper and are threaded; by screwing on two nuts M and N, which have likewise a slightly tapering thread, the ends of the rope will be firmly pressed against the cheeks

A B and *C D*. Instead of having two cheeks, the muff *A D* may be made in a single piece, and its ends slit radially by saw cuts. Sometimes two rope ends are united simply by welding. Of course neither such a joint nor the muff just described can be employed where the joint must bend around a pulley or drum. In such a case nothing remains but to unlay one end of each rope for a distance of 0.5 to 1 meter [20 to 40 ins.], and so interlace the separate wires that the friction between them will prevent the joint from undoing.

FIG. 463.

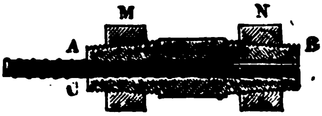


FIG. 464.

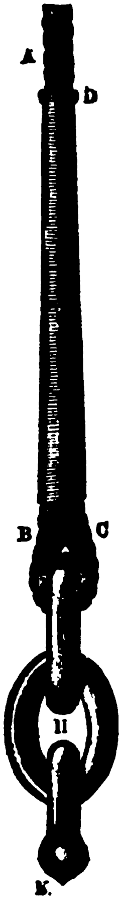


Fig. 464 shows a chain which on one side is attached to a cage and on the other to a wire rope. The end *A B C D* of the rope is bent around the thimble *E F*, through which a link *G* of the chain passes. The wires of the bent back portion *C D* disperse themselves in the portion *A B* of the rope, and the 1.5 meter [4.84 ft.] ring splice is secured by being firmly wound with wire. In order that the rope end may be readily attached and detached, there is inserted a shackle *H K*, whose bolt *K* is screwed into the top of the cage.

An excellent joint for connecting a wire rope with a chain is shown in Fig. 465. Here a conical socket *C C* is placed over the rope *A B*, the projecting end of the latter being unlaid into separate wires, which are bent back over the outer surface of the socket and then fastened by the spirally wound wire *E E*. A second conical socket *F F* surrounds the bundle of bent back wire *C B C*, and when drawn up by means of the stirrup *G G* presses these wires firmly against the surface of the first socket, thus establishing a firm union between the rope and end *F G* of the chain *H*.

Fig. 466 gives two views of a joint for connecting a flat rope with a chain. Here the ring *E E*, which carries the first link of the chain, is prolonged into two plates *D E*, which clamp the end of the flat rope *A B*, being attached to the latter by rivets ***a b***, ***c d***, etc.

There are many devices for locking chains together; a simple one is shown in Fig. 467. Here the link *A B* has a slit in one side large enough to admit the following link *E F* of the chain. The slit is closed by the piece *C*, which is held fast by its bolt *D*. Fig. 468 shows a portion of a chain used to connect a rope with a hoisting cage. It contains two shackles *A B* and *C D*, with two removable bolts *A* and *C*, and a closed loop *E F* which turns around the bolt *L* of the eye-shaped piece *K L*.

FIG. 465.

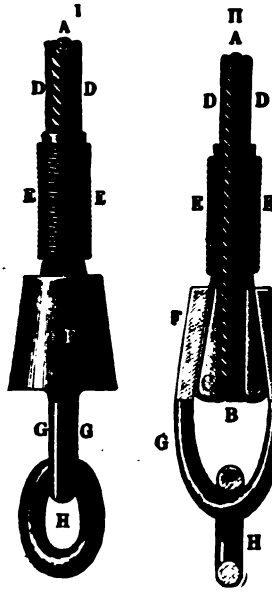
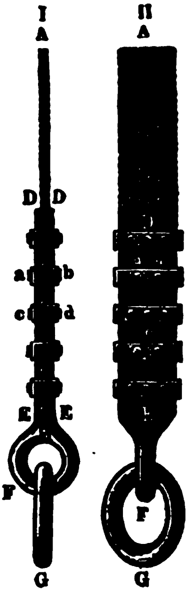


FIG. 466.



The snap *A B C*, Fig. 469, is likewise used for connecting chains. In order to hang in a link *B G*, it is only necessary to push back the sleeve *H* on the shank *D A*, turning the latter about its pin *D* into the position $D A_1$, when the spring *F* will take the position F_1. With the chain hook *A B C*, Fig. 470, the separation is effected by simply lifting out the link on the hook. The cross-section *A D* of the hook is determined according to the rules for bending stress and the cross-section $B\,E$ according to the rules for shearing stress, while the cross-section $H\,K$ is to be determined according to the rules for resistance to tearing. If d again represents the diameter of the iron of the chain, we have, what has already been established for an open-link chain,

FIG. 468.

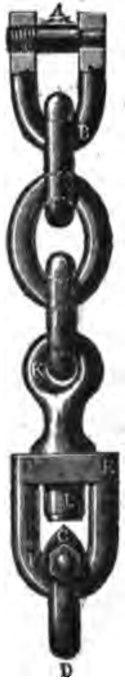


FIG. 467.

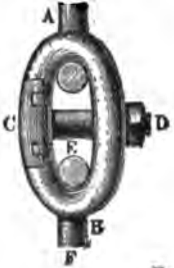


FIG. 469.

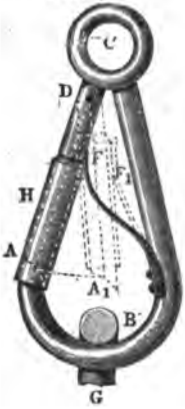
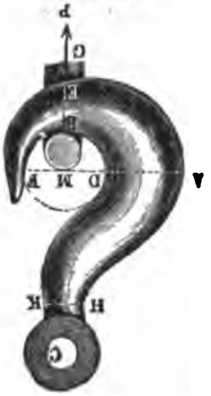


FIG. 470.



$$d = 0.326\sqrt{P}. \quad [d = 0.0087\sqrt{P}.]$$

If we assume for the shearing stress in $B\,E$ and for the tensile stress in $H\,K$ the same value $k = 6$ kilograms [8500 lbs.] used for the links of the chain, then if the hook has everywhere circular cross-sections we shall have, for the diameters at these places,

$$d_1 = \sqrt{\frac{4P}{\pi k}} = 0.461\sqrt{P} = d\sqrt{2}. \quad [d_1 = 0.0122\sqrt{P} = d\sqrt{2}.]$$

If we designate the radius DM of the opening DF by r, and the greatest diameter $A\,D$ of the hook by D, we have for the bending moment at this place the equation

$$P\left(r + \frac{D}{2}\right) = \frac{\pi D^3}{32} k$$

or

$$D^3 = \frac{32\,\pi}{\pi\,k}\left(r + \frac{D}{2}\right).$$

If in this we substitute $P = \frac{\pi d^2}{2} k$ and assume $r = d$, we get

$$D^3 = 8\,d^2(D + 2\,d)$$

or

$$\left(\frac{D}{d}\right)^3 = 8\frac{D}{d} + 16.$$

The solution of this cubic equation gives

$$\frac{D}{d} = 3.54;$$

consequently, for the greatest diameter $A\,D$ of the hook, we have

$$D = 3.54\,d = 1.15\,\sqrt{P}. \quad [= 0.0308\,\sqrt{P}.]$$

§ **122. Sheaves and Guide Pulleys.**—When ropes and chains hang vertically they need no special guide; when they are horizontal and of great length, supporting pulleys, like those described under the head of Wire Rope Gearing, may be employed. But when the direction of a rope or chain is to be altered, guide pulleys must always be used. The diameter of such a pulley depends upon the diameter of the rope and the amount of bending to which it is subjected, wire rope and the large and stiff hempen rope having pulleys 2 to 3 meters [6 to 10 ft.] in diameter, while the sheaves in block and tackle for small hempen rope are sometimes not more than 0.12 meter [4.72 ins.] in diameter. In general, we may say that the resistance due to the stiffness of a rope and that due to friction on the axle of the pulley diminish as the diameters of the pulleys increase. Consequently these large diameters are accompanied by less wear on the rope, and on the journals and their bearings; but as the weight of the pulley increases with its diameter, thus increasing the friction of the journals, there is evidently a limit beyond which it will not be advantageous to increase the diameters of the pulleys. Small sheaves consist of a simple disk of wood or metal, Figs. 471 and 472; the larger ones are formed like a wheel, *ABC*, Fig. 473, and are made of wood or iron. In order that the rope may not run off from the pulley, its circumference is grooved; if the pulley is of cast iron, this groove is lined with wood or gutta-percha, in order to prevent wear of the rope and of the pulley rim.

FIG. 471.

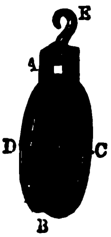


FIG. 472.

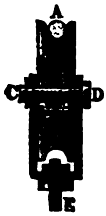
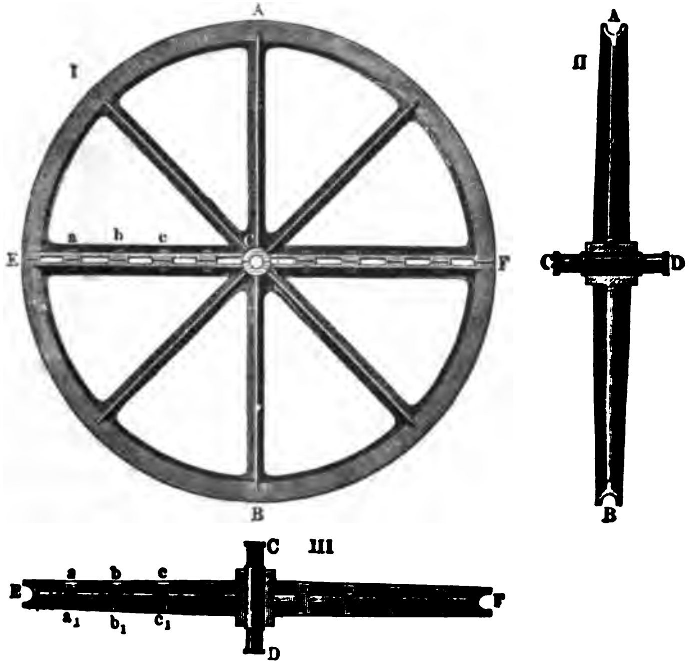


FIG. 473.



The wrought-iron axle *CD* is either fastened to the pulley and turns in fixed bearings, or the pulley is bored so that it can turn loosely on the middle of the fixed axle or arbor. The first arrangement is the more reliable one, and is therefore always used for the larger and stationary rope pulleys, while the second is used only for small and portable guide pulleys where it is desirable to do without bearings for the journals. When several sheaves moving with different velocities are carried by an axle, they must turn loosely on it. When the axle and pulley are rigidly connected, friction takes place at the end of the axle, but with loose pulleys it takes place in the middle. Now, since the axle may be smaller at the ends than in the middle without loss of strength, and as the work of friction increases with the diameter of the axle, the rigidly attached pulley will have less friction on its axle than the loose pulley; but the principal disadvantage of a loose pulley is its uncertain and unsteady motion when the eye and axle are worn.

FIG. 474.

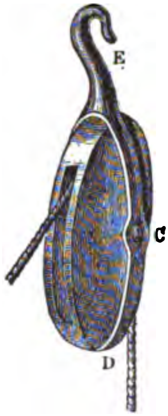
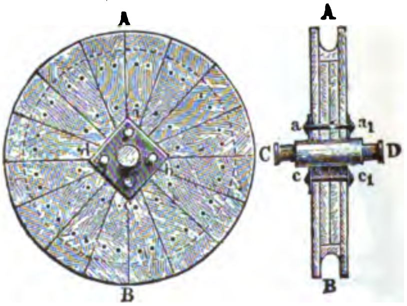


FIG. 475.



In portable sheaves for small hemp rope the axle or bolt CD rests in a *block* CED (Fig. 471 and 472), which in the ordinary block and tackle becomes a casing CDE, Fig. 474, enclosing the sheave, the casing being suspended by the hook E. Fig. 475 shows how a larger wooden guide pulley AB is composed of sector-shaped pieces. Here the pulley is held together by bolts a, b, c, and d, and two iron plates ac and and a_1c_1, through which the axle CD passes.

A simple guide pulley *A C B* made of iron or brass is shown in Fig. 476. Such a pulley rests in a frame *E F*, Fig. 477, I and II, which is provided with a screw *E* for fastening it.

Fig. 476.

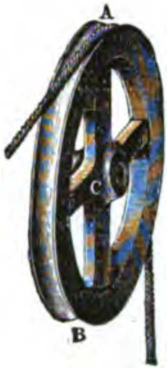


Fig. 477.

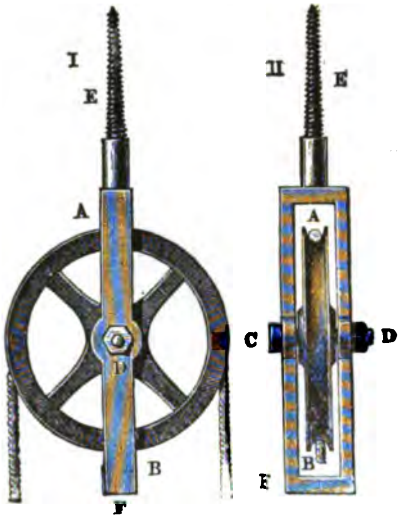
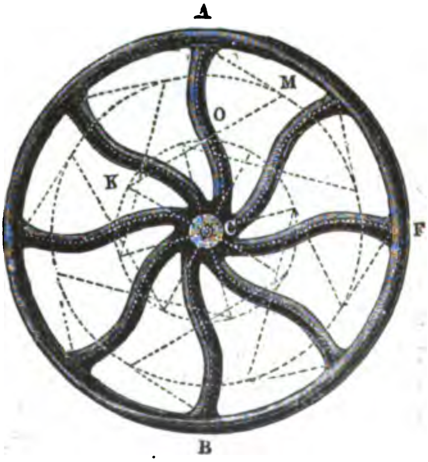
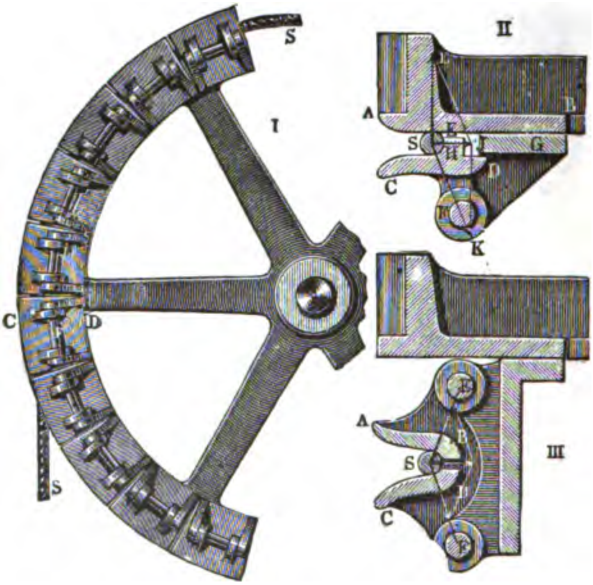


Fig. 478.



The large rope pulleys used in wire-rope transmission or in hoists are composed of two parts, as shown in Fig. 473, I, II, and III; or they may be cast with curved arms, Fig. 478. In the rope pulley shown in Fig. 473 the two halves *E A F* and *E B F* are firmly united by eight pairs of screws, $a a_1$, $b b_1$, $c c_1$, etc., placed along a diameter *E F*. Other methods of joining the parts are also in use; for example, the wrought-iron rings, such as are used in fly-wheels, for holding together the parts of the nave *C*. Here the journals of the wrought-iron shaft *CD* are turned down and rest in finished bearings. When the pulleys turn rapidly, as in rope gearing, the arms are not cross-shaped as in Fig. 473, but elliptical as in Fig. 478, to diminish the resistance offered by the air. In order that the arms may contract during the cooling after they are cast, without cracking near the longer and more slowly cooling rim, it is customary to curve the arms according to two circular arcs

FIG. 479.



A O and *C O*, whose centres *K* and *M* are in the same straight line with the point *O*, at which the arcs pass into each other.

When the rope does not, as in windlasses, completely surround the drum by a series of adjacent coils, but simply makes, say, a half turn around the pulley, a sliding of the rope may be prevented by using *clip pulleys** like those shown in Fig. 479. In Fig. 479 the whole circumference is provided with clips *CD*, Fig. 479, II, which turn about pins *F* carried by little brackets GF fastened to the rim AB of the rope pulley by screws. The rope S enters between the rim A and the lips C of the clips, and presses at H against the foot E of the clip, thus causing an automatic closure of the tongs. The pressure $P = SJ$ of the rope against the foot of the clip calls forth a reaction SK at the journals F, and also a reaction $SL = \frac{P}{\tan SKJ}$ at right angles to the rim or flange of the pulley, both giving rise to friction. It is evident that at the place where the rope leaves the circumference of the pulley, the weight of the clip itself will cause the lips C to open and release the rope, which release would not take place easily if the rope pulley were simply provided with a V-shaped groove for clamping the rope wedge fashion. These rope pulleys have also been constructed with two series of clips AB and CD, Fig. 479, III, each pair of clamps constituting a sort of toggle-joint, in which the rope is firmly grasped between the clips. This sort of rope pulley is used in towing ships, when the rope is stationary and passes over the pulley on board the ship, the ship pulling itself along the rope by means of the clips on the pulley.* In almost all these cases it is found that a semicircumference of contact between rope and pulley suffices to prevent slipping of the rope.

---

* See *Perels*, Die Dampfbodencultur. Verhandlungen des Gewerbvereins, 1862; also, *Ruehlmann*, Allgemeine Maschinenlehre, ii. p. 593.

FIG. 480.

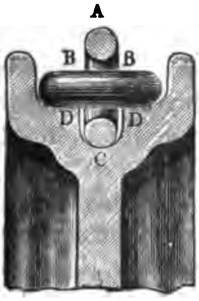
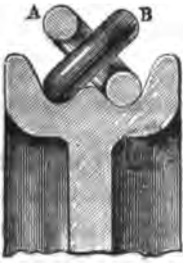


FIG. 481.



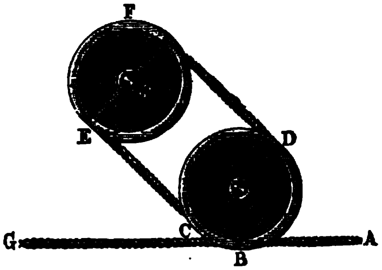
The guide pulleys for chains have their grooves profiled, as shown in Fig. 480, the vertical links lying in the groove C, while the other links rest on the cylindrical surface DD of the larger groove. If the additional groove were not made, the links would be inclined at an angle of 45° with the axis, the motion of the chain would be uncertain, and its strength would be diminished by the unfavorable clamping action. However, where the pulley is profiled as shown in Fig. 481,* the motion of the chain is said to be quite positive.

---

* See Zeitschr. deutsch. Ingenieure, 1867 and 1869.

The position given to the guide pulleys of the ropes or chains is in every case to be determined by the directions of the latter, for the middle plane of the pulley must always contain the advancing as well as the receding portion of the rope or chain. A deviation of the receding portion of the rope or pulley to the extent that it takes place in belt gearing, about 25°, is not permissible here, for the flanges of the pulleys would resist it. At most only very slight deviations of the ropes from the planes of the pulleys may take place, for example in tackle, where the rope is repeatedly passed around sheaves which are placed side by side on the same axle. Another example is found in the towing of ships, and is shown in Fig. 482. Here the rope AG lying in the direction of the ship is passed around a clip pulley J, after first passing over the arc BC of the *upper* groove of the double-grooved pulley H, reaching D by the way EF, and again touching H along the *lower* groove in the arc DB, then passing off to G. In this arrangement the guide pulley H is slightly inclined, so that the points C and D, at which the rope passes off from one groove and on to the other, will lie at the same height and in the horizontal plane of the driving pulley EF. In general the advancing and receding portions of the rope lie in the plane of the rope pulley, consequently one guide pulley will suffice to change the direction of a rope whenever its two directions lie in the same plane. When this is not the case, for example, when a rope is to be

FIG. 482.



---

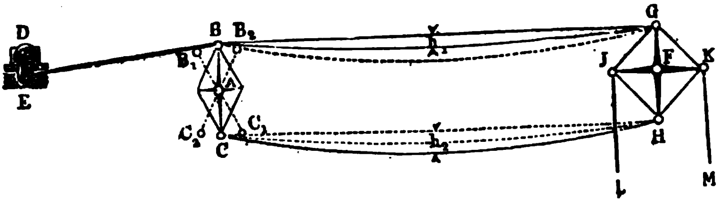
* See *Reuleaux*, Der Constructeur.

led from a given direction L_1 into another direction L_2, askew to the former, two guide pulleys will be needed to effect the change of direction. To find the position of these pulleys we must suppose a third line L drawn, which may have any position whatever, provided it intersects both L_1 and L_2. We may then place one pulley in the plan of L and L_1 and the other in the plane of L and L_2. The freedom allowed in the choice of L permits an endless variety of arrangement. If we choose the position of L so that it coincides with the shortest distance between L_1 and L_2, the portion of rope between the two guide pulleys will have the shortest length possible, but it does not necessarily follow that this arrangement is the best. For this position of L the rope is deflected at each pulley through a right angle, and the pressure on the journals amounts to $R = 2 P \sin \frac{\alpha}{2}$, α representing the angle of deviation, and P the tension in each of the two portions of the rope. We see therefore that the pressure on the journals and the friction to which it gives rise will be greater with this arrangement than if the position of L were so chosen that the angles of contact on the pulleys were acute. In practice, however, it will be found that the consideration of minimum journal friction is of less importance than a convenient and secure arrangement of the guide pulleys.

§ **123. Rope-Rods.**—Although ropes and chains are pure tension organs, transmitting only tensile forces in the direction of their length, nevertheless two of them combined can transmit reciprocating motions, and may then be used as substitutes for rods. Such arrangements we will call *rope-rods*, and in many cases they will be found more advantageous than rods or other rigid means of transmission. One important advantage is that great lengths of rope may hang free, while long rods need frequent support by rocking-arms, etc., and long lines of shaftings require still more frequent support by bearing. It is therefore well to use these rope-rods where it is difficult or impossible to provide such support; for instance, where it is necessary to work the pumps in the caissons of bridge piers by a steam-engine placed on shore. For such places rope-rods possess the further advantages of having comparatively slight hurtful resistances, and a capacity to assume any change of direction with the help of guide pulleys. Consequently in draining excavations and in extensive brick kilns these rope-rods are frequently employed to transmit reciprocating motion.

The arrangement of such rope-work may be seen from Fig. 483. If we suppose the rocker arm $B\ A\ C$ to be pivoted at A and to receive from the crank D by means of the connecting-rod $E\ B$ a swinging motion from $B_1\ C_1$ to $B_2\ C_2$, then the bob F, driving the pump rods $J\ L$ and $K\ M$ at J and K, will likewise have the desired reciprocating motion, the alternate pulls on the wire ropes $B\ G$ and $C\ H$ communicating this motion. It is plain that this transmission of motion is still possible when the direction of the wire ropes is changed by any means, say by guide pulleys. But it is easy to see that a certain loss of stroke always accompanies this mode of transmission, for with a given stroke $B\ B_1$ of the point B of the rocker arm the point G of the bob will always describe a smaller stroke. This may be explained as follows: The two ropes $B\ G$ and $C\ H$ have equal tensions S when they are at rest; now when the point B moves to B_1, the tension S in $B\ G$ will increase to S_1 and the tension in $C\ H$ will diminish to S_2, the difference of tension $S_1 - S_2$ in the two ropes just sufficing to overcome the useful resistance of the axle F of the bob. These alterations of tension S are caused by the rope $B\ G$ becoming tighter when pulling, or by its sag h assuming the smaller value h_1 while the rope $C\ H$ becomes more slack by assuming a greater sag h_2. The distance through which the rocker arm $B\ C$ must move from its position of rest to that which it occupies when S becomes equal to S_1 and S_2 respectively, must be regarded as lost motion: for

FIG. 483.



while this motion is taking place the bob M will not move, the motion of the latter beginning when $H_1 - H_2 = Q$, Q representing the resistance of the bob at the end of the lever arm $F\,G$, and H_1 and H_2 the *horizontal* components of the rope tensions S_1 and S_2. This loss of stroke will be greater the smaller the ratio of the tension S of the rope at rest to its tension when the rope drives, *i.e.*, it will be greater the slacker the ropes are when at rest. The determination of this lost motion can be found approximately as follows: If we know the distance b between the shaft A of the rocker arm and F of the bob, and likewise the greatest tension S of the ropes when at rest, we can determine the sag h of the rope at the centre and the length l of each of the ropes. Let α represent the angle of slope of the rope, *i.e.*, its inclination to the horizon at the ends; then, if we regard the very flat catenary as a parabola, we have from known laws of the funicular polygon (see Vol. I. § 157)

$$\sin \alpha = \frac{G}{2\,S},$$

where $G = l\,q\,\gamma$ is the weight of the rope and $S = q\,k$ the tension at the point of suspension. Here q represents the cross-section, γ the weight of a unit of volume, and k the tension per unit of area. We can also place the weight $G = l\,q\,\gamma = b\,q\,\gamma$, for b and l differ but little, and we then get

$$\sin \alpha = \frac{b\,q\,\gamma}{2\,q\,k} = \frac{b\,\gamma}{2\,k}.$$

Now we can find the sag h at the centre, from α,

$$\tan \alpha = \frac{2\,h}{\frac{1}{2}\,b} = \frac{4\,h}{b} \quad \text{or} \quad h = \frac{b}{4}\tan \alpha,$$

and the length of rope may be obtained with sufficient accuracy from

$$l = b\left[1 + \frac{2}{3}\left(\frac{2\,h}{b}\right)^2\cdot\right]$$

This length must be given to each of the two ropes. During the motion of the rocker arm, one of the ropes will grow tighter and the other more slack. Let the distance described by the end of the rocker arm before the bob begins its motion be represented by e. At this instant the two ropes will have the spans $b_1 = b + e$ and $b_2 = b - e$. Now if we assume that the greatest tension in the tight rope reaches a certain value k_1 per unit of area, we again have for the angle of slope of the driving rope

$$\sin \alpha_1 = \frac{G}{2 S_1} = \frac{l \gamma}{2 k_1},$$

and from α_1 the sag

$$h_1 = \frac{b_1}{4} \tan \alpha_1.$$

The span b_1 may be found from the known length l by means of the expression

$$l = b_1 \left[1 + \frac{2}{3} \left(\frac{h_1}{b_1} \right) \right] = b_1 \left[1 + \frac{2}{3} \left(\frac{\tan \alpha_1}{2} \right)^2 \right],$$

and from this span b_1 we get the distance $e = b_1 - b$. This determines the span b_2 of the slack or driven rope $b_2 = b - e$, and from its known length l we obtain the sag h_2 of this rope from

$$l = b_2 \left[1 + \frac{2}{3} \left(\frac{2 h_2}{b_2} \right)^2 \right] \quad \text{or} \quad h_2 = \tfrac{1}{2} \sqrt{\tfrac{3}{2} (l - b_2) b_2}\,.$$

From h_2 and b_2 we now find the angle of slope at point of suspension by means of

$$\tan {}_2 = \frac{4 h_2}{b_2},$$

and from this the tension $S_2 = q\, k_2$ by means of

$$\sin \alpha_2 = \frac{G}{2 S_2} = \frac{l \gamma}{2 k_2}.$$

Now if the resistance Q of the bob is given, we get from

$$Q = H_1 - H_2 = \frac{G}{2} (\cot \alpha_1 - \cot \alpha_2) = \frac{l q \gamma}{8} \left(\frac{b_1}{h_1} - \frac{b_2}{h_2} \right)$$

the desired cross-section q of the rope.

The loss of stroke is $b_1 = b_2 = 2\, e$, for at every change of stroke the span b_2 of the previously slack rope must be increased to b_1 before it can overcome the resistance Q.

It may be easily seen that the value $e = b_1 - b$, and consequently the loss of stroke, will be smaller the smaller the difference of k and k_1. Therefore, to diminish the loss of stroke as much as possible, the rope must be tightened until it has a great tension k. This will of course increase the friction of the journals, and the necessary cross-section of the rope will also be considerable, for the resistance Q can be overcome only by the difference of tensions k_1 and k_2, which also become smaller as the difference between k_1 and k grows less. For a very small tension k in the rope when at rest, it may happen that though the rocker arm moves, the bob will not move at all—a case which will of course always occur when the motion of the end of the rocker arm is less than $2e$.

Since the tightening or shortening of the rope diminishes the loss of stroke, and slacking or lengthening it increases the lost motion, it follows that variations of temperature will have a like influence, the same rope-rods having greater lost motion in summer than in winter. This effect of changes of temperature must not be disregarded because the changes of length are small; consequently it is desirable to regulate the length of rope by suitable devices for varying the tension of the ropes.

If we neglect the friction and other hurtful resistances always accompanying lost motion, we see that there is no direct loss of work connected with loss of stroke; for the work of stretching the rope is given back again by its elasticity when it becomes slack. The lost motion of the rocker arm may, however, easily give rise to injurious shocks.

EXAMPLE. The distance between the axes of a rocker arm and bob is 60 meters [197 ft.]; the transmission is to be so arranged that the greatest tension in the wire rope when at rest will be 6 kilograms [8500 lbs.], and when in motion 12 kilograms [17,000 lbs.]. What cross-section must be given to the rope when the resistance $Q = 800$ kilograms [1764 lbs.], and how great will the loss of stroke be under these circumstances?

If we assume the specific gravity of wire rope to be 8.5 (taking into account the 10 per cent shortening occurring in its manufacture), we get for the angle of slope α at the point of suspension, when the rope is at rest,

$$sin\ \alpha = \frac{b\gamma}{2k} = \frac{60 \times 8500}{2 \times 6 \times 1,000,000} = 0.0425,$$

or $\alpha = 2° 26'$, and therefore $tan\ \alpha = 0.0426$; consequently, for the sag at the centre

$$h = \frac{b}{4}\, tan\ \alpha = 15 \times 0.0426 = 0.638 \text{ meters } [2.09 \text{ ft.}].$$

From this follows the length of rope

$$l = 60\left[1 + \frac{2}{3}\left(\frac{0.0426}{2}\right)^2\right] = 60.018 \text{ meters.} \quad [196.86 \text{ ft.}]$$

In like manner, when the rope is pulling, we have

$$sin\ \alpha_1 = \frac{l\gamma}{2\,k_1} = \frac{60.018 \times 8500}{2 \times 12 \times 1{,}000{,}000} = 0.02125;\ \alpha_1 = 1° 13',$$

and

$$tan\ \alpha_1 = 0.02125 = 4\,\frac{h_1}{b_1}.$$

Furthermore, for the span we get

$$b_1 = \frac{l}{1 + \frac{2}{3}\left(\frac{2\,h_1}{b_1}\right)^2} = \frac{60.018}{1 + \frac{2}{3}0.010625^2} = 60.0136 \text{ meters } [196.84 \text{ ft.}];$$

consequently

$$e = b_1 - b = 0.0136 \text{ meters } [0.535 \text{ in.}];$$

from which we obtain the loss of stroke $2\,e = 27$ millimeters [1.48 ins.].

The span of the tight rope is therefore

$$b_2 = b - e = 59.9864 \text{ meters } [196.75 \text{ ft.}];$$

and from this we get for the sag at the centre

$$h_2 = \tfrac{1}{2}\sqrt{\tfrac{3}{2}(l - b_2)\,b_2} = \tfrac{1}{2}\sqrt{\tfrac{3}{2}\,0.0316 \times 59.986} = 0.8444 \text{ meters } [2.77 \text{ ft.}].$$

Now follows the angle of slope α_2 from

$$tan\ \alpha_2 = \frac{4\,h_2}{b_2} = \frac{3.3776}{59.986} = 0.056306;\ \text{ or } \alpha_2 = 3° 13'.$$

Hence for the horizontal tensions we have

$$H_1 = \frac{G}{2\ tan\ \alpha_1} = \frac{60\ 018 \times 8500}{2 \times 21{,}250}\,q = 12\,q,\ [= 17{,}100\,q \text{ lbs.}],$$

and

$$H_2 = \frac{G}{2\ tan\ \alpha_2} = \frac{60.018 \times 8500}{2 \times 56{,}306}\,q = 4.53\,q\ [6400\,q \text{ lbs.}];$$

and the cross-section of the rope is

$$q = \frac{Q}{12 - 4.53} = \frac{800}{7.47} = 107.1 \text{ mm.}^2\quad [= 0.166 \text{ sq. ins.}]$$

With 36 wires in the rope, the diameter of wire must correspond to the cross-section

$$\frac{107.1}{36} = 3 \text{ mm.}^2 \left[\frac{0.166}{36} = 0.0046 \text{ sq. in.}\right]$$

If the calculated length of wire rope $l = 60.018$ meters [196.86 ft.] has a temperature 5° C. [41° F.] when the temperature rises to 30° C. [86° F.], the length will be increased by

$$(30 - 5)\,0.0000123\,l \text{ meter}$$
$$= 0.0186 \text{ meter; } [(86 - 41)\,0.00000683 \times 196.86 = 0.061 \text{ ft.;}]$$

consequently the length will then be $(l) = 60.0366$ meters [196.92 ft.].

Substituting this value in the above formulas, we get, for the rope at rest,

$$(h) = 0.909 \text{ meter } [2.98 \text{ ft.}]; \quad (\tan \alpha) = 0.0606; \quad (\alpha) = 3^\circ\, 28';$$

when the rope is stretched and its span $b_1 = 60.0136$ [196.84 ft.],

$$(h_1) = 0.723 \text{ meter } [2.37 \text{ ft.}]; \quad (\tan \alpha_1) = 0.0481; \quad \alpha_1 = 2^\circ\, 45';$$

and when the rope is slack and its span $b_2 = 59.9864$ [196.75 ft.],

$$(h_2) = 1.068 \text{ meter } [3.5 \text{ ft.}]; \quad (\tan \alpha_2) = 0.07105; \quad (\alpha_2) = 4^\circ\, 4'.$$

Accordingly, the horizontal tensions of the two ropes, supposing their spans to be $b_1 = 60.0136$ meters [196.84 ft.] and $b_2 = 59.9864$ meters [196.75 ft.], will be

$$H_1 = \frac{G}{2\,(\tan \alpha_1)} = 5.72\,q, \quad [8136\,q,]$$

and

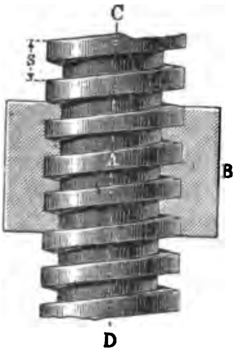
$$H_2 = \frac{G}{2\,(\tan \alpha_2)} = 3.59\,q. \quad [5106\,q.]$$

It is evident that for the case supposed the excess $H_1 - H_2 = 2.13\,q$ [3030 q] amounts to only 0.285 $Q\left(\text{for } \frac{2.13}{7.47} = 0.285\right)$; consequently for this case the loss of stroke will be considerably greater than that calculated for 5° C. [41° F.].

CHAPTER V.

SCREWS.

FIG. 484.



§ **124. Screws in General.**—The pair of elements designated in § 28 of the Introduction as the twisting pair, is very frequently used, and in widely varying machinery. The relative motion of the two elements of the pair, the *screw* and the *nut*, is composed of a rotation about the axis of the screw and a sliding along the same. The possibility of completely inclosing the screw by the nut depends on the condition *that the ratio of these two motions to each other be always the same.* In accordance with this condition, every point of the nut B, Fig. 484, describes relatively to the screw CD a *cylindrical helix* of constant pitch s. Such a helix BLK, Fig. 485, is characterized by the property that it becomes a straight line BH when its cylinder is developed, the line making the angle of inclination $HBG = \alpha$ with the base BG of the cylinder, *i.e.*,

$$tan\ \alpha = \frac{GH}{GB} = \frac{s}{2\pi r},$$

s representing the rise or pitch GH of one coil of the helix, and r the radius AB of the cylinder. Accordingly the length of one coil of the helix is given by

$$BH = l = \sqrt{s^2 + (2\pi r)^2} = \frac{s}{sin\ \alpha} = \frac{2\pi r}{cos\ \alpha}.$$

In like manner we have for a portion BP of the helix, whose ends B and P subtend the angle at the centre $B A O_1 = \omega$, the rise

$$O P = h = r \omega \tan \alpha,$$

and the length

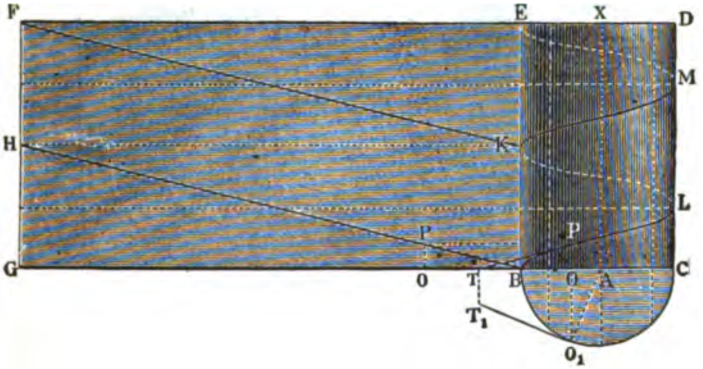
$$B P = l = \frac{r \omega}{\cos \alpha}.$$

From $\tan \alpha = \frac{s}{2 \pi r}$ and $\tan \alpha = \frac{h}{r \omega}$ follows

$$\frac{h}{\omega} = \frac{s}{2 \pi};$$

i.e., the ratio of the sliding to the rotation is constant.

FIG. 485.



Let us suppose a helix $B L K$ . . . , Fig. 486 and Fig. 487, to be drawn on a cylinder $B E D C$, and a straight generatrix $B B_1$ to be so guided along the helix that one of its points B always remains on the helix, and the generatrix $B B_1$ itself constantly intersects the axis $A X$ at the same angle. Then any point B_1 of the generatrix will describe a helix of the pitch s, and the line will generate a surface which we will call a *screw surface* or *helicoid*. According as the helix $B B_1$ is perpendicular or inclined to the axis $A X$, the screw surface will take the form shown in Fig. 486 or Fig. 487. It follows from the manner in which these screw surfaces are generated that any cylinder concentric to $A X$ and with any radius r_1 intersects the screw surface in a radius r_1, and that all these helices have the same pitch. The pitch angle α_1 of this helix is expressed by $\tan \alpha_1 = \frac{s}{2 \pi r_1}$, from which follows that the various helices described by different points of the generatrix BB_1 are less inclined to the base of the cylinder the greater their distance r_1 from the axis.

If instead of the straight line BB_1 we employ a plane figure, bounded by straight or curved lines, as generatrix, guiding the figure so that one of its points remains on the given helix, while its plane constantly passes through the axis of the screw, then will the periphery of this figure likewise generate a screw surface enclosing a sort of screw-wound space. When realized practically, such a screw-shaped space is called a *screw thread*, and may be regarded as the material embodiment of the helix.

FIG. 486.

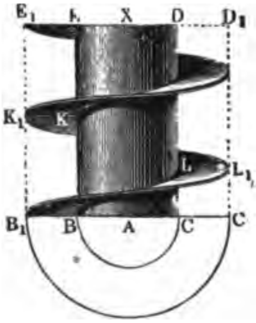
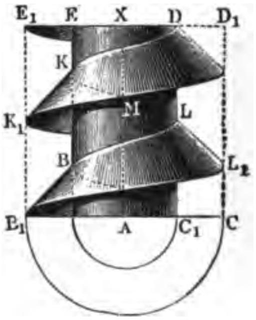


FIG. 487.



The threads vary according to the kind of generating surface chosen, the commonly occurring ones being the *square* thread, Fig. 488, and the *triangular* or V-thread, Fig. 489. The square thread is generated by the motion of a rectangle $BCDE$, Fig. 490, whose height $BC = \frac{s}{2}$; the V-thread is formed by the motion of an equilateral triangle BCD, Fig. 491, whose base $BC = s$. It is evident that the base of the V-threads covers the whole of the inner cylinder or core, while the square threads cover but half of this surface—a circumstance which must be taken into account when the resistance of the threads to shearing, in the direction of the axis, is to be determined. The square threads are principally used in screws which transmit large forces by pressure or shock, as in presses and jacks; for the surfaces of the threads are well adapted to receive great pressures, since they are perpendicular to the axial direction of the pressure. The frictional resistances are also less than with V-threads, for the sharp profiles exert a sort of wedge action which involves more friction. For this latter reason V-threads are almost exclusively used as bolts or *fastenings* where a greater frictional resistance prevents or lessens the liability of the screw joint to loosen of its own accord. For practical reasons the sharp edges of the threads are frequently

Fig. 488.

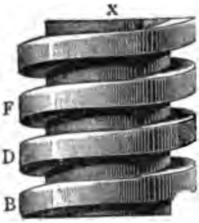


Fig. 489.

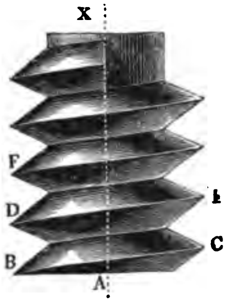


Fig. 490.

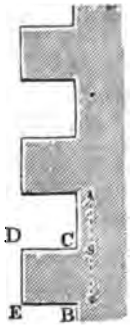


Fig. 491.

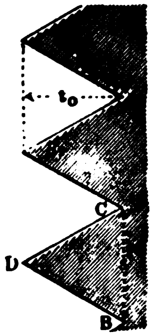


Fig. 492.

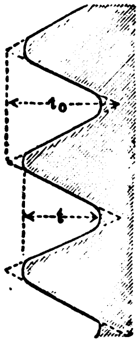


Fig. 493.

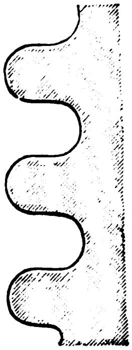
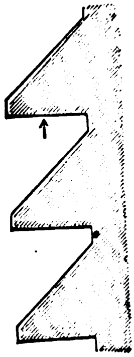


Fig. 494.



rounded, the screws used for fastenings often having the *Whitworth* profile, Fig. 492, while press screws often have round threads of a section like that shown in Fig. 493. Where very great pressures come on the threads, as in press- and stamp-work, it is desirable to increase the strength of the thread by increasing its base. Instead of using a triangular thread which has sufficient strength but causes too much friction, a thread of trapezoidal section is used, as shown in Fig. 494. Of course this form can only be used when the pressure acts in but one direction—that of the arrow.

Screws are further divided into *right-handed* and *left-handed* screws. In a *right-handed* screw the threads on the *front* side of the screw, that is, the side which is *turned toward the observer*, rise from *left to right*, the slant corresponding to that of ordinary writing. Most of the screws occurring in practice are *right-handed*, left-handed threads being used only in special cases.

Finally, we must distinguish between screws with *single*, *double*, *triple* . . . threads. If we suppose a square thread $B_1C_1D_1E_1$, Fig. 495, to be generated by a rectangle whose base is only $\frac{s}{4}$, then the space between the threads, which is $\frac{3}{4}s$ wide, will evidently permit the introduction of another thread, $B_2C_2D_2E_2$ . . ., equal and parallel to the first, the axial distance of the second thread from the first being everywhere equal to $\frac{s}{4}$. In this way a double thread is generated, and

FIG. 495.

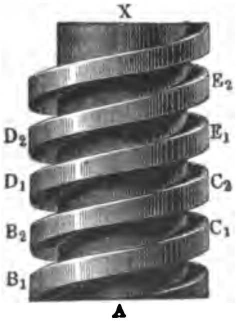
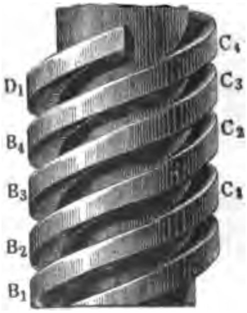


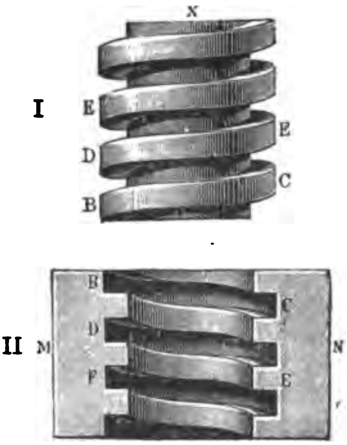
FIG. 496.



from this we may infer what is meant by triple, quadruple . . . threads. Fig. 496 shows a quadruple thread of square section whose generating surface is a rectangle with a base equal to $\frac{s}{8}$. It is clear that screws with many threads are principally to be used where the pitch s is comparatively large, and where a single-threaded screw would have too large a thread and too much friction. The many-threaded screws have a square or trapezoidal section. Under the head of many-threaded screws come *water-screws* used for lifting water and the *screws* for *propelling* ships, both of which will be discussed in the next volume.

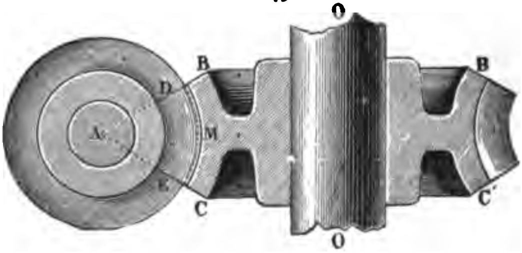
What has thus far been said concerning screws applies principally to the solid element of the twisting pair; the form of the nut or other element of the pair depends on that of the solid screw, for the nut encloses the screw. Consequently, nuts like that shown in Fig. 497, II, always consist of a hollow cylinder M N, whose inner diameter is equal to or slightly larger than the outer diameter of the screw, the cavity being likewise provided with helical threads which exactly fill the spaces between the threads of the screw. From what has been said and from geometry we know that such a complete enclosure of the screw by the nut is only possible with a cylindrical screw of constant pitch, for if the fundamental form were not cylindrical, or if the pitch were variable, the two parts would be locked. It is not indispensable, however, that the nut should completely enclose the screw, a partial enclosure sufficing, as when the nut M of a screw A, Fig. 498, is cut away so as to leave between the planes A B and A C only a prismatic strip for holding the threads D E, which then have the forms of

FIG. 497.



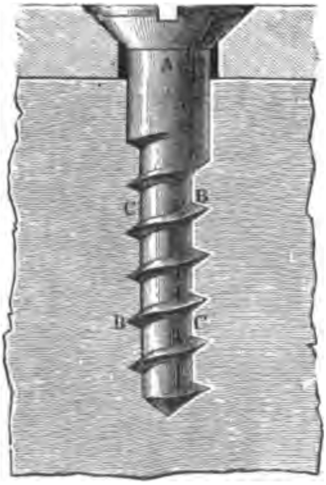
oblique teeth. If this strip and its threads are prevented from leaving the screws, say by means of a guide, this part of the nut will partake of the motion peculiar to the screw as well as a complete nut. The sliding of this piece parallel to the axis then corresponds to the motion of a rack, and if we suppose the strip *B C* to be sufficiently long and bent in the form of a circular wheel *B B′* rotating about the fixed axis *O O*, it is easy to see that it is possible to let the screw *A* work with a toothed wheel, the screw doing duty as a *worm* or endless screw, while the toothed wheel—also called a *worm wheel*—assumes the function of a nut.

Fig. 498.



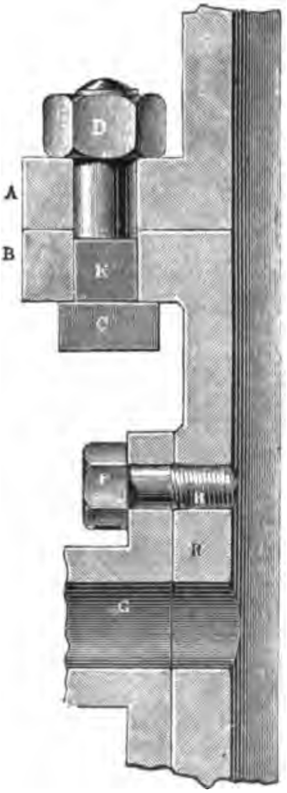
When the screw and nut are of the same material or have the same capacity for resistance, the threads of each have equal sections, as may be seen in Figs. 490 to 494. This rule holds except in cases where the strength of material in the two parts is very different, as in wood screws, Fig. 499, *i.e.*, metallic screws whose nuts are of wood and formed only as the screw enters the wood. In these screws it is customary to make the threads *B* thin and sharp, so that they may have the desired cutting action, and so that the wide spaces *C* between them may be filled by wooden threads of sufficient strength to pre-

Fig. 499.



vent the drawing out of a screw. Wood screws taper toward the end—a form which is not permissible when metallic nuts are used.* Similar principles hold for all screws whose nuts consist of yielding masses, which adapt themselves to the form of the screw. To these belong the screw-propellors and water-screws already mentioned, the screws in screw-ventilators, kneading machines and brick presses, also the screws used in transmitting flour in mills.

FIG. 500.



§ **125. Helical Motion.**—The relative motion of screw to nut has been shown to consist of two simple elementary motions—a rotation about the axis and a sliding parallel to the latter. If we suppose every other motion to be excluded, the form given to the threads necessitates that the ratio of rotation ω to the sliding h be invariable. Therefore, if we give to either screw or nut one of these relative motions, a rotation or sliding, the other motion must appear in the pair to an extent sufficient to satisfy the equation $\frac{h}{\omega} = \frac{s}{2\pi}$. Since here it is only a question of *relative* motion between screw and nut, it does not matter which part has the absolute motion. For example, if one part is held rigid so that it can neither turn nor slide, both motions must be executed by the other part; and if one part is so placed that it can take only one of the motions, the other part must execute the other. The various arrangements corresponding to these cases occur frequently in practice. Thus the bolt CD Fig. 500, used to connect the flanges, *A* and *B* of two pipes, is prevented from rotating by its square shank *E*, and from sliding by the head *C*; consequently the nut must execute both motions. On the other hand, in fastening the branch-pipe *G* to *B R*, the bolt *F H* executes both motions, the nut existing in the stationary pipe *R*. In the *screw-jack*, Fig. 501, the rising motion of the screw *A* lifts the load *Q*, rotation being communicated to the nut *B* by the lever *E*, while the rotation of the screw is prevented by the claw *C* passing through a slit in the standard *D*. On the other hand, in the vise Fig. 502, the jaws *B* have the sliding motion, while the screw *N* is turned by a lever *E*, all sliding motion of the screw being prevented by the collars *D D*.

---

* A rare exception may be seen in the joint for uniting ropes, Fig. 463, p. 567, [and in pipe fittings].

FIG. 501.

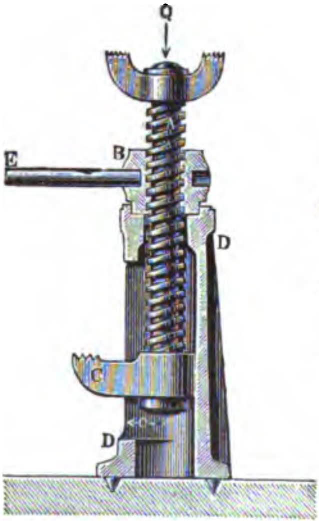
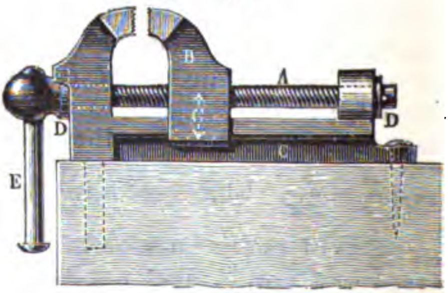


FIG. 502.



We may readily infer from the examples given, which represent the usual constructions, that it is always necessary to provide means for preventing a part from assuming the motion intended for its fellow. For example, in Fig. 501 the screw is prevented from turning and the nut from sliding, and in Fig. 502 the screw is prevented from sliding and the nut from turning, while in Fig. 500 the bolt *C E* is prevented from assuming any motion, and in the other case shown in the same figure, the pipe *R*, containing the thread of the nut, is kept from moving at all. If in Fig. 501 and Fig. 502 the nut and screw were both allowed to rotate the same amount at the same time, the desired helical motion could not take place; but we must not infer from this that both parts, screw as well as nut, may not have the same kind of motion at the same time. For example, it must not be inferred that both may not have a rotation, for such an arrangement is possible provided the two motions are not equal *in amount.* For instance, if the screw is turned through an angle ω_1 and at the same time the nut is turned through an angle ω_2, the relative motion of the two pieces will be represented by $\omega = \omega_1 \pm \omega_2$, according as the directions of rotation are opposite or the same. The relative sliding between nut and screw is given by

$$h = \omega \frac{s}{2\pi} = (\omega_1 \pm \omega_2) \frac{s}{2\pi}.$$

Such screw mechanisms occur in practice, though rarely, two examples being shown in Fig. 134 and Fig. 155, articles 43 and 48. Later on we shall see that this arrangement of a screw with differential motion may under certain circumstances be decidedly advantageous.

From what has preceded we have learned the uses to which screws are put. In the first place, screws are a means of converting rotary into rectilinear motion, and *vice versa;* but screws arranged for this purpose seldom occur, since they are accompanied by very considerable hurtful resistances. A more economical means of converting either of these motions into the other is usually employed; for example, racks and pinions or crank mechanisms. Moreover, the rectilinear motion of screws is slight compared with the rotary motion, for their pitch angle α is usually small. On this account screws are used where it is desirable to reduce a motion in a simple manner and as quickly as possible; in other words, wherever it is desirable to increase the force necessary to overcome a resistance. In all those arrangements in which it is necessary to give very exact positions to pieces, as in *dividing engines*, *measuring apparatus*, *lathes*, *planers*, and many other machines, this screw is almost indispensable. On the other hand, the ease with which a force may be increased causes screws to be used in *jacks*, *presses*, etc.; and for the same reason they are used as fastenings. In the last case the considerable frictional resistances attending the use of screws hinders the unintentional loosening of the screw joint.

§ **126. The Relations of the Forces Acting on Screws.**—If there were no hurtful resistances connected with screws, the relation of the driving force P_0 acting at the end of the lever arm $A\,B = R$, Fig. 503, to the load Q acting in the direction of the axis, could be easily determined by placing the product of the force P_0 and its path $2\,\pi\,R$ during one revolution of the screw equal to the product of the load Q and its pitch s; that is,

$$P_0\,2\,\pi\,r = Q\,s$$

or

$$P_0 = Q\,\frac{s}{2\,\pi\,R} = Q\,\frac{r}{R}\,\frac{s}{2\,\pi\,r} = Q\,\frac{r}{R}\,tan\;\alpha,$$

r representing the radius and α the pitch angle of the middle helix of the thread. In reality, however, the frictional resistances are so large that the requisite driving force P becomes much greater than P_0; often two to three times as great. These frictional resistances consist principally in sliding friction between the threads of the screw and nut, and pivot friction at the surfaces which support the turning piece. When the turn-

FIG. 503.

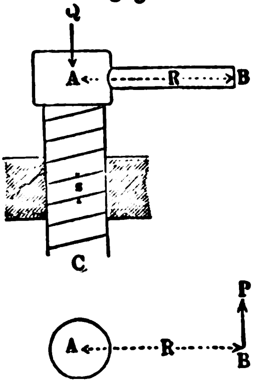
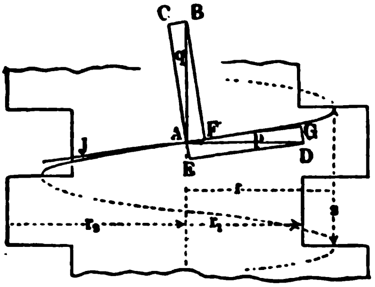


FIG. 504.



ing force acts at one side, as is almost always the case, there is additional friction at the neck journal, and often a sliding friction at the guide, which prevents the sliding part from turning. The latter resistance does not occur when either nut or screw receives both motions, and in this case the pivot friction is also absent, the load taking part in the rotation; this, however, rarely occurs. To determine these resistances let $r = \frac{r_1 + r_2}{2}$, Fig. 504, be the radius of the middle helix of the thread, which we will first suppose to be a square thread. It is sufficiently accurate to assume that the load Q, transferred from the nut to the screw, and the friction of the threads, act at this helix. If s again represents the pitch of the screw, we have for the slope of this middle helix

$$n = \tan \alpha = \frac{s}{2 \pi r}.$$

Moreover, let P_1 be the actual turning force acting perpendicularly to the axis at the *circumference of this helix*, and let us suppose that this force as well as the load Q is distributed uniformly over one coil, one element A of the helix being subjected to the axial pressure $q = \frac{Q}{2 \pi r} = B\,A$ and to the tangential pressure $p = \frac{P_1}{2 \pi r} = A\,D$. These two forces combined produce at A a pressure N normal to the helix, which is

$$N = C\,A + A\,E = q \cos \alpha + p \sin \alpha,$$

this normal pressure causing a sliding friction $F = \mu N$, where μ is the coefficient of friction. During a full revolution of the screw this friction is overcome through a distance equal to the length of one coil, $l = \frac{2 \pi r}{\cos \alpha}$; consequently by equating the quantities of work we have

$$p\, 2 \pi r = q s + \mu q \cos \alpha \frac{2 \pi r}{\cos \alpha} + \mu p \sin \alpha \frac{2 \pi r}{\cos \alpha};$$

or if we introduce $\frac{s}{2\pi r} = \tan\alpha = n$,

$$p = qn + \mu q + \mu p n,$$

i.e.,

$$p = q\frac{n+\mu}{1-n\mu}.$$

Since this relation holds uniformly for every point of the helix, we can put in place of the elementary forces p and q their sums $P_1 = 2\pi r p$ and $Q = 2\pi r q$, and shall then get

$$P_1 = Q\frac{n+\mu}{1-n\mu}.$$

But if the rotation of the screw is not effected by a force P_1 uniformly distributed along its circumference, but is turned by a one-sided force P acting with a lever arm R, the lateral action of this force will cause the journal friction ϕP at the neck journal with radius $\mathfrak{r}$. As the load Q likewise calls forth the friction ϕQ at the pivot or supporting surface whose average radius is $\mathfrak{r}$, we get for the equation of moments

$$PR = P_1 r + \phi P \mathfrak{r} + \phi Q \mathfrak{r}_1.$$

From this follows

$$P(R - \phi\mathfrak{r}) = P_1 r + \phi Q \mathfrak{r}_1,$$

and after substitution of the above values for P_1:

$$P = \frac{r}{R - \phi\mathfrak{r}}\, Q\left(\frac{n+\mu}{1-n\mu} + \phi\frac{\mathfrak{r}_1}{r}\right).$$

Since the force P_0 necessary to lift the load Q, when friction could be neglected, was found to be

$$P_0 = Q\frac{r}{R}\,\frac{s}{2\pi r} = \frac{r}{R}\,Qn,$$

the *efficiency* of the screw, *i.e.*, the ratio of the *force theoretically necessary* to the *force actually exerted*, becomes

$$\eta = \frac{P_0}{P} = \frac{R - \phi\mathfrak{r}}{R}\;\frac{n(1-n\mu)}{n + \mu + (1-n\mu)\phi\frac{\mathfrak{r}_1}{r}}.$$

From this expression for η it is evident that the degree of efficiency increases when the radius $\mathfrak{r}_1$ of the pivot diminishes; and hence we can infer that, other things being equal, an arrangement in which the screw is turned will be more efficient than one in which the nut is turned. For in the latter case the load Q must be supported by a ring-shaped surface of comparatively large radius $\mathfrak{r}_1$, while when the screw turns a small cylindrical pivot can be used as supporting surface. As this pivot friction has considerable influence on the efficiency, it is well, when practicable, to choose an arrangement in which the screw receives the rotation.

The above expressions for P_1 and η are also applicable to the reversed motion, *i.e.*, to the case where the axial force Q becomes the driving force. In applying the above formulas to this case it is only necessary to change the signs of μ and ϕ, for then all the frictional resistances act in the opposite direction. Therefore if (P) represents the force on the lever arm R due to the sinking load Q, we have

$$(P) = \frac{r}{R + \phi \mathfrak{r}} Q \left(\frac{n - \mu}{1 + n\mu} - \phi \frac{\mathfrak{r}_1}{r} \right),$$

and the degree of efficiency for this reversed motion is

$$(\eta) - \frac{(P)}{P_0} = \frac{R}{R + \phi \mathfrak{r}} \frac{n - \mu - (1 + n\mu)\phi \frac{\mathfrak{r}_1}{r}}{n(1 + n\mu)}.$$

When the expression for (P) and consequently that for (η) is negative, it means that the load Q in trying to sink not only fails to exert pressure on the lever arm R, but that a certain force (P) must be exerted on this lever arm in a direction favorable to the sinking of the load Q. In such a case therefore the load cannot sink without assistance, and the whole arrangement is then said to be *self-locking*. Although this property is desirable for many places where convenience and a certain degree of safety are prime considerations, yet it is evident that these are purchased at the expense of economy of work. The limit at which a screw ceases to be self-locking is given by $(P) = 0$, *i.e.*, by

$$n - \mu = (1 + n\mu) \phi \frac{\mathfrak{r}_1}{r},$$

from which the smallest inclination *n* at which the load will sink is still

$$\mu = \frac{\mu + \phi \dfrac{\mathbf{r}_1}{r}}{1 - \mu \phi \dfrac{\mathbf{r}_1}{r}} = \frac{\mu r + \phi \mathbf{r}_1}{r - \mu \phi \mathbf{r}_1}.$$

Most of the screws occurring in jacks, presses, etc., are self-locking, but in some arrangements, stamps for instance, *n* is so great, *i.e.*, the inclination of the threads is so steep, that the screw turns back under the influence of the axial pressure *Q*. Screws used for fastenings must, of course, be always self-locking.

REMARK It has already been mentioned that under certain circumstances sliding friction takes place between the sliding piece and its guide when the latter prevents the piece from turning. This resistance therefore arises only in those cases where each of the two pieces, screw and nut, receives one of the two motions. For instance, in Fig. 501, the screw *A* is kept from turning by the claw *C* pressing against the side of the slit in the standard, while Fig. 502 shows a fixed prism *C* which prevents the nut *B* from turning. As a result of this, sliding friction μT will be generated, *T* representing the pressure on the guide. This frictional resistance during a revolution of a screw is exerted over a path equal to the pitch *s*, and as the load *Q* must be lifted through the same distance, we must take $Q + \mu T$ instead of *Q* as the total resistance to be overcome by the screw. The pressure *T* against the guide is generated by the moment $P_1 r$ of the force P_1 acting at the circumference of the screw. Consequently we have

$$P_1 r = c T; \quad T = P_1 \frac{r}{c},$$

c representing the distance of the middle line of the guide from the axis of the screw. Now if we equate the various quantities of work performed during a revolution, we have

$$P_1 2 \pi r = \left(Q + \mu P_1 \frac{r}{c}\right) \left(s + \mu \cos \alpha \frac{2 \pi r}{\cos \alpha}\right) + \mu P_1 \sin \alpha \frac{2 \pi r}{\cos \alpha}$$

or

$$P_1 = Q \frac{n + \mu}{1 - n\mu - \mu \dfrac{r}{c}\left(n + \mu\right)}.$$

If, as above, we take the friction of the journals into account, we get for the turning force *P* acting on the lever arm *R*

$$P = \frac{r}{R - \phi \mathbf{r}} Q \left(\frac{n + \mu}{1 - n\mu - \mu \dfrac{r}{c}\left(n + \mu\right)} + \phi \frac{\mathbf{r}_1}{r}\right).$$

The expression found above for the circumferential force $P_1 = Q \frac{n \pm \mu}{1 \mp n \mu}$ can be put into another form, if we substitute for $n = \frac{s}{2 \pi r}$ its value *tan* α, and for the coefficient of friction μ the tangent of the angle of friction *tan* ρ. We then get

$$P_1 = Q \frac{\tan \alpha \pm \tan \rho}{1 \mp \tan \alpha \tan \rho} = Q \tan (\alpha \pm \rho),$$

when the *upper* sign refers to the *forward* motion and the *lower* sign to the *reversed* motion.

EXAMPLE. In a screw jack, Fig. 501, where the nut turns, let the pitch $s = 16$ millimeters [0.64 in.], the radius of the neck journal $\mathfrak{r} = 80$ millimeters [3.2 ins.], the average radius of the thread $r = 50$ millimeters [2 ins.], and the radius of the friction on the ring-shaped support $\mathfrak{r}_1 = 65$ millimeters [2.6 ins.]; moreover, let the lever arm of the required turning force P be $R' = 150$ millimeters [6 ins.], the load $Q = 5000$ kilograms [11,200 lbs.], the coefficient of sliding friction along the threads $\mu = 0.1$, the coefficient of journal friction $\phi = 0.08$, and the distance of the edge of the guide from the axis of the screw $c = 0.1$ meter [3.94 ins.]; required the turning force P.

We have here

$$n = \frac{16}{2 \pi \times 50} = 0.051 = \tan 2° 55';$$

hence

$$P = Q \frac{50}{150 - 0.08 \times 80} \left(\frac{0.051 + 0.1}{1 - 0.051 \times 0.1 - 0.1 \frac{50}{100} (0.051 + 0.1)} + 0.08 \frac{65}{50} \right)$$

$$= Q \times 0.348 (0.153 + 0.104) = 0.0894\, Q = 447 \text{ kilograms [983 lbs.]}.$$

When there are no hurtful resistances, we have

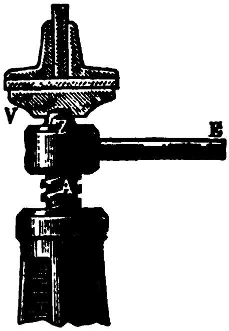
$$P_0 = Q \frac{50}{150} 0.051 = 0.017\, Q = 85 \text{ kgs. [187 lbs.]};$$

consequently the degree of efficiency of the jack is

$$\eta = \frac{0.017}{0.0894} = 0.190, \text{ or about 19 per cent.}$$

If we suppose the screw to turn, but the other circumstances to remain the same, we have the case shown in Fig. 505, where the flat pivot Z supports the load by means of the plate V. With $\mathfrak{r} = 15$ millimeters [0.60 in.] and $\mathfrak{r}_1 = \frac{2}{3} 15 = 10$ millimeters [0.40 in.], we get

FIG. 505.



$$P = Q \frac{50}{150 - 0.08} \frac{}{\times 15} \left(\frac{0.051 + 0.1}{1 - 0.1 \times 0.051} + 0.08 \frac{10}{50} \right)$$

$$= Q\, 0.336 (0.152 + 0.016) = 0.0568\, Q = 284 \text{ kgs. [625 lbs.]}$$

In this case the degree of efficiency is

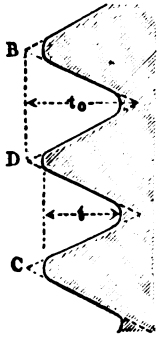
$$\eta = \frac{0.017}{0.0568} = 0.30, \text{ or } 30 \text{ per cent.}$$

This example sufficiently shows the advantage of using an arrangement in which the *screw turns* instead of one in which the *nut turns*.

§ **127. Screws used as Fastenings.**—Screws used for fastenings generally have V-threads, *i.e.*, their profiles are equilateral triangles, Fig. 506, whose vertices are rounded for practical reasons. In these screws either the nut or the screw receives both motions (see Fig. 500), but sometimes cases occur where the turning of the screw draws a nut which is prevented from turning.

The determination of the relation between force and load is similar to that of square-threaded screws, the only difference being that the friction along the threads is ascertained in a somewhat different manner. Of course the load in this case is the pressure with which the parts are pressed together.

FIG. 506.

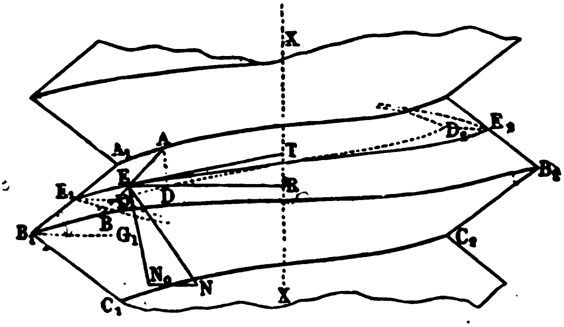


To determine the friction between V-threads, let the equilateral triangle $A_1 B_1 C_1$, Fig. 507, be a section of the thread made by a plane passed through the axis $X X$; $A_1 C_1 = s$ is the base and $A_1 B_1 C_1 = 2 A_1 B_1 G_1 = 2\beta$ the angle at the vertex of the triangle. The line $A_1 B_1$ when moved helically about the axis will generate an oblique screw surface S, upon which the nut presses with the force Q. Let us again suppose that this pressure Q is concentrated along the middle helix $E E E_2$, whose radius is r, and let q represent that part of Q which is supported by an element E of the line of pressure. Also, let p represent that part of the turning force which acts at E perpendicularly to the axis, with the radius r. If as in the square-threaded screw we resolve each of the forces q and p into two components, one in the direction $E T$ of the tangent to the helix at E and the other perpendicular to the first, the sum of the perpendicular components will be

$$E N_0 = N_0 = q \cos \alpha + p \sin \alpha.$$

This force is not, as in the screw thread, perpendicular to the screw surface S pressed upon by the load Q; consequently to determine the friction we must find the component which is perpendicular to the screw surface at E. For this purpose let us suppose a plane passed through E and the axis $X\,X$, which will cut the screw surface S in the straight line $A\,E\,B$, this straight line making the angle $A\,E\,D = \beta$ with the radius $E\,D$ drawn from E. This radius $E\,D$ evidently lies in that normal screw surface S_0 or $E_1\,D_1\,E\,D\,E_2\,D_2$ generated by the perpendicular $E_1\,D_1$, $E\,N_0$ being perpendicular to this normal surface at E. If we now suppose a plane to be passed through the tangent $E\,T$ and the line $E\,A$, this plane will touch the oblique screw surface S at E, and a normal $E\,N$ to this plane will give the direction of that component $E\,N_0$ which is perpendicular to the pressed surface and generates the friction. Evidently these two straight lines $E\,N_0$ and $E\,N$ lie in the same plane with the radius $E\,D$, for these three lines are perpendicular to the tangent $E\,T$. If we designate by δ the angle $N_0\,E\,N$ which the normals to the two screw surfaces S_0 and S make with each other, the force $N_0 = q \cos\alpha + p \sin\alpha$ will resolve itself into two components,

FIG. 507.



$$E\,N = \frac{q \cos\alpha + p \sin\alpha}{\cos\delta} = N$$

perpendicular to the surface S, and

$$N_0\,N = (q \cos\alpha + p \sin\alpha) \tan\delta$$

in a radial direction. The latter component can be neglected because q and p are uniformly distributed around the axis, each element E having an element diametrically opposite, for which this radial component is equal and of opposite direction. In the same manner as before, on account of the uniform distribution of q and p, we can substitute the sums Q and P of these elementary forces, and we then get, as in the preceding article,

$$P 2 \pi r = Q s \pm \mu \frac{Q \cos \alpha}{\cos \delta} \frac{2 \pi r}{\cos \alpha} \pm \mu \frac{P \sin \alpha}{\cos \delta} \frac{2 \pi r}{\cos \alpha};$$

and

$$P = Q \frac{n \pm \dfrac{\mu}{\cos \delta}}{1 \mp \dfrac{n \mu}{\cos \delta}} = Q \frac{n \cos \delta \pm \mu}{\cos \delta \mp n \mu}.$$

It is now only a question of determining the angle δ included between the two normals $E N_0$ and $E N$. This angle is the same as the angle included between the planes $T E D$ and $T E A$ normal to these lines; consequently it is the same as the angle at the edge $E T$ of the spherical triangle $E A T D$. In this triangle one side is $A E D = \beta$, the other side $T E D = 90°$, and the included angle at the edge $E D = 90° - \alpha$. Now according to the formulas of spherical trigonometry we have for the third side $A E T = \gamma$

$$\cos \gamma = \cos \beta \cos 90° + \sin \beta \sin 90° \cos (90° - \alpha) = \sin \beta \sin \alpha,$$

and from this we get for the required angle δ along the edge $E T$

$$\sin \delta = \sin \beta \frac{\sin (90° - \alpha)}{\sin \gamma} = \frac{\sin \beta \cos \alpha}{\sqrt{1 - \sin^2 \beta \sin^2 \alpha}},$$

or

$$\cos \delta = \sqrt{1 - \frac{\sin^2 \beta \cos^2 \alpha}{1 - \sin^2 \beta \sin^2 \alpha}} = \sqrt{\frac{1 - \sin^2 \beta}{1 - \sin^2 \beta \sin^2 \alpha}}.$$

Dividing numerator and denominator by $1 - \sin^2 \beta$, we get

$$\cos \delta = \frac{1}{\sqrt{1 + \sin^2 \beta \dfrac{1 - \sin^2 \alpha}{1 - \sin^2 \beta}}} = \frac{1}{\sqrt{1 + \tan^2 \beta \cos^2 \alpha}}$$

$$= \frac{1}{\cos \alpha \sqrt{1 + \tan^2 \alpha + \tan^2 \beta}}.$$

$n = \tan \theta$
$\alpha = \theta$
$\beta = 90^\circ - \beta$

If we substitute this value for $\cos \delta$, we finally get

$$P = \frac{n \pm \mu \cos \overset{\theta}{\alpha} \sqrt{1 + \tan^2 \overset{\theta}{\alpha} + \tan^2 \beta}}{1 \mp n \mu \cos \alpha \sqrt{1 + \tan^2 \alpha + \tan^2 \beta}} Q.$$

With the proportions common to screws used for fastenings, the pitch angle α of the threads is so small (2° to 3° in *Whitworth's* system*), that we are justified in taking $\cos \alpha = 1$ and $\tan^2 \alpha = 0$, and then get with sufficient accuracy $\cos \delta = \cos \beta$, the above formula for P then becoming

$$P = Q \frac{n \cos \beta \pm \mu}{\cos \beta \mp n \mu}.$$

The angle 2β at the edge of the thread in *Whitworth's* system † is 55°; hence for these screws we have

$$P = Q \frac{0.88\, n \pm \mu}{0.88 \mp n \mu} = \frac{n \pm 1.14 \mu}{1 \mp 1.14\, n \mu}.$$

In all these expressions the upper signs refer to forward motion, while the lower signs give the value (P) for the reversed motion.

---

* The proportions of this much-used system may be found in most engineering pocket-books. The attempts recently made by the "Verein deutscher Ingenieure" to introduce a system of threads based on the French units have not succeeded thus far. See Zeitschr. deutsch. Ingenieure, 1875. The system almost universally used in the United States is the *Sellers* or Franklin Institute thread; its proportions can be found in the American engineering pocket-books of *Nystrom* and *Haswell.*

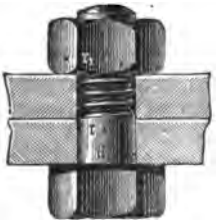
† In Sellers' system the angle $2 \beta = 60°$; hence

$$P = Q \frac{0.866\, n \pm \mu}{0.866 \mp n \mu} = \frac{n \pm 1.15 \mu}{1 \mp 1.15\, n \mu}.$$

The remarks on journal friction when square-threaded screws were discussed, hold also for screws used as fastenings: in this case the surface supporting the pressure Q is almost always a ring surface, *i.e.*, the bearing surface of the nut or of the head; while the neck-journal friction generated by the turning force P is not borne by a special journal, but by the threads themselves or the shank of the bolt. Sliding friction in the guides does not occur in this case.

Example. What turning force must be exerted upon a wrench at a distance 0.2 meter [7.87 ins.] from the axis, when applied to an inch bolt which unites two plates by pressing them together with force of 500 kilograms [1110 lbs.], the coefficient of friction for the slightly lubricated surfaces being taken at 0.16?

FIG. 508.



According to *Whitworth's* system, an inch (25.4 millimeters) bolt has a core 0.75 inch = 19 millimeters in diameter, and a pitch $s = \frac{1}{8}$ inch = 3.2 millimeters; consequently the average radius $r = 11.1$ millimeters [0.43 in.], and the ratio 0 $n = \frac{3.2}{2 \times 3.14 \times 11.1} = 0.046$. ($\alpha = arc\ tan\ 0.046 = 2°\ 38'$.) If, according to the ordinary proportions of a hexagon nut, Fig. 508, we take for the average radius of its bearing surface $\mathfrak{r}_1 = 0.7d = 17.8$ millimeters [0.7 in.] and for the radius of the journal friction $\mathfrak{r} = r = 11.1$ millimeters [0.43 in.], and the angle at the edge of the thread $2\beta = 55°$, we have for the required turning force

$$P = Q\frac{r}{R - \phi\mathfrak{r}}\left(\frac{n + 1.14\mu}{1 - 1.14n\mu} + \phi\frac{\mathfrak{r}_1}{r}\right)$$

$$= 500\frac{11.1}{200 - 0.16 \times 11.1}\left(\frac{0.046 + 1.14 \times 0.16}{1 - 1.14 \times 0.046 \times 0.16} + 0.16\frac{17.8}{11.1}\right)$$

$$= 28.0(0.230 + 0.257) = 13.6 \text{ kgs. } [29.9 \text{ lbs.}].$$

On the other hand, in order to loosen the nut there is needed a force

$$(P) = 500\frac{11.1}{200 + 0.16 \times 11.1}\left(\frac{0.046 - 1.14 \times 0.16}{1 + 1.14 \times 0.046 \times 0.16} - 0.16\frac{17.8}{11.1}\right)$$

$$= 27.5(-0.135 - 0.257) = -10.78 \text{ kilograms } [23.8 \text{ lbs.}]$$

Without frictional resistances there would be needed a turning force

$$P_0 = Q\frac{r}{R}n = 27.8 \times 0.046 = 1.28 \text{ kilograms } [2.82 \text{ lbs.}];$$

consequently the degree of efficiency for screwing up will be

$$\eta = \frac{1.28}{13.6} = 0.094, \text{ or less than 10 per cent,}$$

while the security against the nut's loosening itself is given by

$$(\eta) = \frac{10.78}{1.28} = 8.42.$$

§ **128. Check Nuts.**—The foregoing investigations have shown among other things that screws of the ordinary proportions, *i.e.*, those in which the pitch angle α is not very great or the threads very steep, do not usually possess the property of assuming a reverse motion, under the sole influence of an axial force Q. We can easily determine for every individual case the limit at which this turning back or reversal commences, by placing $(P) = 0$. The value of N corresponding to this limit depends not only on the frictional resistances of the threads, but in a great degree also on the pivot friction at the supporting surface. It is only when the load takes part in the turning that this latter resistance can be neglected, and that the often-given condition holds, which for square threads is

$$n - \mu = 0; \ \alpha = arc\ tan\ \mu, \ i.e., = \text{angle of friction};$$

and for V-threads is

$$n - \frac{\mu}{cos\ \beta} = 0; \ \alpha = arc\ tan\ \frac{\mu}{cos\ \beta}.$$

In all cases where the pivot friction does actually occur during the reversed motion, the limit is given by the condition

$$\frac{n - \mu}{1 + n\mu} = \phi\frac{\mathfrak{r}_1}{r} \text{ for square threads}$$

and

$$\frac{n\ cos\ \beta - \mu}{cos\ \beta + n\mu} = \phi\frac{\mathfrak{r}_1}{r} \text{ for V-threads,}$$

the angle at the vertex of the thread being 2β.

It has been frequently mentioned that square threads usually and V-threads always have such small pitch angles that the screws cannot be reversed by the axial pressure alone. But when these screws are subject to shocks or vibrations they behave differently; at least experience shows that oft-repeated vibrations, even though small, inevitably cause a loosening of the nuts, unless prevented by special locking devices.* Among the devices proposed for and applied to this purpose, we shall examine only check nuts, a locking arrangement which consists simply of using two nuts instead of a single one. When two pieces B and C are to be connected by a bolt A, in cases where the parts may be firmly pressed together, and where the joint is not subject to particular shock, it will be sufficient to use only one nut D; for, according to what has preceded, its friction is sufficient to prevent loosening, however great the axial pressure Q may be. But if this joint is subject to shocks, a certain amount of living force will be communicated to the masses of the different parts, and it may easily happen that the living force thus communicated to the nut will cause the nut to turn. The work transferred to the nut by such a shock will be expended in producing elastic effects, compression or extension, in the material, which will cause corresponding stresses in the latter. Now when the resultant of these stresses has a turning moment with respect to the axis of the screw, which is equal to the moment of friction of the nut, further shock or vibration will cause the nut to yield. Although this process cannot be followed by calculation, the explanation just given will make clear the fact often observed in practice, that nuts will loosen under repeated *shocks* and *vibrations* when they will not loosen under the greatest *statical* pressure. Moreover, it is evident that the work contained in such a shock is greater as the frictional resistance of the nut is greater, *i.e.*, the more firmly the nut is screwed home, for loosening cannot take place till the moment of the elastic stresses corresponds to that of the resistance. The effect of small shocks on nuts which are firmly screwed home is therefore mostly confined to producing elastic oscillations in the material of the nut. But in cases where only a moderate pressure can be exerted by the nut, as in cap bolts for shaft bearings, a firm screwing down of the cap hindering the rotation of the shaft, it is evident that with the slight frictional resistance of the nut, the limit will soon be reached, at which the moment of the elastic stresses will exceed the frictional resistance of the nut. On this account locking arrangements are specially necessary for nuts which can only be pressed with moderate force against their support. On the other hand, loosening of the nuts of bolts used in fastening parts of frames is less apt to occur, for they are screwed home with considerable force.

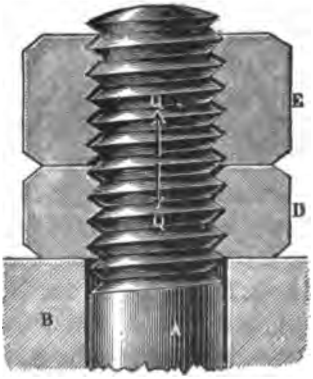
---

* See Zeitschr. des Vereins deutsch. Ingen. 1871.

But even in this latter case, in spite of the great friction of the nut, it may be loosened by comparatively slight vibrations, provided they follow each other quickly, as is shown by the frequency with which the tightly screwed nuts used on the *fish-plates of rails* loosen. Perhaps this effect of frequent repetition of vibrations may be explained in the same manner as the destruction of a bridge by the repeated small vibrations caused by the marching of troops, *i.e.*, by the summing up of small oscillations following each other regularly. That these small frequently recurring vibrations may under certain circumstances overcome very considerable frictional resistances, is shown by an example well known to practical men. The cap screws of bearings for fans making some 2000 revolutions per minute often loosen when provided with check nuts, consequently greater security must be obtained by the use of more positive locking devices, as keys, pins, etc. But for most cases where such extraordinary velocities do not occur, check nuts are a convenient and sufficient locking device.

It is easy to explain the action of check nuts. Let *A*, Fig. 509, be a bolt used for fastening the cap *B* to the pedestal of a bearing. In this case the nut *D* can be pressed against the cap *B* with just enough pressure to prevent any play of the shaft in its bearing. As a consequence the frictional resistance of the nut will be small, and the danger of loosening by shocks, considerable. This danger can be diminished by increasing the frictional resistances at the threads. This can be done by adding a second or check nut *E* to the bolt, screwing it as tightly against the lower nut *D* as is consistent with the strength of the bolt. In this way the upper nut acts as a sort of press-

Fig. 509.

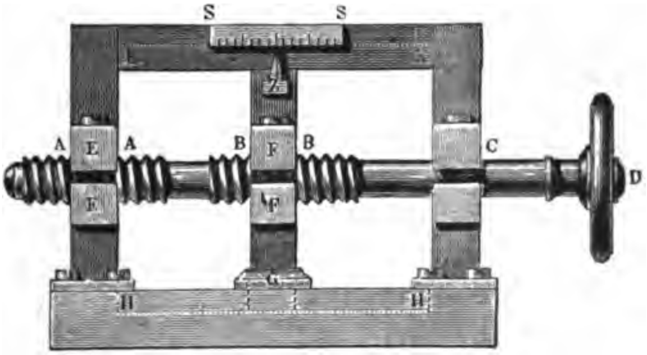


ing device exerting a certain pressure Q on the lower nut, the latter reacting, of course, with an equal and opposite pressure against the upper nut. These two forces Q are communicated by the bolt A, but are not transmitted to the cap B, only that portion of the bolt which is enclosed by the nuts being subject to tension by the force Q. It is evident that with this arrangement the lower nut D presses against the upper, and the upper nut E against the lower surfaces of the threads of the bolt, as shown by the section lines of the figure. Now if we suppose the lower nut to be acted upon by a vibration of the cap which tends to loosen the nut, it is clear that the upper nut must take part in such a backward motion, and the frictional resistance along the threads of both nuts will have to be overcome. This explains the action of check nuts. An idea of the magnitude of the frictional resistance that can be developed in check nuts can be obtained from this fact. If a bolt is fastened in a vise by its head, it is possible to twist off the shank by a wrench placed over the two nuts after they have been firmly screwed against each other, which is proof that in such a case the moment of the frictional resistances exceeds the torsional strength of the material. This device is often employed by workmen for screwing up ***stud bolts*** having no head to which a wrench can be applied. It is clear that in order to loosen checks nuts it is necessary to exert a backward pressure on the *upper nut only*, after which each nut can be easily unscrewed.

Another matter of some interest may here be mentioned. It often happens in practice that in using check nuts the two nuts are of different heights. A certain height must be given to a nut in order that the threads may not strip (see § 132 on dimensions). Now it is evident from the figure that the upward pressure P of the piece B (cap or plate) partly unloads the lower nut, for the tendency of this pressure is to press the nut against the lower surfaces of the threads of the bolt. Consequently the lower nut is subjected to a force of only $Q - P$, while the upper nut is subjected as before to Q, Q being the force with which the nuts were originally pressed together. If P were equal to Q, the lower nut would be entirely unloaded, or subject to no pressure, while the upper one would be acted upon by the force $P = Q$. With a further increase of P the threads of the bolt would be subjected to P, only the smaller part of this force being transmitted by the lower nut in any case. It follows from this that if the nuts are to be of different heights it is well to give the outer or check nut the greater height, placing the smaller one between the check nut and the plate to be fastened, and not in the inverse order, as is so often seen. Generally, however, it seems advisable to make both nuts of the same height.

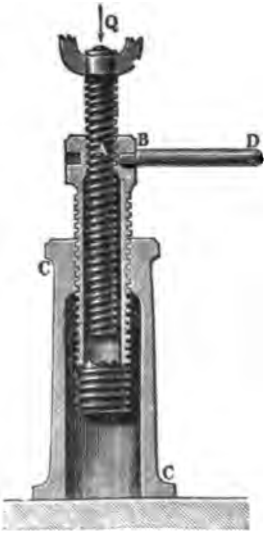
§ **129. The Differential Screws.**—If the ratio $n = \frac{s}{2 \pi r} = \tan \alpha$ of the two velocities in the direction of the axis and in the circumference of the screw be very small, with a given diameter of screw the pitch s, and consequently the dimensions of the thread, will be small, and not strong enough to resist a great pressure Q. In such a case we may overcome the disadvantage of fine threads accompanying the small axial motion, by using a combination of two screws differing in pitch. Such an arrangement is called a *differential screw*. A screw of this kind used as a micrometer screw is shown in Fig. 510. Here the screw is provided at A and B with threads of different pitch s_1 and s_2, the nut E of the screw A with pitch s_1 being fixed to the frame HK, while the nut of the screw B with pitch s_2 is a slide bar FG, which is prevented from turning by the prismatic guides H and K. As the screw AB can slide endwise in its

FIG. 510.



bearing C, it will enter the fixed nut E a distance s_1 for every revolution, while the nut F at the same time moves in the *opposite direction* a distance s_2. The motion of the nut F and its index hand Z is therefore $s_1 - s_2$; hence the effect of the differential screw is the same as if a screw with a pitch $s_1 - s_2$ had been used. It is clear that this difference can be made as small as desired without assuming very small values for the pitches s_1 and s_2.

FIG. 511.



Again, when it is a question of effecting a great multiplication of force by a screw, as is often desirable in jacks, the differential screw may be used, but then it should be given the more convenient form shown in Fig. 511. Here the screw A, on which the load Q rests, has its nut on the inside of the tube BB, which on the outside is formed into a screw whose nut is in the frame CC. As both screws are at once right-handed or left-handed, but their pitches s_1 and s_2 different, it follows that one rotation of the tube by the lever BD will cause the tube to screw into its nut CC a distance s_2, while the screw A, which is prevented from rotating, unscrews from its nut B a distance s_1. Hence the load Q will be raised through a distance $s_1 - s_2$ during one revolution, and if we neglect hurtful resistances we can find the theoretical driving force P_0 acting on the lever arm $AD = R$ from

$$P_0 \times 2\pi R = Q(s_1 - s_2),$$

or

$$P_0 = Q\frac{s_1 - s_2}{2\pi R},$$

or

$$P_0 = \frac{Q}{R}(r_1 n_1 - r_2 n_2) = \frac{Q}{R}(r_1 \tan \alpha_1 - r_2 \tan \alpha_2),$$

n_1 and n_2 representing the tangent of the pitch angles α_1 and α_2. In order to determine the turning force P actually needed, we may remark that there is no pivot friction, the load Q being supported only by the threads. Besides the frictional resistances at the two threads, there is only a slight journal friction at the outer surface of the threads, caused by the one-sided pressure P of the lever D. Moreover, it should be noticed that in the forward motion of the jack as a whole, *i.e.*, in lifting the load Q, the screw A unscrews from its nut B in a direction opposite to the load Q, while the screw-tube B enters the stand C in the direction of the load Q. This motion corresponds to the forward motion of the screw A in B and to the reversed motion of the tube B in C. Now if we let r_1 represent the average radius of the screw A, r_2 the average radius of the thread on the surface of the tube B, and $n_1 = \tan \alpha_1 = \frac{s_1}{2\pi r_1}$, also $n_2 = \tan \alpha_2 = \frac{s_2}{2\pi r_2}$, we can calculate the turning force P needed at D as follows: The load Q, during the reverse motion of the tube B, generates a force (P) on the lever-arm $AD = R$, which may be determined from the formula

$$(P) = Q\frac{r_2}{R}\,\frac{n_2 - \mu}{1 + n_2\mu},$$

previously established. This force (P) together with the turning force P must be able to raise the screw A; therefore for this forward motion we must have

$$P + (P) = P + Q\frac{r_2}{R}\,\frac{n_2 - \mu}{1 + n_2\mu} = Q\frac{r_1}{R}\,\frac{n_1 + \mu}{1 - n_1\mu};$$

i.e.,

$$P = \frac{Q}{R}\left(r_1\frac{n_1 + \mu}{1 - n_1\mu} - r_2\frac{n_2 - \mu}{1 + n_2\mu}\right)$$

$$= \frac{Q}{R}\Big(r_1 \tan(\alpha_1 + \rho) - r_2 \tan(\alpha_2 - \rho)\Big).$$

Here the slight journal friction on the outer surface of the threads has been neglected. Finally, for the degree of efficiency we find

$$\eta = \frac{P_0}{P} = \frac{r_1 n_1 - r_2 n_2}{r_1 \frac{n_1 + \mu}{1 - n_1 \mu} - r_2 \frac{n_2 - \mu}{1 + n_2 \mu}}$$

$$= \frac{r_1 \tan \alpha_1 - r_2 \tan \alpha_2}{r_1 \tan (\alpha_1 + \rho) - r_2 \tan (\alpha - \rho)}.$$

The expressions here developed apply also to the reverse motion when μ is given an opposite sign.

EXAMPLE. Suppose that the screw jack calculated in the example in § 126, for a load of 5000 kilograms [11,200 lbs.] is to be provided with a differential screw, how great a turning force P is needed at the lever arm $R = 0.5$ meter [19.69 ins.] when the radii and pitches of the screws are taken as follows: $r_1 = 30$ mm. [1.2 ins.], $r_2 = 50$ mm. [2.0 ins.], $s_1 = 30$ mm. [1.2 ins.] (double-threaded), and $s_2 = 14$ mm. [0.56 in.]? Here, as in the previous example, the rise of the load for one revolution is $s_1 - s_2 = 30 - 14 = 16$ mm. [0.64 in.]; consequently the theoretical turning force is

$$P_0 = Q \frac{16}{2\pi \times 500} = 25.5 \text{ kilograms [56.2 lbs.]}.$$

Moreover,

$$n_1 = \frac{30}{2\pi \times 30} = 0.1592 = \tan 9^\circ\ 3',$$

and

$$n_2 = \frac{14}{2\pi \times 50} = 0.0446 = \tan 2^\circ\ 33'.$$

Therefore the actual driving force needed is

$$P = \frac{5000}{500} \left(30 \frac{0.1592 + 0.1}{1 - 0.0159} - 50 \frac{0.0446 - 0.1}{1 + 0.00446} \right)$$

$$= 10\,(7.902 + 2.757) = 106.6 \text{ kilograms [235 lbs.]},$$

and the degree of efficiency

$$\eta = \frac{25.5}{106.6} = 0.239, \text{ or nearly 24 per cent.}$$

This arrangement is, therefore, intermediate in efficiency between the two kinds of jacks discussed in § 126, the jack in which the screw turns and that in which the nut turns.

§ 130. Screws with Differential Motion.—From what has preceded, it is sufficiently plain that screws generally have only a slight efficiency, and that it is not advisable to use them in machinery which is continually running, because they are not economical of power. We nevertheless find them used in jacks, presses, etc., principally because of the ease with which they effect a considerable reduction of speed, *i.e.*, increase of force, the relation n giving directly the ratio between the velocity of the force acting at the circumference of the screw to the velocity with which the load is lifted.

The low degree of efficiency is shown by investigation to be due to the friction between the threads and the friction at the supporting surface of the screw or nut, as the case may be. With respect to this latter resistance, the efficiency is found to be greater when the screw turns, for its pivot can be made of smaller diameter than the ring-shaped bearing of the nut. The loss of work caused by this pivot friction is smaller the smaller the radius $\mathfrak{r}_1$ of the pivot in comparison with the average radius r of the thread. On the other hand, the neck-journal friction has generally but little influence.

Taking first the friction along the threads, we have

$$\eta = n \frac{1 - \mu n}{n + \mu};$$

we can now find the highest degree of efficiency of which the screw is capable when $\phi = 0$ by making $\frac{\partial \eta}{\partial n} = 0$; that is,

$$(n + \mu)(1 - 2\mu n) - (n - \mu n^2) = 0, \quad \text{or} \quad n^2 + 2\mu n = 1.$$

From this follows

$$n = -\mu \pm \sqrt{1 + \mu^2},$$

that value of the velocity ratio of the screw which will give the best result for a given coefficient of friction μ. When μ is only a small fraction, which is the case when the threads are well oiled, this value of n differs but little from unity; for example, for $\mu = 0.1$; $n = 0.905$, corresponding to a pitch angle of the screw

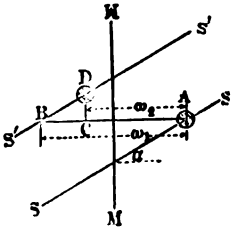
$$\alpha = arc\ tan\ 0.905 = 42° 9'.$$

For these proportions the degree of efficiency is

$$\eta = 0.905 \frac{0.9095}{1.005} = 0.819.$$

If a screw were constructed with so steep a pitch, the loss of work due to friction along the threads would be a minimum; but the advantage of quickly reducing the velocity, the main point in machines where screws are used, would be sacrificed, for with so great a pitch angle the paths s of the load and $2\pi r$ of the circumferential force would be nearly equal. Now the differential screw described in the preceding article has a considerable velocity ratio in spite of the steep threads employed, for the path of the load for every revolution of the screw is expressed by the difference $s_1 - s_2$. If this is to be regarded as an advantage of the differential screw, it is, on the other hand, a disadvantage that the friction of the threads must be overcome at *two places*, which, to be sure, is partly made good by the absence of pivot friction.

FIG. 512.



There is still another way of obtaining a considerable velocity ratio with steep screws, by likewise employing differential action, the nut and screw being both turned in the same direction but unequally. For instance, if we give to the screw a rotation ω_1 in the same time that the nut takes to turn through ω_2, the relative sliding of the two parts will, according to § 125, be given by $h = \frac{\omega_1 - \omega_2}{2\pi} s$. The friction between the threads during this motion must be overcome through a distance represented by $\frac{\omega_1 - \omega_2}{\cos \alpha}$. This may be proved by Fig. 512. Let SS be a helix making a pitch angle α with the axis MM, and let A be any point of the nut. This point will pass from A to D if we give the screw a motion $AB = \omega_1$ and the nut a simultaneous motion $AC = \omega_2$ in the same direction. The axial motion of the nut will then be

$$CD = h = (\omega_1 - \omega_2) \tan \alpha,$$

and the relative sliding of the nut along the thread of the screw will be

$$BD = \frac{\omega_1 - \omega_2}{\cos \alpha}.$$

The ratio between the path $n(\omega_1 - \omega_2)$ of the load Q and the path ω_1 of the force acting at the circumference of the screw, can here be made as small as desired. It will be seen that this arrangement is better than the differential screw described in the preceding article, in that the friction along the threads is very small, for its path is proportional to the small difference $(\omega_1 - \omega_2)$. On the other hand, the pivot friction which is absent in the differential screw, Fig. 511, exists here both at the nut and at the screw. Such an arrangement as this will give a good result only when the radii of the pivots are made as small as possible. Such arrangements frequently occur in practice, for example, in *boring machines.* In connection with Fig. 134 and Fig. 155, the mode of imparting unequal rotations to the screw and nut was explained.

To determine the efficiency of the screw last described, let $n = \tan \alpha$ again represent the velocity ratio of the screw whose middle helix has the radius r, let Q be the axial load, and P_1 the turning force acting at the circumference of the screw. We then find, when journal friction is neglected, for a rotation ω_1 of the screw and a simultaneous rotation ω_2 of the nut,

$$P_1 r_1 \omega_1 = Q r(\omega_1 - \omega_2) n + \mu Q r(\omega_1 - \omega_2) + \mu P_1 n r(\omega_1 - \omega_2),$$

and from this the circumferential force

$$P_1 = Q \frac{n + \mu}{\dfrac{\omega_1}{\omega_1 - \omega_2} - n\mu}.$$

Now if $\mathfrak{r}$ again represents the radius of the neck journal, and $\mathfrak{r}_1$ and $\mathfrak{r}_2$ the lever arms of friction of the pivot surfaces on screw and nut, we can find the turning force P needed by the screw at the lever R, from

$$PR\omega_1 = P_1 r\omega_1 + \phi P\mathfrak{r}\omega_1 + \phi Q(\mathfrak{r}_1\omega_1 + \mathfrak{r}_2\omega_2),$$

or

$$P = \frac{P_1 r\omega_1 + \phi Q(\mathfrak{r}_1\omega_1 + \mathfrak{r}_2\omega_2)}{(R - \phi\mathfrak{r})\omega_1}$$

$$= \frac{r}{R - \phi\mathfrak{r}} Q \left(\frac{n + \mu}{\dfrac{\omega_1}{\omega_1 - \omega_2} - n\mu} + \phi \frac{\mathfrak{r}_1\omega_1 + \mathfrak{r}_2\omega_2}{r\omega_1} \right).$$

If we represent the ratio $\frac{\omega_2}{\omega_1}$ by ν, this expression can be written thus:

$$P = \frac{r}{R - \phi\mathfrak{r}} Q \left(\frac{n + \mu}{\dfrac{1}{1 - \nu} - n\mu} + \phi \frac{\mathfrak{r}_1 + \mathfrak{r}_2\nu}{r} \right).$$

Without hurtful resistances, we have $P_0 R\omega_1 = Qr(\omega_1 - \omega_2)n$, or $P_0 = \frac{r}{R} Q \frac{\omega_1 - \omega_2}{\omega_1} n$, and then $\eta = \frac{P_0}{P}$ may be calculated.

Example. In the differential screw jack, Fig. 513, let the average radius r of the threads equal 125 millimeters [4.92 ins.], and the tangent of the pitch angle $n = 1$ or $\alpha = 45°$. If the radius of the wheel on the screw is $R_1 = 0.248$ meter [9.176 ins.], and for the wheel on the nut $R_2 = 0.25$ meter [9.84 ins.], and the radii of the corresponding pinions M and L, $r_1 = 0.052$ meter [2.05 ins.], and $r_2 = 0.050$ meter [1.97 ins.], we have

$$\nu = \frac{\omega_2}{\omega_1} = \frac{50 \times 248}{250 \times 52} = 0.954.$$

Moreover, if the diameter of each of the steel pivots D and H is 30 millimeters, [1.2. ins.], $\mathfrak{r}_1 = \mathfrak{r}_2 = \frac{2}{3}\,15 = 10$ millimeters [0.39 in.], and if the radius of the neck journal at E is $\mathfrak{r} = 20$ millimeters [0.79 ins.], we shall have for the turning force P, needed at the circumference of the wheel N with the lever arm R_1, when the load $Q = 5000$ kilograms [11,200 lbs.], $\mu = 0.1$, and $\phi = 0.08$,

$$P = \frac{125}{248 - 0.08 \times 20}\,5000 \left(\frac{1 + 0.1}{\dfrac{1}{1 - 0.954} - 0.1} + 0.08\,\frac{10 + 10 \times 0.954}{125} \right)$$

$$= 2536.5\,(0.0508 + 0.0125) = 160.6 \text{ kilograms [354 lbs.].}$$

When there are no hurtful resistances, we have

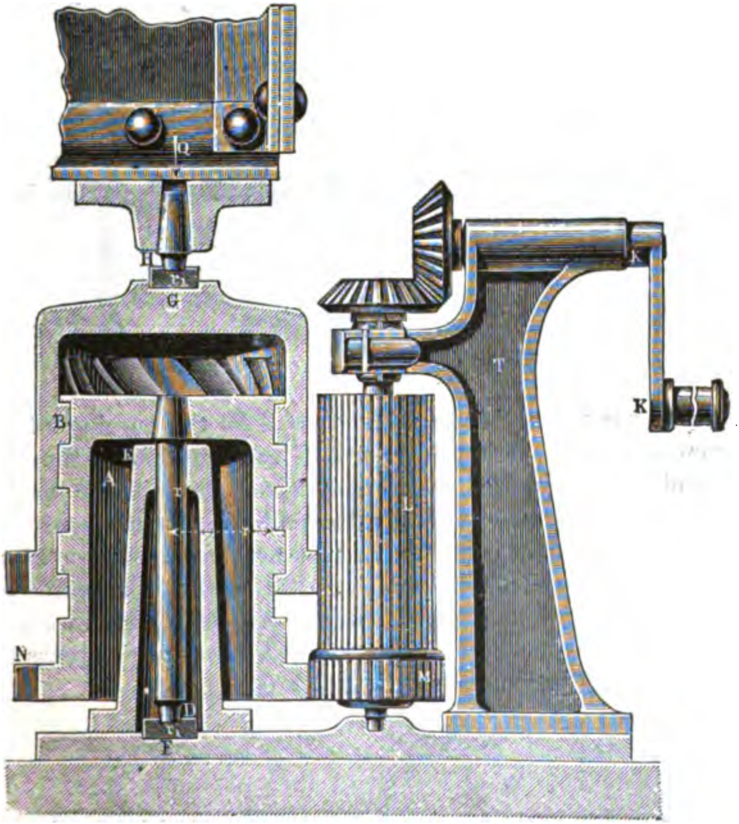
$$P_0 = \frac{125}{248} 5000 (1 - 0.954) = 114.9 \text{ kilograms } [253 \text{ lbs.}];$$

consequently the efficiency of the present mechanism is

$$\eta = \frac{114.9}{160.6} = 0.715,$$

which is considerably greater than that found for the screw jacks examined in § 126 and § 129.

FIG. 513.



§ 131. Right- and Left-handed Screws.—A construction often employed with the tie rods of iron roofs for regulating the length of the rods in accordance with the variations of

FIG. 514.

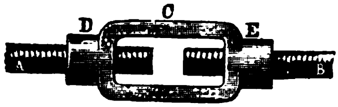
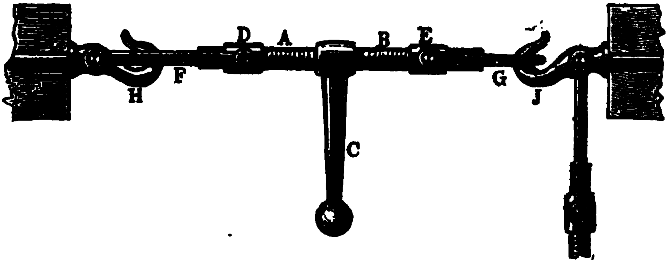


FIG. 515.



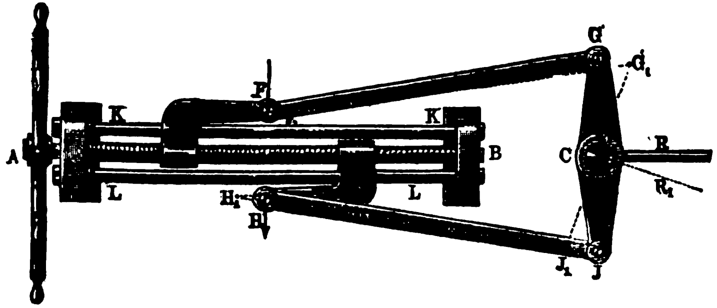
temperature is shown in Fig. 514. Here the ends *A* and *B* of the rods are provided with threads of the same pitch but opposite in direction, their nuts *D* and *E* being united by the frame *C*. According as this connecting piece is turned in one or the other direction, the ends of the rods recede from or approach each other, the corresponding lengthening or shortening of the rods for every revolution of the nuts amounting to $2s$, when s is the pitch of each of the screws. A somewhat similar device is employed in the car-coupling shown in Fig. 515. Here the turning bar *C* is welded to the screw *A B*; the turning of the screw causes the stirrups *F* and *G* connected with the nuts *D* and *E* to draw the hooks nearer together till the buffers press upon each other with the desired force. In these arrangements friction only occurs at the threads, pivot friction disappearing as in the differential screw of § 129. Therefore, if r, n, μ, and ρ represent the same quantities as before, we shall have, for the force P at the lever arm R necessary to press the coupled parts together with the force Q,

$$P = 2\frac{r}{R} Q \frac{n+\mu}{1-n\mu} = 2\frac{r}{R} Q \tan(\alpha+\rho),$$

when we neglect the slight friction at the cylindrical surface of the screw, due to the one-sided action of P.

The screw with right- and left-handed thread is also used in mechanisms for moving the rudders of steam-ships. An example is given in Fig. 516, where the two nuts *D* and *E*, jointed at *F* and *H* to the two connecting-rods *FG* and *HJ*, communicate the sliding motion of the screw to the pins of the double lever *GJ*, which is fastened to the upper end of the vertical rudder post *C*. By turning the screw *A B*, the opposite motions of the nuts in this arrangement give to the rudder every desired position, and the arrangement has the further advantage of maintaining this position without any additional locking device, for the screws used are never so steep as to permit any moving of the mechanism by pressure on the rudder. The angle through which the rudder is turned, when the screw makes one revolution, depends on the position of the arm *GJ* at the time and on the ratio of the length of the connecting rod $FG = HJ$ to the length of the arms $CG = CJ$. For further information on this point, reference may be made to the following chapter on Crank Trains. It is plain from the figure that if the mechanism is to be capable of communicating any motion when the two screws have the same pitch, it is necessary that the lever arms *CG* and *CJ* be equal, and likewise that the connecting rods *FG* and *HJ* have the same length. Moreover, the direction of the screw *A B* must pass through the axis *C* of the rudder, for it is only with such a symmetrical arrangement where the connecting rods form equal angles with the direction of the screw that the equal shifting motions needed at *F* and *H* to turn the rudder can be communicated. The usually inclined positions of the connecting rods relatively to the screw subject their nuts to certain lateral pressures per-

FIG. 516.



pendicular to the screw, these two pressures having always the same direction. For example, if the rudder is to be moved from *R* to R_1, the forces in the connecting rod will act in the direction from *G* to *F* and from *H* to *J*; consequently the lateral pressures mentioned must have the direction given by the arrows at *F* and *G*. These lateral pressures are borne by the guide rods *K* and *L*, thus preserving the screw from all bending action.

This same mechanism has been modified in the manner shown in Fig. 517, the two opposite threads being placed on the same portion of the screw spindle, the threads crossing each other and the nuts taking the form of half-cylinders so arranged that they do not interfere. The mechanism then takes the form shown in Fig. 518, and we see that the guide rods not only take the lateral pressures, but also serve to keep the nuts in contact with the screw. Such screws with opposite and intersecting threads are also occasionally employed in other machinery with reciprocating motion, for example, in the ink-distributing devices of certain printing-presses, etc.

FIG. 517.

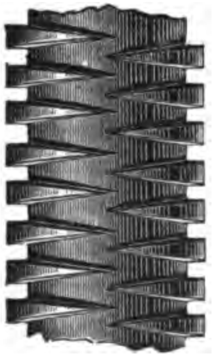
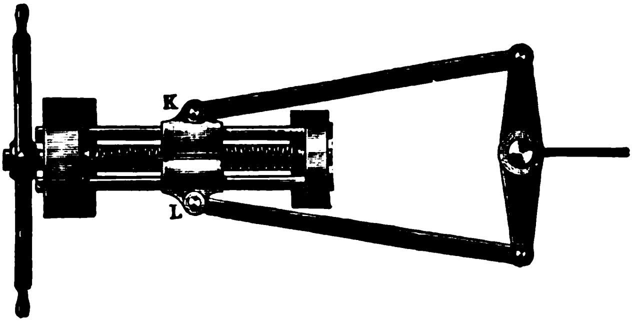
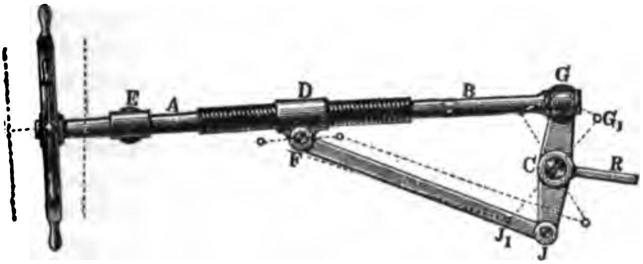


FIG. 518.



In this connection we may mention an interesting screw mechanism designed by *Rogers* for moving the rudder, in which only *one* screw spindle with the ordinary thread is used. Here, Fig. 519, the screw spindle $A B$ can slide endwise in its bearing pivoted at E, and is so united to the end G of the arm $C G$ that it can only turn in the bearing G, all other motion being prevented by collars. As in the preceding mechanism, a connecting rod $F J$ connects the nut D with the other end of the lever $G J$. To understand the action of this mechanism, let us suppose the arm $G J$ and the lever R to be turned through a small angle, say till the arm takes the position $G_1 J_1$; we then see that the motion of the point G to G_1 causes the screw to slide in its bearing E a certain distance s_1 in the direction $A B$, while a motion of the other end of the lever from J to J_1 causes the nut D to likewise shift a certain distance F_2 in the opposite direction $B A$. The ratio of these two shifting motions s_1 and s_2, for every position of the rudder or lever $G J$, depends on the lengths and directions of the parts, reference being made to the following chapter for the determination of this ratio. Whatever the value of this may be, the sum of the two shifting motions $s_1 + s_2$ is always exactly equal to the pitch s representing the relative sliding between nut and screw for every revolution of the latter. Therefore, if $n = tan\ \alpha$ represents the velocity ratio of the screw of average radius r, and ω its angle of rotation, we shall always have $r \omega n = s_1 + s_2$. In this mechanism, therefore, we have an arrangement in which the screw only is turned, while screw and nut have each a sliding motion, the ratio of one motion to the other constantly varying. We have here another proof of the many applications of which the screw mechanism with differential action is capable

FIG. 519.



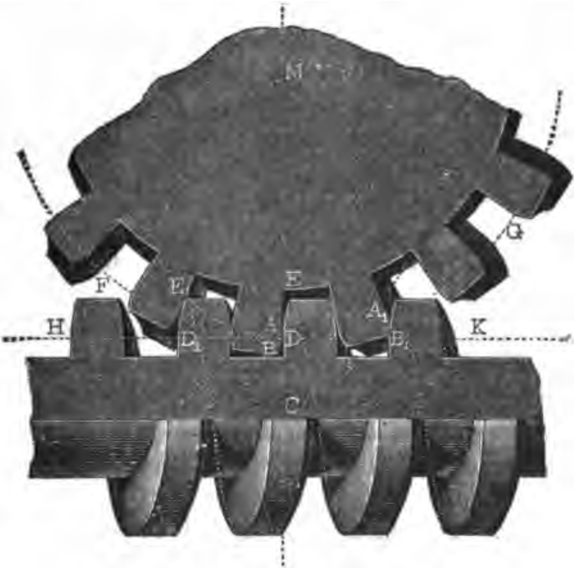
when the screw and the nut simultaneously but unequally take part in the turning or sliding.

§ **132. The Endless Screw.**—It has been mentioned, § 124, that a screw can work with a toothed wheel, in which case the screw is called an *endless screw* or *worm* and the wheel a *screw* or *worm wheel.* An idea of how the worm wheel may be developed from the nut can be obtained by supposing a strip cut from a long nut by two planes passing through the axis of the screw, which strip is bent in the form of a wheel. When, as is usually the case, the screw is single-threaded, one revolution will cause the wheel to turn through one tooth, each tooth corresponding to a thread. The velocity ratio between the axes of the screw and wheel is therefore given by the number z of the teeth. If the screw instead of being single-threaded were provided with 2, 3, or v threads, the velocity ratio would be expressed by $\frac{z}{2}, \frac{z}{3} \cdots \frac{z}{v}$. Since the number z of the teeth may be as large as desired, it is evident that the endless screw allows of a considerable velocity ratio between two shafts. This ratio is greater than is practically attainable by a pair of ordinary spur wheels. This mechanism is mostly used where, as in measuring instruments, a very small magnitude is to be measured by a perceptible motion, or where a considerable resistance is to be overcome by a small force. That the worm gear will not be economical of power in the latter case may be inferred from the considerable frictional resistances attending ordinary screws.

In worm gearing the worm is almost always the driver: it is only in rare cases that the worm is driven by the wheel, for example, it is sometimes done with the fly of a clock. In such a case the screw must have a number of threads, for the pitch angle must be large or motion is impossible.

In the endless screw the cross-section of the threads cannot be assumed at pleasure, as in the screws heretofore considered, but must be determined with reference to the principles governing the teeth of racks and their pinions. If we pass a plane through the axis C of a screw, Fig. 520, perpendicular to the axis of the wheel M, the sections obtained will give the profiles of the screw threads and of the teeth of the worm wheel. A little thought will show that these profiles are to be drawn like those of a rack $H\,K$ and a pinion of radius $M\,A$, for which the pitch $A\,A_1 = D\,D_1$ is equal to the pitch s of the screw. Let the circle $F\,A\,G$ represent the pitch circle of the worm wheel and $H\,K$ the pitch line of the rack. Then by properly rolling the describing circle $A\,M$ and the straight line $A\,K$ (see § 71, Fig. 244) we get the radial flanks $A\,E$ and $D\,C$, for the faces $A\,B$ of the wheel we get parts of the involute of the pitch circle $F\,A\,G$, and for the outer portions $D\,E = D_1\,E_1$ of the thread we get the cycloids generated by the describing circle $M\,A$. The line of contact is, of course, $B_1\,A\,E_1$ as was more fully shown in earlier articles. Of course other forms of profiles could be employed as well; for example, the involute profiles given in Fig. 253, § 74. The profiles thus found are exact only for the middle plane of the wheel; on each side of this plane the profiles of the teeth are different, for their planes do not pass through the axis of the screw. To determine theoretically the geometrical character of the surfaces

FIG. 520.



bounding these teeth would take too long; we will therefore only state that in practice these surfaces, when carefully made, are cut by a helical milling tool (see § 89) whose form is exactly like the worm which is to work with the worm wheel. In this process it is only necessary to continuously turn the milling tool in contact with the wheel, when the latter will of itself assume the rotation needed for the proper formation of its teeth.

It is evident that the worm wheel can be replaced by a sector when the shaft is to turn through only a part of the rotation. This device is used in calico printing for exactly adjusting the rolls containing the parts of the pattern.

In what has preceded it was assumed that the middle plane of a worm wheel passed through the axis of the worm, and then the inclination of the teeth of the wheel was the same as the angle of inclination of the helix of the worm. But the teeth of a worm wheel can be arranged perpendicularly to this middle plane, provided the axis of the worm is inclined to this plane at an angle equal to the pitch angle of the screw threads. *Olivier* * first called attention to this arrangement, which at times is a convenient one, as it economizes space.

To determine the relation between the driving force and load in the endless screw, let Q represent the resistance to be overcome at the circumference of the worm wheel, s the pitch and r the average radius of the threads, a the radius of the pitch circle of the worm wheel, and ρ the radius of its journals. Strictly speaking, there are two kinds of friction in worm gearing—friction along the threads, which is to be estimated as in the ordinary nut and screw; and tooth friction, like that occurring in a rack. This latter resistance is so much smaller than the former that it will only be necessary to take into account the friction along the threads. Now let P_1 represent the pressure which must be exerted at the circumference of the screw in order to perform the work $P_1 \times 2 \pi r$; this will cause (see § 126) a frictional loss of work:

$$\mu P_1 \sin \alpha \frac{2 \pi r}{\cos \alpha} = \mu P_1 s.$$

---

* See *Olivier*, Théorie géométrique des engrenages.

The pressure Q at the circumference of the worm wheel generates the friction ϕQ at its journals, which reduced to the circumference becomes $\phi Q \frac{\rho}{a}$. We must therefore substitute for the resistance of the screw in the direction of its axis

$$Q + \phi Q \frac{\rho}{a} = Q\left(1 + \phi \frac{\rho}{a}\right);$$

then, as before, we get

$$P_1 = Q\left(1 + \phi \frac{\rho}{a}\right)\frac{n + \mu}{1 - n\mu}.$$

If z represents the number of teeth in the wheel, from $2\pi a = zs$ we have

$$a = \frac{zs}{2\pi} = nzr,$$

which makes

$$P_1 = Q\left(1 + \phi \frac{\rho}{nzr}\right)\frac{n + \mu}{1 - n\mu}.$$

If this pressure P_1 at the circumference of the screw is to be generated by a force P acting on one side of the screw at the lever arm R, we get, as in § 126,

$$PR = P_1 r + \phi P \mathfrak{r} + \phi Q\left(1 + \phi \frac{\rho}{a}\right)\mathfrak{r}_1 \text{[*]}$$

or

$$P = \frac{r}{R - \phi \mathfrak{r}}\left(P_1 + \phi Q\left(1 + \phi \frac{\rho}{a}\right)\frac{\mathfrak{r}_1}{r}\right)$$

$$= \frac{r}{R - \phi \mathfrak{r}} Q\left(1 + \phi \frac{\rho}{a}\right)\left(\frac{n + \mu}{1 - n\mu} + \phi \frac{\mathfrak{r}_1}{r}\right),$$

where $\mathfrak{r}$ represents the radius of the neck journal and $\mathfrak{r}_1$ the lever arm of the friction at the pivots or collars of the worm.

---

[*] In this formula the friction at the journals of the worm caused by the one-sided or *eccentric* action of the worm wheel is neglected.

Here it may be mentioned that the obliquity of the teeth of the worm wheel occasions a pressure in the direction of its axis, causing friction at the collars, but this is so small on account of the slight motion, that it may be neglected. If frictional resistances were absent, the driving force P_0 would be

$$P_0 = \frac{r}{R} Q n;$$

consequently the efficiency is given by

$$\eta = \frac{P_0}{P} = \frac{R - \phi r}{R} \frac{n}{\left(1 + \phi \frac{\rho}{a}\right)\left(\frac{n + \mu}{1 - n\mu} + \phi \frac{\mathfrak{r}_1}{r}\right)}.$$

If in these expressions for P and η the coefficients of friction μ and ϕ be given opposite signs, the formulas will apply to the reverse motion, *i.e.*, to the case where the worm-wheel drives. From the expression

$$(P) = \frac{r}{R + \phi \mathfrak{r}} Q \left(1 - \phi \frac{\rho}{a}\right)\left(\frac{n - \mu}{1 + n\mu} - \phi \frac{\mathfrak{r}_1}{r}\right)$$

we obtain the condition that must be satisfied before the reverse motion can take place:

$$\frac{n - \mu}{1 + n\mu} - \phi \frac{\mathfrak{r}_1}{r} = 0 \quad \text{or} \quad n = \frac{\mu r + \phi \mathfrak{r}_1}{r - \mu \phi \mathfrak{r}_1},$$

as in the ordinary screw.

EXAMPLE. Suppose the screw jack calculated in § 126 to have its turning nut formed into a worm wheel of 0.15 meter [5.9 ins.] diameter; suppose this wheel subjected to the pressure of 447 kilograms [985 lbs.]: how great a force must be exerted at a crank 0.20 meter [7.87 ins.] long, fixed to the endless screw whose average radius $r = 40$ millimeters [1.57 ins.] and whose pitch $s = 15$ millimeters [0.59 in.], the shaft of this worm having a neck journal 40 millimeters [1.57 ins.] in diameter, and a pivot 24 millimeters [0.94 in.] in diameter? Here $Q = 447$ kilograms [985 lbs.], $\mathfrak{r} = 20$ millimeters [0.79 in.], $\mathfrak{r}_1 = \frac{1}{3} \times 24 = 8$ millimeters [0.32 in.], and

$$n = \frac{15}{2\pi \times 40} = 0.0597 = \tan 3^\circ 25'.$$

As the friction of the neck journal of the worm wheel was considered in determining the resistance Q and is contained in the value 447 kilograms [985 lbs.], the required force P becomes

$$P = \frac{r}{R - \phi r} Q \left(\frac{n + \mu}{1 - n u} + \phi \frac{r_1}{r}\right)$$

$$= \frac{40}{200 - 0.08 \times 20} Q \left(\frac{0.0597 + 0.1}{1 - 0.1 \times 0.0597} + 0.08 \frac{8}{40}\right)$$

$$= 0.2016\, Q\, (0.1607 + 0.016) = 0.0356\, Q = 15.9 \text{ kilograms } [35 \text{ lbs.}].$$

The number of teeth of the worm wheel is determined from

$$z = \frac{2\pi \times 150}{15} = 62.8 = \text{about } 63.$$

Without frictional resistance we should have

$$P_0 = \frac{r}{R} Q n = \frac{40}{200}\, 0.0597\, Q = 0.0119\, Q = 5.32 \text{ kilograms } [11.7 \text{ lbs.}]$$

consequently the efficiency of the worm gear is

$$\eta = \frac{0.0119}{0.0356} = 0.334, \text{ or about } \tfrac{1}{3}.$$

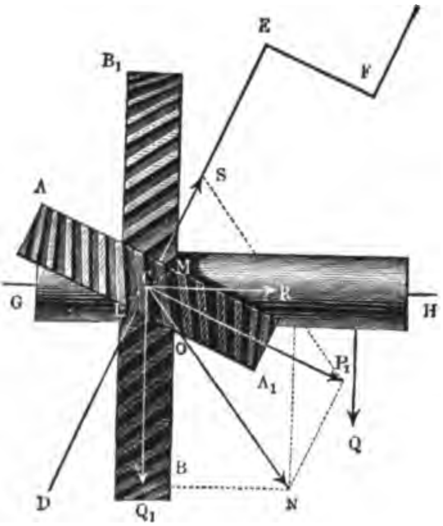
The efficiency of the screw jack in § 126 was found to be 0.191, consequently the whole combination here considered has an efficiency

$$\eta = 0.191 \times 0.334 = 0.064,$$

i.e., it has only the minute efficiency of 6½ per cent. This example clearly shows how very uneconomical the ordinary screw jacks are.

§ 133. Screw Wheels.—The endless screw is closely related to screw wheels. Screw gearing consists essentially of two screws which mesh into each other. That the wheel of the worm gear may be regarded as a screw will be readily seen from the following considerations: If we suppose the endless screw to have a number of threads, it will assume the form of a wheel with as many teeth as the screw has threads. For example, if the screw has as many threads as the worm wheel has teeth, and if the radius r of the screw is equal to the radius R of the wheel, there will be no difference of form between the screw or worm and the worm wheel, and either can be exchanged for the other. In this case we have two wheels on shafts set at right angles to each other, which wheels may be regarded as screws. In like manner we may suppose any two shafts that are askew to each other provided with screws of different diameters, and shall then have the general case of screw wheels, Fig. 521, of which the endless screw is only a special case, characterized by perpendicular axes, and by the fact that one of the wheels, the worm, has usually only a single tooth.

FIG. 521.



In order that we may determine the relations between the driving force and the load as well as the resistances of two screw wheels, let r_1 and r_2 represent the average radii of the screws $A\,A_1$ and $B\,B_1$ (Fig. 521), and α_1 and α_2 the pitch angles of the middle helices; these angles are estimated in such a way that when the teeth are in contact at LM, $\alpha_1 = LMA$ and $\alpha_2 = LMB$, *i.e.*, so that the angle included by the axes $ECH = 180° - (\alpha_1 + \alpha_2)$. When the circumference of the wheel A turns through the distance $v_1 = r_1\,\omega_1$, the circumference of the wheel B will describe the path $v_2 = r_2\,\omega_2$, so that it is easy to see that these paths are to each other as follows:

$$v_1 : v_2 = L\,O : M\,O = \sin\alpha_2 : \sin\alpha_1 .$$

Now if P_1 and Q_1 represent the tangential forces acting in the plane of the wheels at the point of contact C, we may replace these forces by their components CS, CR, and CN, which are parallel to the axes and perpendicular to the helix LM. When friction is neglected and the normal pressure CN is represented by N, we have

$$P_1 = N \cos NCP_1 = N \sin\alpha_1$$

and

$$Q_1 = N \cos NCQ_1 = N \sin \alpha_2;$$

hence with friction neglected

$$P_1 = Q_1 \frac{\sin \alpha_1}{\sin \alpha_2}.$$

When friction is taken into account the resolution of the forces must not be along the normal CN, but along a line deviating from this by the angle of friction ρ, the reaction of the driven tooth against the driving tooth taking place along this line; under these conditions we have

$$P_1 = Q \frac{\sin (\alpha_1 + \rho)}{\sin (\alpha_2 - \rho)}.$$

The latter expression can also be found in the following manner: When the circumference of A turns through a distance $LO = v_1 = r_1 \omega_1$, the circumference of B turns through the distance $MO = v_2 = r_2 \omega_2$ and the teeth slide on each other a distance $LM = v$. The principle of virtual velocities therefore gives

$$P_1 v_1 = Q_1 v_2 + \mu N v.$$

Now since

$$\frac{v_1}{v_2} = \frac{\sin \alpha_2}{\sin \alpha_1}, \qquad N = \frac{P_1}{\sin \alpha_1} = \frac{Q_1}{\sin \alpha_2},$$

and

$$v = v_1 \cos \alpha_1 + v_2 \cos \alpha_2,$$

we can also write

$$P_1 v_1 = Q_1 v_1 \frac{\sin \alpha_1}{\sin \alpha_2} + \mu \frac{Q_1}{\sin \alpha_2} v_1 \cos \alpha_1 + \mu \frac{P_1}{\sin \alpha_1} v_2 \cos \alpha_2$$

or

$$P_1 (1 - \mu \cot \alpha_2) = Q_1 \frac{\sin \alpha_1}{\sin \alpha_2} (1 + \mu \cot \alpha_1).$$

If we substitute $\mu = \tan \rho$ and multiply both sides by $\sin \alpha_2 \cos \rho$, we shall have

$$P_1 (\sin \alpha_2 \cos \rho - \cos \alpha_2 \sin \rho) = Q_1 (\sin \alpha_1 \cos \rho + \cos \alpha_1 \sin \rho),$$

which agrees with the expression given above,

$$P_1 = Q_1 \frac{\sin(\alpha_1 + \rho)}{\sin(\alpha_2 - \rho)}.$$

For ordinary worm gear $\alpha_1 + \alpha_2 = 90°$; then, as before,

$$P_1 = Q_1 \frac{\sin(\alpha_1 + \rho)}{\sin(90° - \alpha_1 - \rho)} = Q_1 \tan(\alpha_1 + \rho).$$

If we put $\alpha_1 + \alpha_2 = 180°$, we get for two wheels with parallel shafts and screw teeth

$$P_1 = Q_1 \frac{\sin(\alpha_1 + \rho)}{\sin(180° - \alpha_1 - \rho)} = Q_1.$$

In this case there is no friction along the threads, for there is no sliding of the teeth along the helix, since

$$v = v_1 \cos\alpha_1 + v_2 \cos\alpha_2 = 0.$$

We must not infer from this that there is no friction whatever in this kind of gearing; it is only the friction *along the threads* that disappears; there still remains the *tooth friction*, neglected above, which is due to the radial sliding of the teeth discussed in § 88.

When the driving force P acts on the wheel A at a distance $EF = a$ and the resistance Q at a distance b from the axis GH, we have

$$P = \frac{r_1}{a} P_1 \quad \text{and} \quad Q_1 = \frac{b}{r_2} Q;$$

hence

$$P = \frac{r_1}{a} P_1 = \frac{r_1}{r_2} \frac{b}{a} Q \frac{\sin(\alpha_1 + \rho)}{\sin(\alpha_2 - \rho)}.$$

The journal friction of the shafts can now be determined from the forces P, P_1, Q and Q_1 as with ordinary spur wheels. There are other frictional resistances at the journals of screw wheels, proportional to the components S and R parallel to the axes and due to the obliquity of the teeth. These lateral forces not only generate frictional resistances at the collars of the shafts, which are to be estimated like ordinary pivot friction, but the excentric action of these forces causes additional friction in the neck journals. These lateral resistances are greater the shorter the lengths l and l_2 of the axes, measured between the journals relatively to the radii; for the reactions called forth by S and R in *each* of their respective journals is given by

$$S_1 = S_2 = S\frac{r_1}{l_1} = \frac{P_1}{tan\ \alpha_1}\frac{r_1}{l_1}$$

and

$$R_1 = R_2 = R\frac{r_2}{l_2} = \frac{Q_1}{tan\ \alpha_2}\frac{r_2}{l_2}.$$

These axial, lateral pressures constitute one of the principal disadvantages of screw wheels,

ω_1 and ω_2.

For the angular velocities of the wheels we found above,

$$\frac{v_1}{v_2} = \frac{r_1\,\omega_1}{r_2\,\omega_2} = \frac{sin\ \alpha_2}{sin\ \alpha_1};$$

consequently the velocity ratio is

$$n = \frac{\omega_1}{\omega_2} = \frac{r_2\,sin\ \alpha_2}{r_1\,sin\ \alpha_1}.$$

If we designate the pitches of the teeth measured along the circumference of the wheels by t_1 and t_2 and the number of teeth by z_1 and z_2, we see from the figure that

$$t_1\,sin\ \alpha_1 = t_2\,sin\ \alpha_2$$

hence

$$\frac{2\,\pi\,r_1}{z_1}\,sin\ \alpha_2 = \frac{2\,\pi\,r_2}{z_2}\,sin\ \alpha_2$$

or

$$\frac{z_2}{z_1} = \frac{r_2\,sin\ \alpha_2}{r_1\,sin\ \alpha_1} = n.$$

Consequently here, as in ordinary spur wheels, the velocity ratio is inversely proportional to the number of teeth. For the thickness s of the tooth in a direction normal to the helix LM, we can assume, as in ordinary toothed wheels,

$$s = \tfrac{19}{40}\, t_1 \sin \alpha_1 = \tfrac{19}{40}\, t_2 \sin \alpha_2 .$$

EXAMPLE. In a pair of screw gears let $r_1 = 0.2$ meter [7.87 ins.] and $r_2 = 0.6$ meter [23.62 ins.], and let the inclination of a tooth to the planes of the wheels be $\alpha_1 = \alpha_2 = 60^\circ$. How great a force is needed at the lever arm $a = 1.2$ meter [47.24 ins.], to overcome a resistance $Q = 1800$ kilograms [3967 lbs.] acting with a lever arm of 0.4 meter [15.75 ins.]?

When hurtful resistances are neglected,

$$P_0 = \frac{r_1}{r_2}\frac{b}{a}\, Q \frac{\sin \alpha_1}{\sin \alpha_2} = \frac{0.2}{0.6}\frac{0.4}{1.2}\, 1800 = 200 \text{ kilograms, [441 lbs.].}$$

When friction is taken into account, and $\mu = 0.1$, also $\rho = arc \tan 0.1 = 5^\circ\ 42'$, we have

$$P = \frac{0.2}{0.6}\frac{0.4}{1.2}\, 1800 \frac{\sin 65^\circ\ 42'}{\sin 54^\circ\ 18'} = 200 \frac{0.913}{0.812} = 224.8 \text{ kilograms [496 lbs.].}$$

Neglecting journal friction, the efficiency of the wheels is

$$\eta = \frac{P_0}{P} = \frac{200}{224.8} = 0.889, \text{ or about 89 per cent.}$$

For the velocity ratio we have

$$n = \frac{r_2 \sin \alpha_2}{r_1 \sin \alpha_1} = 3.$$

If for the same radii and same position of the axes this velocity ratio is to be equal to 5, we must alter the angles α_1 and α_2 to correspond. To determine these angles we have the conditions

$$\alpha_1 + \alpha_2 = 120^\circ \quad \text{and} \quad \frac{r_2 \sin \alpha_2}{r_1 \sin \alpha_1} = 5,$$

or

$$\frac{\sin \alpha_2}{\sin \alpha_1} = \frac{5 \times 0.2}{0.6} = 1.667 = \nu.$$

These angles may now be found from

$$\sin (\alpha_1 + \alpha_2) = \sin \alpha_1 \cos \alpha_2 + \cos \alpha_1 \sin \alpha_2 = \sin \alpha_1 (\cos \alpha_2 + \nu \cos \alpha_1)$$

$$\begin{aligned} \cos (\alpha_1 + \alpha_2) &= \cos \alpha_1 \cos \alpha_2 - \sin \alpha_1 \sin \alpha_2 = \cos \alpha_1 \cos \alpha_2 - \nu \sin^2 \alpha_1 \\ &= \cos \alpha_1 (\cos \alpha_2 + \nu \cos \alpha_1) - \nu. \end{aligned}$$

By dividing one of these equations by the other, we get

$$\frac{\sin (\alpha_1 + \alpha_2)}{\cos (\alpha_1 + \alpha_2) + \nu} = \frac{\sin \alpha_1}{\cos \alpha_1} = \tan \alpha_1 ;$$

therefore

$$\tan \alpha_1 = \frac{\sin 120^\circ}{\cos 120^\circ + 1.667} = \frac{0.866}{-0.5 + 1.667} = \frac{0.866}{1.167} = 0.7420;$$

hence

$$\alpha_1 = 36^\circ\ 35' \quad \text{and} \quad \alpha_2 = 83^\circ\ 25'.$$

Were there no hurtful resistances, the driving force would be

$$P_0 = \frac{0.2}{0.6}\,\frac{0.4}{1.2}\,1800\,\frac{\sin 36^\circ\ 35'}{\sin 83^\circ\ 25'} = 120 \text{ kilograms [265 lbs.];}$$

but when journal friction is considered,

$$P = \frac{0.2}{0.6}\,\frac{0.4}{1.2}\,1800\,\frac{\sin 42^\circ\ 17'}{\sin 77^\circ\ 43'} = 137.8 \text{ kilograms [304 lbs.];}$$

hence the efficiency of these wheels,

$$\eta = \frac{P_0}{P} = \frac{120}{137.8} = 0.871, \text{ or about 87 per cent.}$$

The normal pressure N is

$$N = \frac{a}{r_1}\,\frac{P}{\sin \alpha_1} = \frac{1.2 \times 137.8}{0.2 \times 0.596} = 1387 \text{ kilograms [3060 lbs.].}$$

From this we can calculate, according to § 78, the *normal* pitch t of cast-iron teeth:

$$t = 1.73\sqrt{N} = 1.73\sqrt{1387} = 64.4 \text{ millimeters [= about } 2\tfrac{1}{2} \text{ ins.];}$$

consequently the *circumferential* pitches are

$$t_1 = \frac{t}{\sin \alpha_1} = \frac{65}{0.596} = 109.1 \text{ millimeters [4.3 ins.];}$$

$$t_2 = \frac{t}{\sin \alpha_2} = \frac{65}{0.993} = 65.5 \text{ millimeters [2.58 ins.].}$$

Finally we obtain the number of teeth:

$$z_1 = \frac{2\pi r_1}{t_1} = \frac{6.28 \times 200}{109.1} = 11.5;$$

$$z_2 = \frac{2\pi r_2}{t_2} = \frac{6.28 \times 600}{65.5} = 57.6;$$

for which we can properly assume 12 and 60 teeth, the circumferential pitches now becoming $t_1 = \frac{6.28 \times 200}{12} = 104.67$ millimeters [4.12 ins.] and $t_2 = \frac{6.28 \times 600}{60} = 62.8$ millimeters [2.47 ins.].

§ **134. The Dimensions of Screws.**—The dimensions of screws are generally referred to the diameter d of the shank, as a unit; at least this is the case with the screws used as fastenings. A greater freedom of dimensioning is allowed in square-threaded screws for communicating motion, as in jacks, presses, etc., the pitch and pitch angle of the thread in particular being determined by the velocity ratio employed.

The diameter of the bolt can be determined from the force acting on the screw, according to the formulas for strength established in Vol. I. Almost all screws (all those used as fastenings) are subject to *tensile* stress; it is only in jacks, presses, and sometimes in set screws that we find *compressive* stress. In no case should the design permit a *bending* stress on the screw. Of course the turning of the screw or nut causes a torsional stress, the force needed at the *circumference* of the screw to effect the turning being regarded as the twisting force. The frictional resistance experienced by the nut along its base has no influence on this twisting moment.

It follows from what has just been said that the screw must be regarded as a body subject to a combined axial and twisting stress, use being made of the formula, Vol. I. § 2.

$$Q = Fk\left[1 - \left(\frac{Me}{kW}\right)^2\right];$$

where Q is the pressure acting in the direction of axis, k the greatest working stress, M the twisting moment, F the area of cross-section at a point of fracture, W the twisting moment at this point, and e the distance of the outermost fibre from the axis. It is, however, customary to calculate the screw by the more simple formula $Q = Fk$, the torsional resistance being allowed for by assuming a small value for the working stress k.

The cross-section to be assumed in the calculation is that of the *core* of the screw, *i.e.*, the cross-section of the cylinder to which the bottom of the threads are attached; then calling d_0 the diameter of this core, we have

$$Q = \frac{\pi d_0^2}{4} k.$$

In ordinary bolts with V-threads, we have on an average

$$\frac{d_0}{d} = 0.75;$$

substituting above, we get the formula

$$Q = \frac{0.75^2 d^2 \pi}{4} k = 0.441\, d^2 k,$$

containing the diameter d of the shank of the bolt.

Assuming for wrought-iron screws the small value $k = 2$ kilograms per square millimeter [2850 lbs. per sq. in.], we obtain

$$Q = 0.88\, d^2,\ [Q = 1250\, d^2,]\ \text{or}\ d = 1.07 \sqrt{Q}.^*\ [d = 0.029 \sqrt{Q}.]$$

According to Redtenbacher the pitch s of screws used as fastenings may be determined from

$$z = \sqrt{48 + 16.8\, d}; \quad [z = \sqrt{48 + 426\, d};]$$

where $z = \frac{d}{s}$ represents the *number* of threads or coils for a distance equal to the diameter d of the shank this gives us for diameter of core

$$d_0 = \frac{z - 2}{z} d; \dagger$$

---

* *Redtenbacher* gives for the diameter of screw the formula

$$d = \tfrac{1}{9} \sqrt{P}\ \text{cm.} = 1.11 \sqrt{P}\ \text{mm.} \quad [d = 0.03 \sqrt{P}.]$$

† In Sellers' system, which is now the standard thread in the United States, the diameter of the core is given by the formula

$$d_0 = d - \frac{1.299}{z'},$$

where z' is the number of threads or coils *per inch* of bolt. The pitch of the screw in this system is given by the empirical formula

$$s = 0.24 \sqrt{d + 0.625} - 0.175.$$

For fuller information see Journal of the Franklin Institute, May 1864, and Jan. 1865.

Reuleaux gives, for the pitch s of the *Whitworth* screws, the empirical formula

$$s = 1 + 0.08d. \qquad [s = 0.04 + 0.08d.]$$

In Whitworth's system the angle 2β at the vertex of the threads is 55°, which would make the depth t_0, Fig. 523, of the threads when not rounded equal to

$$t_0 = \frac{s}{2} \cot \frac{55^\circ}{2} = 0.96\,s;$$

when rounded, as in Fig. 522, the depth becomes

$$t = \frac{2}{3}\,t_0 = 0.64\,s.^*$$

The average pitch angle of the threads in Whitworth's system varies from about 2° to 4° 30′.

The nuts are usually made in the form of a hexagonal prism, Fig. 524, and according to *Redtenbacher* the width of the wrench, *i.e.*, the distance D across the flats of the nut, should be

FIG. 522.

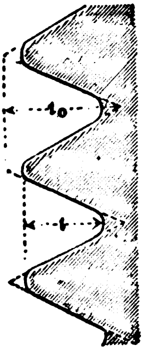


FIG. 523.

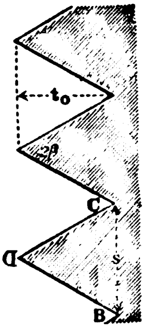
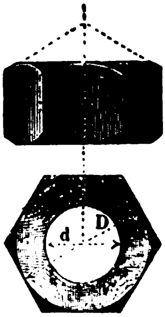


FIG. 524.



be $$D = 5 + 1.4\,d, \qquad [D = 0.2 + 1.4\,d,]$$

and the height of the nut

$$h = \tfrac{2}{3}D = 3.33 + 0.93\,d. \qquad [h = 0.13 + 0.93\,d.]\dagger$$

---

* In Sellers' system $t_0 = 0.866\,s$ and $t = \frac{3}{4}\,t_0 = 0.65\,s$.

† In *Sellers'* system

$$D = 1.5\,d + \tfrac{1}{8} \text{ for rough nuts}$$

When, as is often the case, $h = d$, the resistance to stripping is great, for the shearing area $\pi d h = \pi d^2$ is large, giving for the working shearing stress

$$\frac{Q}{\pi d^2} = \frac{Q}{\pi \times 1.07^2 Q} = 0.28 \text{ kilogram [400 lbs. per sq. in.]}.$$

The hexagonal head of the screw is like that of the nut, with the exception that the depth is smaller, *i.e.*, $h_1 = 0.7\,d$.

For screws with square threads, the number z of threads for a length equal to the diameter may, according to *Redtenbacher*, be

$$z = \tfrac{1}{2}\sqrt{48 + 16.8\,d}, \qquad [z = \tfrac{1}{2}\sqrt{48 + 426\,d},]$$

and for the diameter of the core

$$d_0 = \frac{z - 1}{z}\,d.$$

The nut is made of the height h in order that the wear may be kept down, for which reason the pressure on the surfaces of the threads should not exceed 0.5 kilogram [700 lbs. per sq. in.]; the heights of the nuts for these screws is generally $h = 1.5\,d$.

In all screws which are much worked, as in presses, etc., the nuts must press against well-fitting bearing surfaces in order that the screws may not be subject to bending stress.

Example. What pressure Q can be exerted in the direction of the axis by the one-inch bolt, calculated in § 127, when the maximum working stress $k =$ 6 kilograms per square millimeter [8500 lbs. per sq. in.]?

We have here $d = 25.4$ millimeters [1 in.]; $d_0 = 19$ millimeters [0.75 in.]; hence

$$F = \frac{\pi d_0^2}{4} = 283.5 \text{ mm}^2 \quad [0.44 \text{ sq. in.}], \quad e = 9.5 \text{ mm.} \quad [0.375 \text{ in.}],$$

and

$$\frac{W}{e} = \frac{\pi d_0^3}{16} = 1346 \quad \left[\frac{W}{e} = \frac{\pi d_0^3}{16} = 0.0827\right].$$

---

and

$$D = 1.5\,d + \tfrac{1}{16} \text{ for finished nuts.}$$

Also

$h = d$ for rough nuts,
$h = d - \tfrac{1}{16}$ for finished nuts.

The force acting at the circumference of the middle thread is

$$P_1 = Q \frac{n + 1.14\mu}{1 - 1.14 n \mu},$$

and when $\mu = 0.16$ and $n = 0.046$ is substituted, we shall have

$$P_1 = Q \frac{0.046 + 1.14 \times 0.16}{1 - 1.14 \times 0.046 \times 0.16} = 0.230\, Q;$$

consequently for the twisting moment we get

$$M = P_1 r = 0.230\, Q \times 11.1 = 2.55\, Q \text{ mm. kg. } [0.10\, Q \text{ in. lbs.}].$$

From these values therefore we obtain

$$Q = Fk\left(1 - \left(\frac{Mc}{k\, W}\right)^2\right) = 283.5 \times 6\left(1 - \left(\frac{2.55\, Q}{6 \times 1346}\right)^2\right) = 1360 \text{ kgs. } [3000 \text{ lbs.}].$$

According to the usual mode of calculating,

$$Q = 0.88\, d^2 = 568 \text{ kilograms } [1250 \text{ lbs.}],$$

from which we see that the formulas

$$Q = 0.88\, d^2 \quad [Q = 1250\, d^2] \quad \text{and} \quad d = 1.07\sqrt{Q} \quad [d = 0.029\sqrt{Q}]$$

afford a large margin for safety.

REMARK. The theory of screws is given in most of the text-books of applied mechanics; for example, *Poncelet* in his Cours de mécanique appliquée aux machines, *Navier* in his Résumé des Leçons sur l'application de la mécanique, etc., and *Coriolis* in his Calcul de l'eget des machines. See also *Ritter's* Lehrbuch der technischen Mechanik, and *Wiebe's* Lehre von den einfachen Maschinentheilen. Screw wheels are treated by *Olivier* in his geometrical theory of toothed gearing. The Principles of Mechanism by *Willis* may also be consulted. A full description of the methods of making screws is given by *Karmarsch* in Vol. XIII. of *Prechtl's* Technologische Encyklopaedie. The proportions of screws are given by *Redtenbacher* in Resultate fuer den Maschinenbau, *Reuleaux* in Der Constructeur, and *Unwin* in his Elements of Machine Design.

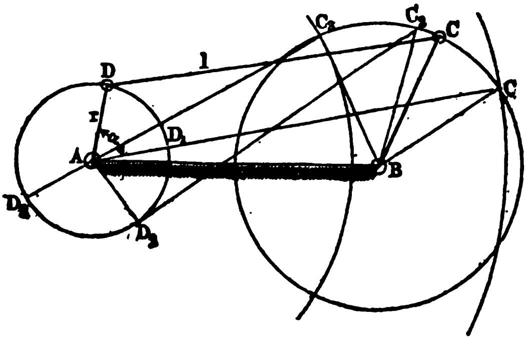
CHAPTER VI.

CRANK TRAINS.

§ **135. The Quadric Crank Train.**—*Crank trains* are principally used for converting continuous rotary motion into reciprocating rectilinear motion, and *vice versa*. In its most general form the mechanism consists of four parallel turning pairs, A, B, C, D, Fig. 525, connected by four links, one of these links being held fast, as $A\,B$, to form the frame or support of the mechanism. Now if any one of the adjacent links, say the crank $A\,D$, be turned through a certain angle $D\,A\,D_1 = \alpha$, the rocker $B\,C$ will be turned around the axis B by the connecting rod $D\,C$, the angle $C\,B\,C_1$ being determined from the geometrical dimensions of the quadrilateral $A\,B\,C\,D$. While the two links adjacent to the fixed link $A\,B$ have a purely turning motion around their centres, the connecting rod $D\,C$ will have a compound motion whose character has been given in § 8 of the Introduction. The mechanism here described is called by *Reuleaux*, the crank quadrilateral or quadric chain, in which the rotating levers $A\,D$ and $B\,C$ are the cranks, and the connecting link $D\,C$ is the *coupler or connecting rod*. To distinguish between the two levers, we will call the lever $A\,D$, which makes a complete rotation about its axis, the *crank*, and the lever $B\,C$, which swings to and fro, the *rocker*.

An examination of the figure shows that during the rotation of the crank $A\,D$ the latter gets into a position $A\,D_1$, whose direction coincides with that of the connecting rod $D_1\,C_1$, the point C_1 being found by describing an arc from A with a radius equal to $D\,C + A\,D = l + r$. This point C_1 will be the extreme position for the end of the rocker $B\,C$, for a further turning of the crank from D_2 to D_3 causes the lever $B\,C$ to return from its position C_1 by passing through C_2 to another position C_3, which likewise lies in a straight line with A and D_3. This point C_3 which is the other limit to the swing of the rocker $D\,C$, is found at the intersection of the circle C with a circle described from A as a centre with a radius $A\,C_3 = D\,C - A\,D = l - r$. We see from this that when the crank $A\,D$ makes a

FIG. 525.



complete revolution, the rocker $B\,C$ will make one forward and return motion through the angle $C_1 B\,C_3$. The ends C_1 and C_3 of the arc described by the rocker are called its *change points*, for at these points the motion changes its direction. The corresponding positions D_1 and D_3 of the crank pin are called the *dead points* of the crank, for in these positions no motion of the crank can be produced by the action of the rocker $B\,C$.

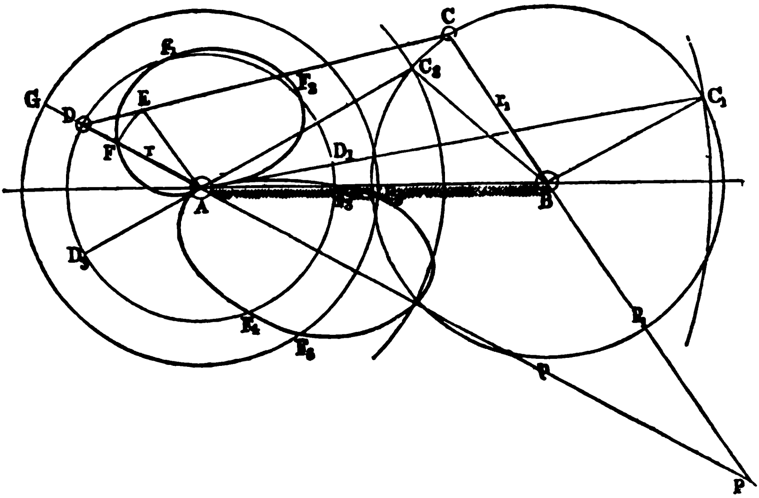
The relation of the velocities of the two cranks $A\,D$ and $B\,C$ at every instant can be found as follows: Suppose that the link $A\,B$, Fig. 526, is fixed, and that the mechanism in any position $A\,B\,C\,D$ is given a very slight displacement, in consequence of which $A\,D$ is turned through $\delta\,\alpha$ and $B\,C$ through $\delta\,\beta$. The simultaneous motion of the coupler $D\,C$ may be regarded as an infinitely small rotation about the instantaneous centre. The latter is evidently at the intersection P of the directions of the two cranks $D\,A$ and $C\,B$. Let the distances of the points D and C from the instantaneous centre be respectively $D\,P = p$ and $C\,P = p_1$, and let $\delta\,\omega$ represent the instantaneous rotation around P; then will the elementary paths v and v_1 of the points D and C be given by

$$v = D\,P \times \delta\,\omega = p\,\delta\,\omega\,;\quad v_1 = C\,P \times \delta\,\omega = p_1\,\delta\,\omega.$$

Since these paths are also given by

$$v = r\,\delta\,\alpha \quad \text{and} \quad v_1 = r_1\,\delta\,\beta,$$

FIG. 526.



where r and r_1 are the lengths AD and BC of the cranks, we have from these relations

$$\frac{v}{v_1} = \frac{r\,\delta\,\alpha}{r_1\,\delta\,\beta} = \frac{p}{p_1}.$$

To represent this expression graphically, draw from A the straight line AE parallel to the corresponding position BC of the rocker; then will

$$DP : CP = DA : EA,$$

or when EA is represented by x,

$$\frac{p}{p_1} = \frac{r}{x}.$$

Substituting this value, we have

$$\frac{v}{v_1} = \frac{r}{x} \quad \text{and} \quad \frac{\delta\,\alpha}{\delta\,\beta} = \frac{r_1}{x}.$$

These expressions permit of a very simple diagram for showing the ratio of the velocities of the two cranks at every instant of the motion. For if the distance $A\,E = x = r\frac{p_1}{p}$ is laid off on the crank from A and equal to $A\,F$, and if this construction is repeated for all the various crank positions, the points F will form a curve in which the radius vector coinciding with the crank, gives the above value x. If in addition we describe about A two circles with radii $A\,D = r$ and $A\,G = B\,C = r_1$, we shall get for any crank position, as $A\,D$, the distances $A\,F$, $A\,D$, and $A\,G$, which will give the ratio of the circumferential velocities and the ratio of the velocities of rotation of crank and rocker. The following relations always hold:

$$\frac{v}{v_1} = \frac{A\,D}{A\,F} \quad \text{and} \quad \frac{\delta\,\alpha}{\delta\,\beta} = \frac{A\,G}{A\,F}.$$

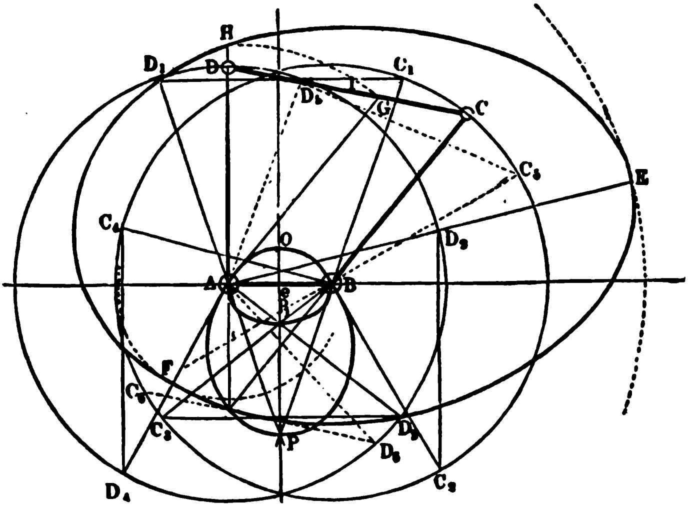
It is evident from the meaning and mode of generation of this curve, that, corresponding to the change points C_1 and C_3 of the rocker, the curve will have two *points of return* which must coincide with the centre A, since the corresponding velocities of the rocker are equal to O. It likewise follows that the tangents to the curve at these return points must coincide with the corresponding positions $A\,D_1$ and $A\,D_3$ of the crank at the dead points. When the curve cuts the crank circle D, as at F_1, F_2, F_3 and F_4, it means that, for the corresponding positions of the cranks, the circumferential velocities v and v_1 are equal.

Similar remarks may be made with reference to the angular velocities $\delta\,\alpha$ and $\delta\,\beta$ and the intersections F_5 and F_6 of the curve with the circle G. Finally, those circles about A which touch the curve between F_1 and F_2 and between F_5 and F_6, determine by their points of contact the crank positions cor responding to the maximum velocities of the rocker for a uniform rotation of the crank. These relations are perfectly general, and apply equally to a uniform and to a variable rotation of the crank.

The quadric crank train has many applications in machinery, several examples of which have already been discussed in Chapter III, under the head of straight-line motions. The lemniscate linkage, Fig. 381, and Evans' parallel motion, Fig. 356, are in reality quadric crank trains. We will not enter upon the analytical investigation of the general case, as it is complex, and would take longer than its practical interest would justify; consequently in the following we shall only discuss the more frequently occurring construction.

§ **136. The Drag-link Coupling.**—The quadric train is frequently used to communicate rotation from one shaft to another parallel shaft. When thus used, give to two shafts

FIG. 527.



A and *B*, Fig. 527, two levers or cranks *A D* and *B C*, which are generally of equal length *r*. The distance between the shafts $A B = e$ and the length of the coupler $D C = l$ must be so chosen that change points cannot occur, thus rendering possible the continuous rotation of both shafts. From the figure we see that when the shaft *A* turns and the pin *D* successively takes the positions D_1, D, D_2, D_3, D_4, the coupler will cause the pin *C* to take the positions C_1, C, C_2, C_3, C_4, thereby insuring the transmission of rotation from *A* to *B*.

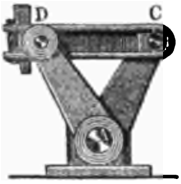
The ratio of the angular velocities α and β of the shafts, which is here equal to the ratio of the circumferential velocities v and v_1 of D and C, because the cranks are equal, varies at every instant, as may be easily shown. In the first place, there are two particular positions of the system, in which both velocities are equal. These positions, $D_1\ C_1$ and $D_3\ C_3$, are the ones in which the points C and D are equally distant from the respective instantaneous centres P and P_1 of the coupler, *i.e.*, they correspond to the two positions in which the cranks $A\ D$ and $B\ C$ make equal angles with the line $A\ B$ joining the shafts. The instantaneous centre can always be found at the intersection of two crank directions, and if we construct the centroids for the motion of the coupler we shall obtain a curve which will contain thê instantaneous centres P and P_1 and will pass twice through the axial points A and B corresponding to the two positions in which each crank coincides with the direction $A\ B$. Consequently the centroid is a curve $P\ A\ O\ B\ P_1\ A\ O\ B\ P$, which is symmetrical with respect to the straight line $P P_1$. To get an idea of the variation of the velocity ratio $\frac{\delta\ \alpha}{\delta\ \beta} = \frac{v}{v_1}$, suppose the closed curve $D_1\ H\ E\ D_3\ F$ to be drawn in the manner described in the preceding article. The radius vector of this curve, which starts at A and coincides with the crank position, has a length x, from which we may obtain $\frac{v}{v_1} = \frac{r}{x}$, as before. The curve accordingly cuts the circle described from A with radius $A\ D = r$ at those points D_1 and D_3 which correspond to the particular positions of equal velocity mentioned above. Furthermore, if we suppose two circles to be described about A, one touching the inside of the curve at F and the other the outside of the curve at E, we shall get at $A\ F$ and at $A\ E$ those positions of the crank $A\ D$ for which the velocity ratio $\frac{v}{v_1}$ is respectively a maximum and minimum. Consequently if the crank $A\ D$ is turned with uniform velocity, the crank $B\ C$ will attain its maximum velocity when the system is in the position $A\ D_2\ C_2\ B$, and its minimum velocity for the position $A\ D_4\ C_4\ B$, corresponding to the crank positions $A\ E$ and $A\ F$. These two velocities are to each other as the rays $A\ E$ and $A\ F$.

It follows that this mechanism is not suitable for a uniform transmission of motion, but is applicable to cases in which the shaft must alternately move quickly and slowly, for example, in planers, where to economize time it is desirable to make the tool move faster on the return stroke than on the forward or cutting stroke. The ratio of the times occupied in forward and return strokes can easily be found from the diagram, as follows: If we take that position $B\ C_2$ of the driven crank which corresponds to its maximum velocity, as the middle position for the return stroke, and lay off on each side of this position on the crank circle $B\ C$ the quadrants $C_2\ C_5$ and $C_2\ C_6$, the corresponding positions $B\ C_5 D_5 A$ and $B\ C_6 D_6 A$ of the system will give at $A\ D_5$ and $A\ D_6$ the positions of the driving crank $A\ D$ which correspond to the beginning and end of the return stroke. Consequently for a uniform rotation of the driving shaft A the duration of the quick return stroke to the duration of the slow forward stroke will be as the ordinary angle $D_5 A\ D_6$ is to the re-entrant angle $D_5 A\ D_6$.

An analytical investigation of the variable velocity is too complex, and in practical cases it is always easier and sufficiently exact to employ the graphical method here given. The mechanism just discussed is known as the ***drag-link coupling.***

If in the mechanism under examination we suppose the distance e of the shafts to gradually diminish until it finally becomes equal to 0, the centroid will gradually diminish until it reduces to the point at which the axes coincide. In this case the transmission of motion will of course be uniform, for the ratio of the distances of the instantaneous centre from the crank ends will constantly be equal to the ratio of the lengths of the cranks. This constant ratio will still exist even when the cranks are of unequal lengths. This mechanism is shown in Fig. 528, and serves to connect a shaft A, provided at its free end with a crank D, with a second shaft A whose axis is in line with the first, and likewise provided with a crank C coupled to the first crank D by a ***drag-link*** $D\ C$. If the two axes were always exactly in line, this coupling could be dispensed with by *rigidly* uniting the two shafts. But when, on account of imperfect setting up or unavoidable vibration in the foundations, there is danger of the two shafts getting slightly out of line, this drag-link coupling may be used with advantage, as it avoids the excessive bending action and even fractures accompanying rigid connection. If the deviation of the axes from a straight line were considerable, the resulting variation in the uniformity of the transmission would be slight, and in most cases hardly perceptible.

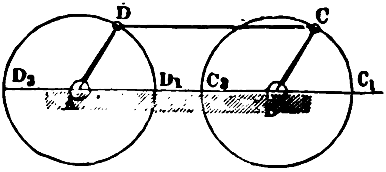
FIG. 528.



In the foregoing discussion of the coupling it was tacitly assumed that the length l of the couples is greater than the distance e of the axes, for when $l < e$ there will be change points in the mechanism and no rotary motion whatever, but only an oscillatory motion, whose angle of swing diminishes as the ratio of l to e and r diminishes. The mechanism would, of course, be locked and motion impossible if $l = 0$, *i.e.*, when the crank pins coincided. The particular case in which $l = e$, however, leads to an important arrangement which will now be considered.

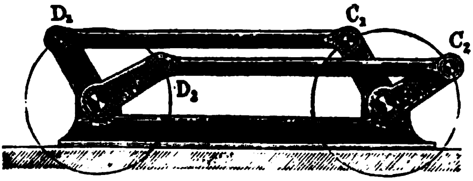
§ **137. Parallel Cranks.**—When the crank quadrilateral becomes a parallelogram $A\ B\ C\ D$, Fig. 529, by the coupler becoming equal to the distance between the axes, the parallelism between the cranks will be maintained for every instant of the motion. It follows from this that the velocities of the cranks will constantly be the same in amount and direction. A uniform rotation of one crank will therefore cause a similar rotation of the other. An example of this is given in § 107, Fig. 414. In this case it was only a question of transmitting oscillations of a certain magnitude, but when continuous rotary motion is to be transmitted, care must be taken that the cranks pass the *dead points*, *i.e.*, the positions $D_1\ C_1$ and $D_3\ C_3$ of the coupler pass-

FIG. 529.



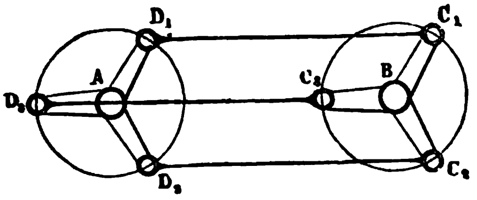
ing through the axes A and B. This is accomplished in the simplest manner by employing two pairs of parallel cranks $D_1 C_1$ and $D_2 C_2$, Fig. 530, which make with each other some angle $D_1 A D_2 = C_1 B C_2$. Provided this angle is not 0 or 180°, it will happen that when one of the systems is at its dead points the other system will be in action. In locomotives where the use of this kind of coupling for the driving axles is universal, the cranks on each axle are at right angles. This sort of coupling is a comparatively simple one, and is accompanied by small frictional resistances when the corresponding links are made as exactly alike as possible.

Fig. 530.



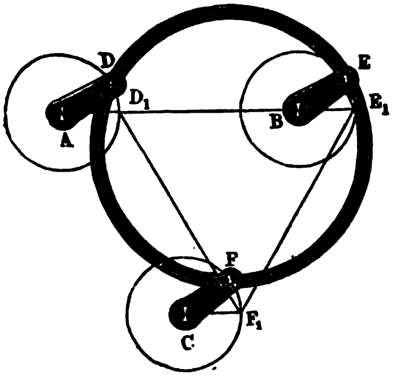
It has also been suggested that the shafts be provided with three cranks so placed as to make angles of 120° with each other, Fig. 531, where the coupling rods might be replaced by a wire rope or thin rods, for with this arrangement at least one of the rods would be always subject to *tension* and would suffice to transmit the motion. This would make it suitable for transmissions to great distances were it not for the necessity of placing the cranks in different planes so that the rods could work without interfering, a condition giving rise to difficulties of construction, for instance, the employment of bent axles. Then, too, the alternate straining of the ropes would give rise to irregularities of transmission like those found in § 123, where rope rods were discussed.

Fig. 531.



Parallel cranks have also been used for transmitting an invariable rotation from one shaft to several others parallel to the first. Let $A D$, $B E$, and $C F$, Fig. 532, be three equal parallel cranks on the shafts A, B, and C; then if we connect their crank pins by the connecting rods $D E$, $E F$, and $F D$ the rotation of any one of the shafts A will be communicated unchanged to the remaining shafts B and C. The mechanism can act in every position, for where two cranks are at their dead points, for example at $A D_1$ and $B E_1$, the rotation of A will be directly transmitted to C, and from the latter rotation will be transmitted to B. Moreover the three coupling rods can be united so as to form a rigid frame $D E F$, an example of which has already been given in Fig. 444, § 112, where it takes the form of a ring.

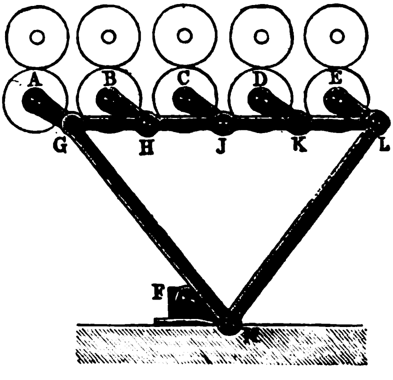
FIG. 532.



This mechanism is repeatedly used in machinery; for example, in communicating circular motion to the disks or basins used in grinding plate-glass. In Fig. 533 is shown the mechanism for communicating rotation to the feed rollers A, B, C, D, E of a certain flax-break, one crank F driving a series of parallel cranks by means of a frame $G L M$.

Kirchmeyer's coupling for locomotives, Fig. 534, is a combination of parallel crank trains, which is intended to satisfy the condition that the distance of the coupled driving axles A and B may vary without disturbing the transmission of motion. Such a variation in the position of the axles would be of advantage in passing around sharp curves. In this arrangement the two perpendicular cranks D and D_1 of the axle A are not directly connected with E and E_1 of the axle B, but are united by a third intermediate axle C and its cranks G and G_1. This axle C is allowed to rise or fall a

FIG. 533.



little in its bearings, thus permitting a variation in the distance between the driving axles.

By the above assumption that the opposite sides of this quadrilateral are equal, another arrangement* is obtained, Fig. 535, in which the cranks $A\,D$ and $B\,C$ cross. Here also there are two positions, $A\,B\,C_1\,D_1$ and $A\,B\,C_2\,D_2$, in which the cranks make equal angles with the base $A\,B$, and in which the coupler $C\,D$ becomes parallel to $A\,B$. For these positions the angular velocities of the two axles are equal, for the distances of the instantaneous centre (P_1 or P_2) from the ends of the coupler, are then equal. To find the velocity ratio for other positions of the system we may construct the centroids. For any position $A\,B\,C\,D$ of the system the instantaneous centre P is at the distances $P\,A = p$ and $P\,B = p_1$ from the axes. Prolong $A\,B$ and $C\,D$ till they intersect at O; then because the triangles $O\,D\,A$ and $O\,B\,C$ are equal, we find $P\,B = P\,D$; hence $P\,A + P\,B = A\,D$, or

$$p + p_1 = r,$$

FIG. 534.

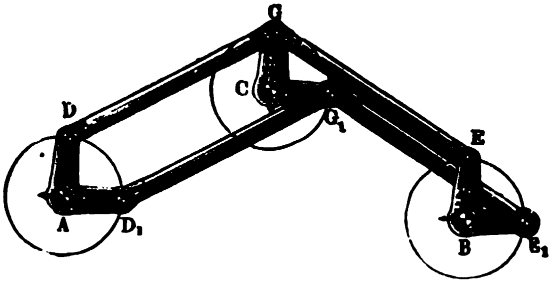
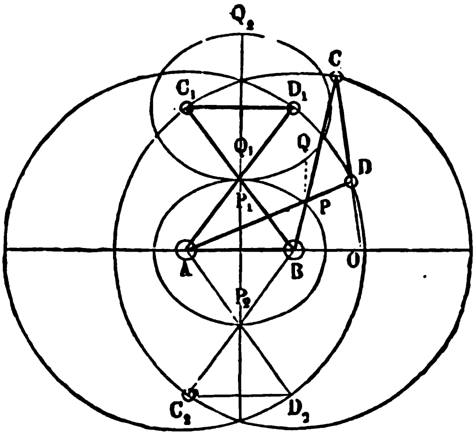


FIG. 535.



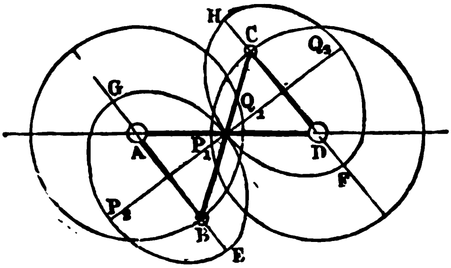
---

* See *Reuleaux*, "Kinematics of Machinery," § 47 and § 67.

where r represents the length $A\,D$ of the crank. Now the constant sum of the distances of the instantaneous centre from the axes means, as is well known, that the instantaneous centre lies on an ellipse whose foci are at the axes, and whose transverse axis $A\,D = r$.

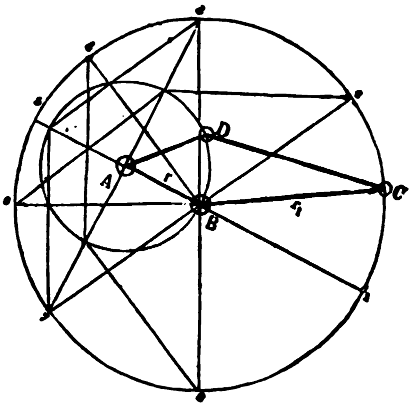
This ellipse is the fixed centroid, and from the symmetry of the mechanism it is evident that the moving centroid attached to the coupler $C\,D$ is also an ellipse equal to the fixed centroid. The foci of the moving ellipse are at the centres of the crank pins. To see this it is only necessary to suppose the coupler $C_1\,D_1$ fixed, and then reason as before. From the known properties of centroids it is evident that we can replace the motion of the whole system by rolling the elliptical centroid $Q_1\,Q\,Q_2$ attached to $C\,D$ on the ellipse $P_1\,P\,P_2$ fixed to the frame, to which case the principles established in § 50 for elliptical wheels are applicable. To bring out more clearly the resemblance to the elliptical gears, let us suppose the whole system in Fig. 535 to receive an additional rotation about A which is at every instant equal and opposite to the rotation of the driving crank $A\,D$. This will bring the crank $A\,D$ to rest, while the formerly fixed link or frame $A\,B$ rotates uniformly around A, *i.e.*, it now becomes the crank. By this *inversion* (see § 31 of the Introduction) there is obtained a new mechanism, Fig. 536, in which the rotating crank $A\,B$ drives the crank $D\,C$ by means of the coupler $B\,C$. Since the additional motion communicated to the whole system has not altered the relative motion of the individual links, the centroids have the same meaning as before, but it is necessary to suppose that the originally stationary ellipse P, attached to $A\,B$, now rotates about A while the moving ellipse fastened to the coupler $C\,D$ is confined to a rotation around its fixed focus D. The angles of velocity $\delta\,\alpha$ and $\delta\,\beta$ of the two axes A and D are equal

FIG. 536.



when the ellipses touch at the vertices P_1, Q_1 or P_2, Q_2 of the conjugate axes. The maximum value of the ratio $\frac{\delta\alpha}{\delta\beta} = \frac{r+e}{r-e}$ obtains when E is in contact with F, while its minimum value $\frac{r-e}{r+e}$ corresponds to contact of the vertices G and H. The two mechanisms Fig. 535 and Fig. 536 differ principally in this, that in Fig. 535 the rotations of the axes take place in the same, and in Fig. 536 in opposite, directions. As the mechanism has two dead points, means must be provided for passing them whenever the mechanism is to be used. *Reuleaux* has shown how this may be done in a simple manner by two pairs of teeth, and has given to the mechanism the name of *anti-pár-allel cranks* (see Der Civil Ingenieur, vol. X. p. 99, 1859).

FIG. 537.



REMARK. An interesting mechanism by *Galloway* * may be obtained by making two pairs of adjacent links equal and the opposite ones unequal, Fig. 537, where $AD = AB = r$ and $DC = BC = r_1$. Here a complete rotation of the crank AD causes only half a rotation in the crank BC, and *vice versa*, as is evident from the positions of the cranks designated by 1, 2, 3, 4, 5 . . 8.

§ 138. The Ordinary Crank Train.

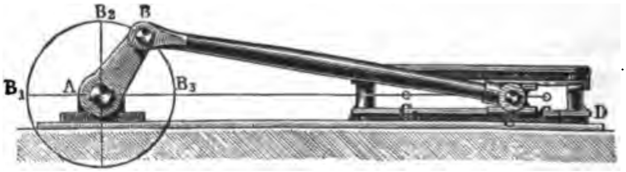
—If, in the most general case of the quadric crank train, we gradually increase the length of the rocker arm, the circular arc in which the end of the connecting rod oscillates will gradually become flatter, and will finally become a straight line when the length of the rocker arm is infinite. Then the rocker arm may be replaced by guides which compel the end of the connecting rod to move in a straight line, and we thus get the ordinary crank train, Fig. 538, which is so extensively used in practice. Here it is almost invariably the case that the direction of the straight path along which the end C of the rod is guided passes through the axis A of the crank, and then the stroke $C_1 C_3$ is exactly equal to the diameter $B_1 B_3 = 2r$ of the crank circle, and the connecting rod swings equally to each side of the stroke. The end C of the connecting rod is usually provided with a *cross-head* sliding in prismatic guides, or it may be guided by any of the other straight-line motions discussed in Chapter III.

---

* See *Reuleaux*, Kinematics of Machinery, p. 197.

When the bearing for the crank shaft A and the guides are rigidly attached to the machine, we have the ordinary crank train so extensively used in steam-engines and pumps, and almost every case where it is desirable to convert rotary motion into rectilinear motion, or the reverse. When used in steam-engines the steam piston becomes a part of the cross-head, which then acts as driver, while in pumps the crank drives the cross-head and the pump piston attached to the latter.

FIG. 538.

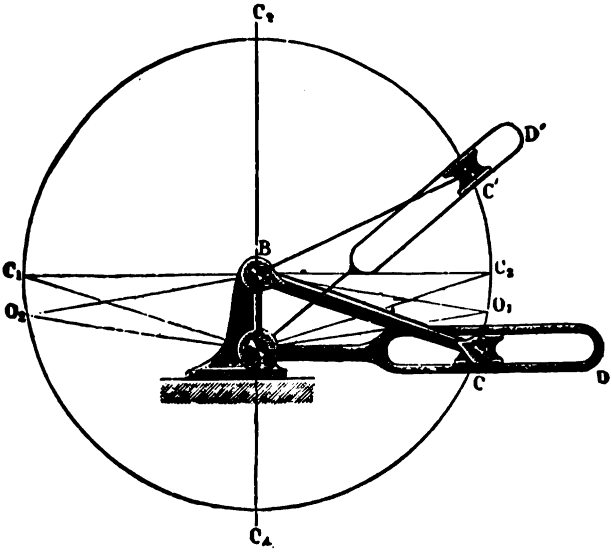


By applying the principle of inversion, § 50 of the Introduction, we may easily deduce from the present mechanism three others, by successively holding fast the three moving links, crank, cross-head, and connecting rod ; it being of course understood that the frame $A\ D$ moves with the guides in these three cases. To obtain these inversions, let us suppose, for example, that the whole system, Fig. 538, receives a rotation equal and opposite to that formerly possessed by the crank. This will render the crank $A\ B$ stationary. The frame $A\ D$ whose form is essentially that of a slotted bar $A\ D$, Fig. 539, now rotates uniformly around the axis A. Evidently the connecting rod $B\ C$ can only rotate about the fixed axis B, but this rotation is not a uniform one when $A\ D$ has a uniform velocity. Finally the cross-head C has a rotation about A in addition to its original reciprocating sliding motion in the bar $A\ D$. If we suppose the bar $A\ D$ to turn about A with the angular velocity

α, the connecting rod $B\,C$ will have at any instant an angular velocity β which depends on the ratio $\frac{A\,C}{B\,C}$ increasing as the distance $A\,C$ increases.

We see at once from the figure that this ratio of the velocities varies between the maximum $\frac{l + r}{l}$ at C_2 and the minimum $\frac{l - r}{l}$ at C_4, r again representing the length of the crank $A\,B$ and l that of the connecting rod $B\,C$. The velocities of the two axes A and B are only equal at the points O_1 and O_2, whose distances from A and B are equal to l. If we draw perpendicular to $A\,B$ the diameter $C_1\,B\,C_3$ of the circle described by C, the ratio of the ordinary angle $C_1\,A\,C_3$ to the reëntrant angle $C_1\,A\,C_3$ will evidently measure the ratio of the times needed by the rod $B\,C$ to respectively traverse the upper and lower half of its circle, provided the slotted bar $A\,D$ turns uniformly. On account of this property it has been used as a quick return motion for planing machines. In like manner by

FIG. 539.



communicating a rotation around *B* equal and opposite to the rotation of the connecting rod *B C*, the latter is rendered stationary, and a new mechanism, Fig. 540, arises, in which the continuous rotation of the crank *B A* communicates to the slotted bar *A D* an oscillating motion around *C*, and at the same time a reciprocating sliding motion. This mechanism constitutes the main train of oscillating steam-engines, the alternate sliding of the bar through its centre of oscillation *C* being then represented by the piston motion. But this same mechanism is also employed as a quick return motion in planers, use being made of the oscillating motion of the slotted link *C D*, Fig. 541, to communicate to the tool-holder a slow advance and a quick return by the help of the rod *E*. The fact that the slotted bar *C D* encloses the crank pin *A* with its slot, instead of the centre of oscillation *C*, has no influence on the motion, for the object of the slot is only to render possible the variation of distance between *A* and *C*. Moreover, when the crank *B A* turns uniformly we see that here also the times needed for the forward and return motions of the tool-holder *E* are to each other as the reëntrant angle $A_1 B A_2$ to the ordinary angle $A_1 B A_2$.

FIG. 540.

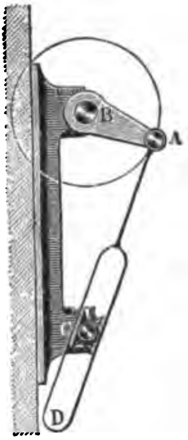
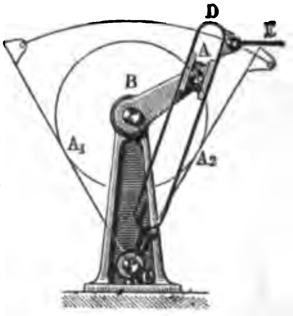


FIG. 541.



Finally, if we render the cross-head stationary by giving to the crank mechanism, Fig. 538, a reciprocating rectilinear motion which at every instant is equal and opposite to the motion of the cross-head, we obtain the mechanism sketched in Fig. 542, where the crank *A B* in addition to its rotation about *A* has an oscillation $A_1 A_2 = 2r$ in the direction *A C*. The connecting rod *B C* is compelled to swing about the stationary cross-head, the latter having here the form of a hollow cylinder, so that the rod *A D* has no longer the form of a slotted bar, but now appears as a solid cylinder sliding in a hollow one. This particular form of the parts of the mechanism is not essential, and evidently does not alter the character of the motion. One of the very few applications of this mechanism is shown in Fig. 543, which represents a disk-polishing machine in which the polishing wheel *W* has a reciprocating as well as a rotary motion. The shaft *E* of the wheel slides in the bearing *F*, and is provided with an endless screw *S* which communicates continuous rotation to the wheel or shaft *A*. This causes the crank *A B* to give the shaft *A* a reciprocating motion whose stroke is $2r$, the traversing motion being communicated to the polishing wheel by the fork-shaped piece *G E*.

FIG. 542.

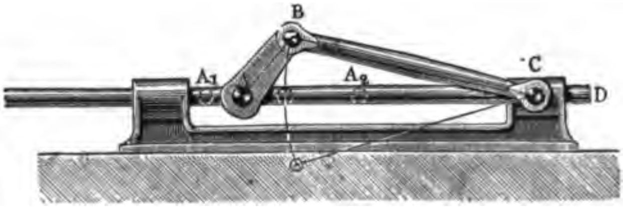
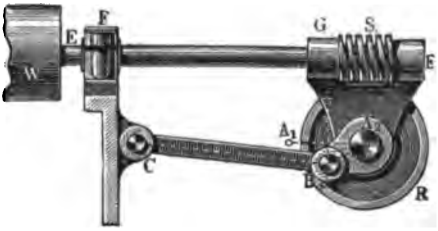
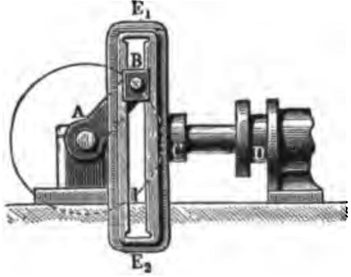


FIG. 543.



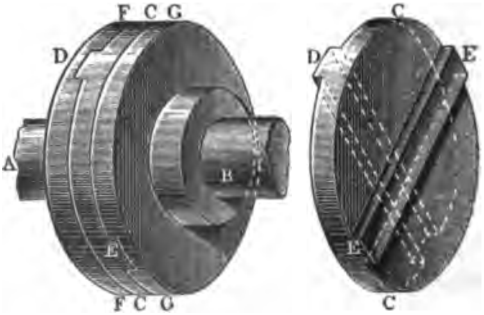
In a previous article the ordinary crank train was deduced from the quadric crank train by assuming the rocker arm infinite, *i.e.*, by substituting for the rocker arm a sliding block whose direction passed through the crank shaft. In a similar manner the connecting rod may be assumed to be infinite by compelling the crank pin to move in a straight path, *i.e.*, by guiding it in a slot perpendicular to the cross-head. In this way is obtained the mechanism shown in Fig. 544, which is often applied in steam pumps and elsewhere. Here the crank shaft-bearing *A* and the guide *D* are united by the frame, and the rotating crank *A B* communicates to the slotted rod *C* the reciprocating motion 2 *r* in the direction *A D*. This is a very compact arrangement of the crank train, but involves a great deal of friction at the slot *E*.

FIG. 544.



By fastening the different links of this crank train we get other mechanisms. For example, if we render the crank *A B* stationary by giving to the whole system a rotation about *A*, we obtain a mechanism in which the frame *A D* and the slide *C* turn about the crank *A B* in such a way that the axis of the cylinder *D* always passes through the fixed point *A* and the direction of the slot *E* always passes through the fixed point *B*. Consequently this mechanism, Fig. 545, is the same as that described in § 25 as *Oldham's* coupling. Here the disk fastened to the shaft *A* and containing the groove *D* corresponds to the frame *A D* rotating about *A*, Fig. 544, while the disk, fastened to the other shaft *B* and containing the groove *E*, corresponds to the sliding block on the crank pin *B*, Fig. 544. Finally, the middle disk *C*, carrying the projecting bars that fit the grooves *D* and *E*, is identical with the slotted link *E* and its rod *C*. It was proved in § 25, in discussing the motions of this mechanism, that a uniform rotation of the shaft

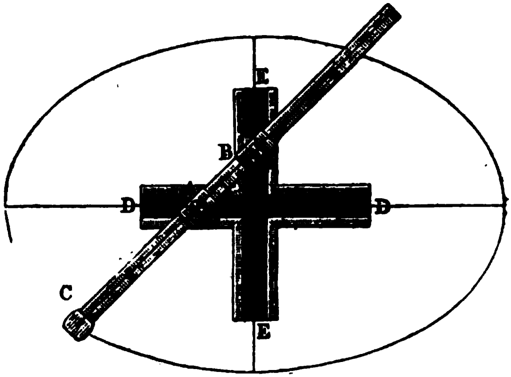
FIG. 545.



A was accompanied by a uniform rotation of the other shaft B.

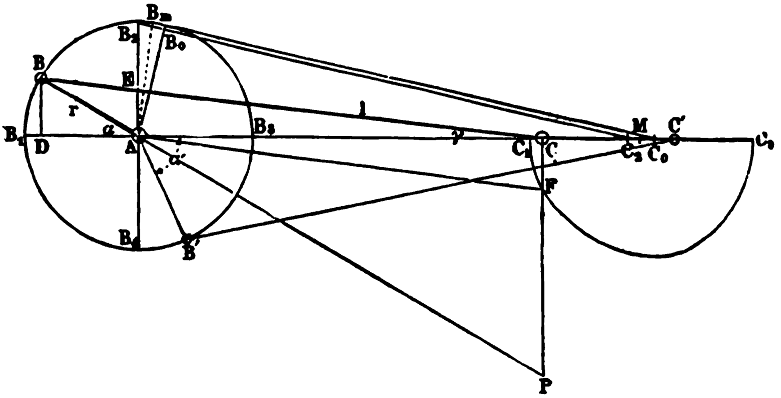
Again, if in the present mechanism we hold fast the slotted link $C E_1 E_2$, Fig. 544, which may be regarded as a link having two prisms at right angles to each other, we get the mechanism shown in Fig. 546. Here the two pins A and B are at a constant distance apart, and are constrained to move in two guides $E E$ and $D D$ at right angles to each other. This form of motion was examined in § 11 of the Introduction, where it was shown that every point C in the straight line A B describe an ellipse, whose axes are $2a$ and $2 a + r$, and whose directions coincided with that of the guides. It is evident that this mechanism is the same as that of a well-known ellipto-graph; the "elliptic chuck" invented by *Leonardo da Vinci* is a very remarkable application of this mechanism.

FIG. 546.



§ **139. The Motions in the Crank Train.**—In order to investigate the motions of the crank train, let A $B = r$, Fig. 547, be the length of the crank and B $C = l$ the length of the connecting rod. Moreover, suppose the direction $C_1 C_3$ of the stroke of the cross-head C to pass through the axis A, and let the dead point B_3, which lies between the axis A and the guide $C_1 C_3$, be known as the *inner* and B_1 as the *outer* dead point. Furthermore, suppose the crank A B to have the right-handed rotation $B_1 B_2 B_3 B_4$, in the direction of the hands of a watch, and let the half rotation $B_1 B_2 B_3$, during which the cross-head *recedes* from the crank shaft, be known as the *forward* motion and the half rotation $B_3 B_4 B_1$, during which the cross-head *approaches* the crank shaft, as the *return* motion. The angle of rotation α of the crank for any position $A B$ of the crank is to be estimated from the dead point from which the crank starts,—from B_1 for the forward motion and from B_3 for the return motion. Finally, let s be the stroke of the cross-head, that is, the distance that it travels beyond C_1 during the forward motion and the distance that it recedes from C_3 during the return motion. At present no attention will be paid to the masses of the crank, connecting rod, etc., the influence of the masses on the motion being left for special investigation later.

FIG. 547.



If the crank $A B$ has turned through the angle $B_1 A B = \alpha$ during the forward motion, the path $C_1 C = s$ of the cross-head may be found from the relation

$$r \cos \alpha + A C_1 + s = l \cos \gamma,$$

where γ is the angle at which the connecting rod is inclined to the stroke $A C$. Since $A C_1 = l - r$, the above expression may also be written as follows:

$$r \cos \alpha + l - r + s = l \cos \gamma$$

or

$$s = r (1 - \cos \alpha) - l (1 - \cos \gamma).$$

It is easy to see that for the return motion, when the crank has moved through the angle $B_3 A B' = \alpha'$, the path of the cross-head $s' = C_3 C'$ may be found from

$$r \cos \alpha' + l \cos \gamma' + s' = r + l$$

or

$$s' = r(1 - \cos\alpha') + l(1 - \cos\gamma').$$

We can therefore write for the general formula

$$s = r(1 - \cos\alpha) \mp l(1 - \cos\gamma),$$

the upper sign referring to the forward stroke and the lower one to the return stroke. To determine γ, we have from the figure

$$r\sin\alpha = l\sin\gamma,$$

from which follows

$$\sin\gamma = \frac{r}{l}\sin\alpha \quad \text{and} \quad \cos\gamma = \sqrt{1 - \frac{r^2}{l^2}\sin^2\alpha}.$$

As l is always considerably greater than r in actual constructions, for instance, $\frac{r}{l}$ is seldom greater than $\frac{1}{5}$, we can with sufficient exactness make

$$\cos\gamma = \sqrt{1 - \frac{r^2}{l^2}\sin^2\alpha} = 1 - \frac{r^2}{2l^2}\sin^2\alpha;$$

and after substitution we obtain

$$s = r(1 - \cos\alpha) \mp \frac{r^2}{2l}\sin^2\alpha.$$

Of course when $\alpha = 0$ we also have $s = 0$. But when $\alpha = 90°$ we have

$$s = r \mp \frac{r^2}{2l} = r\left(1 \mp \frac{r}{2l}\right).$$

The path described by the cross-head during a quarter revolution on the forward motion is smaller than the half stroke r by the amount $MC_2 = \frac{r^2}{2l}$; on the other hand, the path of the cross-head for a quarter revolution on the return motion is greater than a half stroke by the same amount, so that the cross-head always occupies the same position C_2 when the crank passes the points B_2 and B_1, *i.e.*, when it passes the diameter perpendicular to the stroke. We see from this that the motion of the cross-head differs during the forward and return stroke, the difference increasing with the ratio $\frac{r}{l}$. For instance, when $\frac{r}{l} = \frac{1}{5}$, the paths described during a quarter revolution on the forward and return motion respectively are $0.9\,r$ and $1.1\,r$. For an infinitely long connecting rod the formula becomes simply $s = r\,(1 - \cos\alpha)$ both for the forward and the return stroke; in this case the two motions are alike.

For the middle position M of the cross-head, *i.e.*, for $s = r$, we obtain from

$$r \cos\alpha \pm l = \pm l \cos\gamma = \pm \sqrt{l^2 - r^2 \sin^2\alpha}$$

or

$$r^2 \cos^2\alpha \pm 2\,l\,r \cos\alpha + l^2 = l^2 - r^2 + r^2 \cos^2\alpha$$

the angle of rotation α:

$$\cos\alpha = \mp \frac{r}{2\,l}.$$

If we substitute $\frac{r}{l} = \frac{1}{5}$, we find from this that $\alpha = 95°\,45'$ for forward stroke and $\alpha = 84°\,15'$ for return stroke.

The crank position B_0, in which the connecting rods are at right angles to the crank, is of some interest. For this case we have from the figure

$$\tan\gamma = \mp \cot\alpha_0 = \frac{r}{l}.$$

For example, when $\frac{r}{l} = \frac{1}{5}$, $\alpha_0 = 101°\,20'$ for the forward motion and $78°\,40'$ for the return motion. The path $C_1\,C_0$ or $C_3\,C_0$ which is traversed by the cross-head before reaching this crank position can be found from the general formula by substituting these values of α_0; we then have, when $\frac{r}{l} = \frac{1}{5}$,

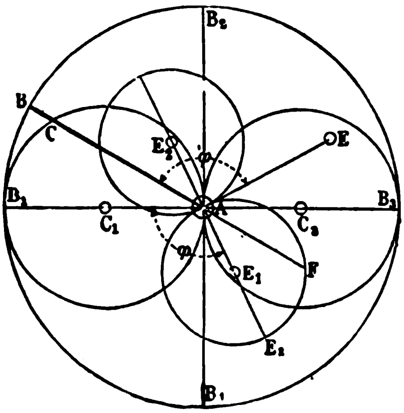
$$s = 1.10\,r \text{ for the forward motion,}$$
$$s = 0.90\,r \text{ for the return motion.}$$

In this case the point C_0 lies as far back of the centre M as the point C_2, corresponding to the 90° crank position, lies in front of the centre. With an infinitely long connecting rod, the corresponding crank positions B_m and B_0 coincide with the middle position B_2, and C_2 and C_0 coincide with the centre M of the stroke.

In many of the constructions occurring in practice the length l of the connecting rod is so great relatively to the length r of the crank that we may neglect the small angle of deviation γ, whose greatest value is given by $sin\ \gamma = \frac{r}{l}$. This holds especially for the eccentric used in moving the valves of steam-engines, for eccentrics are really nothing but cranks. In such cases, therefore, we may always assume that the connecting rod is infinitely long, the motion of the cross-head or valve rod being expressed by the simple formula $s = r(1 - cos\ \alpha)$. We now see that this travel can be directly found from the crank circle for any position $A\ B$ of the crank, by finding the distance of the projection D of the crank pin on the diameter $B_1 B_3$ from the dead point, for in every case $B_1 D = r(1 - cos\ \alpha)$.

The travel of the cross-head for every crank position may also be found graphically, the required distance being picked off from the diagram without dropping any perpendicular $B\ D$. For if we suppose two circles C_1 and C_2, Fig. 548, to be described on those two radii $A\ B_1$ and $A\ B_3$ of the crank circle, which correspond to the dead points of the crank, then for any position $A\ B$ of the crank, the chord $A\ C$ will give the distance of the cross-head from its *middle* position, for $A\ C = A\ B_1\ cos\ \alpha = r\ cos\ \alpha$. The circle C_1 can thus be used for the motion of the crank in the semicircle $B_4 B_1 B_2$, *i.e.*, while the cross-head moves to one side of its middle position and the second circle C_2 can be used for the semicircle $B_2 B_3 B_4$ of the crank, *i.e.*, for the motion on the other

FIG. 548.



side of the middle position. Now if we suppose an eccentric to be placed on the crank shaft A or a second crank $A E$ placed there, which makes with the crank $A B$ an angle $B A E = \phi$, we can determine in a similar manner the travel of the cross-head or of the valve rod connected with the second crank. If for this purpose we pass through A two circles whose centres lie on $B_1 B_3$, on opposite sides of A, and whose diameters equal the length of the crank $A E$, *i.e.*, equal the eccentricity, the path of the valve for every crank position $A B$ will be found in that chord of these circles which is in advance of the crank position $A B$, by the angle $B A E = \phi$. To avoid laying off this angle ϕ for every crank position $A B$, the two circles of the eccentric may be set back an amount equal to this angle ϕ, whereupon their centres E_1 and E_2 will fall into that diameter $E_1 A E_2$ of the crank circle which makes with the stroke $B_1 B_3$ the angle $B_1 A E_1 = \phi$. We now see that the chord $A F$ of these circles, which coincides with the direction of the crank $A B$, at once gives the distance of the valve moved by the eccentric $A E$ from its middle position, at the instant when the crank is in the position $A B$. We also see from the figure that the length of this chord is given by $A F = A E_3 \cos F A E_3 = - e \cos (\alpha + \phi)$, where e is the eccentricity $A E = A E_3$ of the eccentric. But this quantity gives the distance of the valve rod from its middle position when the shaft or its crank has moved through its angle α, and the angle of advance of the eccentric is ϕ. This property has been used with great advantage by *Zeuner* to represent graphically the motion of the valves of steam-engines.* The travel of the valve from its centre is of course a maximum when it is equal to e, *i.e.*, when the crank has turned through the angle $B_1 A E_2 = 180° - \phi$, and becomes equal to 0 when the crank has turned through 90° more.

From the general formula for the travel s of the cross-head.

$$s = r(1 - \cos \alpha) \mp \frac{r^2}{2 l} \sin^2 \alpha,$$

---

* See *Zeuner's* Valve Gears.

we can obtain the velocity c of the cross-head when the crank A turns uniformly. For according to § 20, Vol. I,

$$c = \frac{\delta s}{\delta t} \quad \text{and} \quad \omega = \frac{\delta \alpha}{\delta t} = \text{angular velocity;}$$

hence, by differentiation and substitution,

$$c = \omega \frac{\delta s}{\delta \alpha} = \omega \left(r \sin \alpha \mp \frac{r^2}{2\,l} 2 \sin \alpha \cos \alpha \right)$$

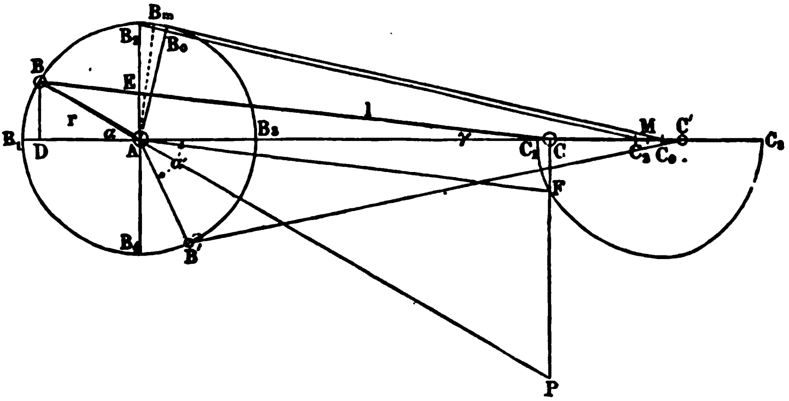
$$= \omega\, r \left(\sin \alpha \mp \frac{r}{2\,l} \sin 2\,\alpha \right).$$

As the constant circumferential velocity v of the crank pin is expressed by $r\,\omega$, the ratio of the velocity of the cross-head to that of the crank pin, for any angle α, will be given by

$$\nu = \frac{c}{v} = \sin \alpha \mp \frac{r}{2\,l} \sin 2\,\alpha.$$

The velocity c is equal to 0 when $\alpha = 0$ and when $\alpha = 180°$,

FIG. 549.



i.e., when the crank is at its dead points. For $\alpha = 90$, *i.e.*, for the crank positions B_2 and B_4, Fig. 549, we have $\frac{c}{v} = 1$. In other words, for the crank positions perpendicular to the stroke, the cross-head has the same speed as the crank pin, whatever the ratio $\frac{r}{l}$ may be. Moreover, it is clear that this holds even when the crank shaft has a variable rotation.

When the velocity c for the middle position M of the cross-head is to be found, we substitute in the above formula the corresponding value of α, which value was found above to be

$$\cos\alpha = \mp\frac{r}{2l} \quad \text{and} \quad \sin\alpha = \sqrt{1 - \frac{r^2}{4l^2}}.$$

Substituting above, we have

$$v = \sin\alpha \mp \frac{r}{2l}\sin 2\alpha = \sin\alpha\left(1 \mp \frac{r}{l}\cos\alpha\right)$$

$$= \sqrt{1 - \frac{r^2}{4l^2}}\left(1 + \frac{r^2}{2l^2}\right).$$

Neglecting the higher powers of $\frac{r}{l}$, this becomes

$$v = \frac{c}{v} = 1 + \frac{3}{8}\frac{r^2}{l^2};$$

for example, when $\frac{r}{l} = \frac{1}{5}$ we have $v = 1.015$.

The maximum piston velocity c can be found from

$$\frac{\delta c}{\delta \alpha} = 0,$$

i e., from

$$\cos\alpha - \frac{r}{l}\cos 2\alpha = 0.$$

This may be transformed into

$$\cos\alpha = \frac{r}{l}(2\cos^2\alpha - 1),$$

hence

$$\cos\alpha = \frac{l}{4r} - \sqrt{\left(\frac{l}{4r}\right)^2 + \frac{1}{2}}.$$

It will be sufficiently exact to substitute for this

$$\cos\alpha = -\frac{r}{l};$$

for example, when $\frac{r}{l} = \frac{1}{5}$, we have $\alpha = 101^\circ 30'$.

Substituting the value of *cos* α, we get for the maximum velocity of cross-head or piston

$$c \max = v\left(\sin\alpha - \frac{r}{2l}\sin 2\alpha\right)$$

$$= v\sqrt{1-\left(\frac{r}{l}\right)^2}\left(1+\frac{r^2}{l^2}\right) = v\left(1+\frac{r^2}{2l^2}\right).$$

Consequently when $\frac{r}{l} = \frac{1}{5}$ we have

$$c \max = v\left(1 + \tfrac{1}{2}\tfrac{1}{25}\right) = 1.02\,v.$$

The average piston speed per revolution can evidently be found from

$$\frac{c}{v} = \frac{4r}{2\pi r} = 0.6366.$$

A clear idea of the piston speed c at different points can be obtained by graphical means, for if we erect a perpendicular to the stroke C_1C_3 at any position C of the cross-head, the intersection P of this normal with the corresponding direction of the crank will give the instantaneous centre about which the connecting rod BC turns at the instant. As the velocities c and v of the ends C and B of the connecting rod are to each other as the instantaneous radii PC and PB, we have in every case

$$c : v = PC : PB = FC : AB,$$

when AF is drawn through the axis A parallel to the direction of the connecting rod. Therefore, if we assume the radius $AB = r$ of the crank as a measure of the circumferential velocity v of the crank pin, we shall get at CF or AE the value of the instantaneous piston speed c. As the line AE lies on the radius perpendicular to the stroke C_1C_3, *i.e.*, on the line parallel to the infinitely long rocker that has been replaced by the slide C_1C_3, we see that this relation agrees with that established in § 135 for the general crank train. Furthermore, we see that the velocity c is not only equal to the velocity v of the crank pin when the latter is at B_2, but also for another position B_5, for which the direction of the connecting rod likewise passes through B_2. For all positions of the crank pin between

B_2 and B_5 the piston speed e is greater than the speed of the crank pin, because the distance AE for all these points is greater than the crank arm r. In all the remaining positions of the crank along the semicircle $B_1 B_2 B_3$ the speed c is smaller than v. The same remarks apply to the return motion of the crank.

If the lines $CF = c$ be found as above for every possible crank position, the points F will form a continuous curve $C_1 F C_3$, each of whose ordinates directly measures the velocity of the piston at the instant when the cross-head passes through the foot C of the ordinate. From what has preceded, we know that this curve becomes a semicircle described on the stroke $C_1 C_3$ when the connecting rod is infinitely long, and that this semicircle will cut at two points the curve F obtained for a finite length of connecting rod. In all those cases in which length of connecting rod relatively to crank is not very small, we can assume that the maximum piston speed corresponds very closely to that crank position for which the connecting rod is tangent to the crank circle, *i.e.*, to the position B_0 of the crank pin, for which we found above

$$\cot \alpha = -\frac{r}{l}.$$

Calculation gave for this angle, corresponding to the maximum of velocity, the approximate value

$$\cos \alpha = -\frac{r}{l};$$

but, as this angle differs but little from 90°, it will be sufficiently accurate to place the cosine equal to the cotangent. How close this approximation is, becomes evident when we substitute $\frac{r}{l} = \frac{1}{5}$, for then

$$arc \cot \alpha - \frac{r}{l} = 101^\circ 20'$$

and

$$arc \cos \alpha - \frac{r}{l} = 101^\circ 30'.$$

Of course the foregoing remarks hold only under the supposition that the crank turns with a constant angular velocity, and when this is not the case the velocity c must be determined from the variable velocity v, according to the proportion

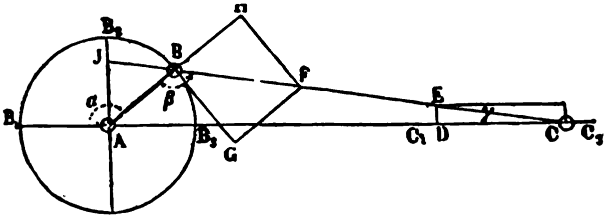
$$\frac{c}{v} = \frac{AE}{AB}.$$

The variation in the action of the force or resistance Q of the piston when transmitted to the crank pin is closely connected with the conversion of the velocity v of the crank pin into the velocity c of the piston rod. If we suppose a piston force $Q = CD$, Fig. 550, to act at the cross-head C in the direction of the stroke $C_1 C_3$, this force can be resolved into a component $ED = Q \tan \gamma$ perpendicular to the guide, and another component

$$CE = \frac{Q}{\cos \gamma},$$

coinciding with the connecting rod, γ again representing the inclination of the connecting rod to the guide. The latter force CE at the crank AB can be resolved into two components, one radial and the other tangential, the radial component $HB = FB \cos \beta$ pressing the shaft A against its bearings and generating friction, while the tangential component $GB = FB \sin \beta$ becomes the circumferential force U, which tends to help or hinder the rotation of the crank shaft according as the piston pressure Q drives as in steam-engines, or resists as in pumps. In all cases the circumferential force U will perform work during the motion, which work, friction being neglected,

Fig. 550.



must equal that performed by the force at the piston. This circumferential force $G B = U$ is according to the figure:

$$U = F B \times \cos(\beta - 90°) = \frac{Q}{\cos \gamma} \sin \beta.$$

If we prolong the connecting rod $C B$ to its intersection J with the radius perpendicular to the stroke, we have from the triangle $A J B$

$$A J = A B \frac{\sin \beta}{\cos \gamma} = r \frac{\sin \beta}{\cos \gamma};$$

and from this we may conclude that the distance $A J$ measures the circumferential force U, provided the radius r of the crank is regarded as the measure of the instantaneous force Q at the piston. When this force Q is constant, the circumferential force may be found by the same construction before given for determining the velocity c from the constant velocity v of the crank pin. When the force on the piston varies, it must be reduced according to the proportion

$$\frac{U}{Q} = \frac{A J}{r},$$

in the manner already indicated for the velocity c. Consequently the velocity curve found in Fig. 549 will also measure the variation of the circumferential force U. This agreement between the laws of force and of velocity may be at once deduced from the principle of virtual velocities, according to which $Q c = U v$ even when the velocities and forces are not uniform. From this at once follows that when there is a constant force Q the average circumferential force becomes

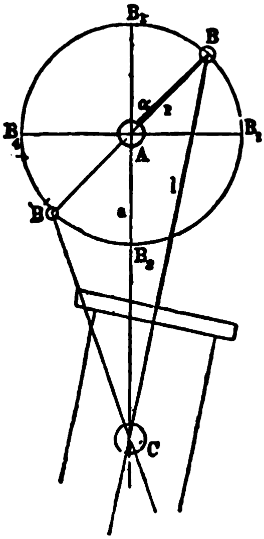
$$U = Q \frac{4 r}{2 \pi r} = 0.6366 \, Q.$$

For the case in which the force Q on the piston is variable, it is of course necessary that the work A during a revolution be determined according to the laws of variation of the force on which it depends, and then the average circumferential force may be found from

$$U = \frac{A}{2 \pi r}.$$

This will be more fully discussed hereafter.

FIG. 551.



§ 140. The Oscillating Crank Train.—The above developments for the ordinary crank train apply also to the crank train with swinging slot or block used in oscillating steam-engines; this may be shown as follows: Let $a = A\,C$, Fig. 551, be the distance of the crank shaft A from the axis of oscillation C of the cylinder. When the crank turns through the angle $B_1 A\,B = \alpha$ estimated from the outer dead point, it will cause a travel of the piston represented by

$$s = B_1 C - B\,C = a + r - \sqrt{a^2 + 2\,a\,r \cos \alpha + r^2},$$

or approximately

$$s = a + r - a\left(1 + \frac{r \cos \alpha}{a} + \frac{r^2}{2\,a^2} - \tfrac{1}{2} \times \tfrac{1}{2}\,\frac{4\,r^2 \cos^2 \alpha}{2\,a^2}\right)$$

$$= r(1 - \cos \alpha) - \frac{r^2}{2\,a}(1 - \cos^2 \alpha) = r\,(1 - \cos \alpha) - \frac{r^2}{2\,a} \sin^2 \alpha.$$

In like manner for the return stroke of the piston, when the crank has moved from the inner dead point B_3 through the angle $B_3 A\,B' = \alpha$, we find

$$s = B'C - B_3\,C = \sqrt{a^2 - 2\,a\,r \cos \alpha + r^2} - (a - r),$$

or approximately,

$$s = r(1 - \cos \alpha) + \frac{r^2}{2\,a} \sin^2 \alpha;$$

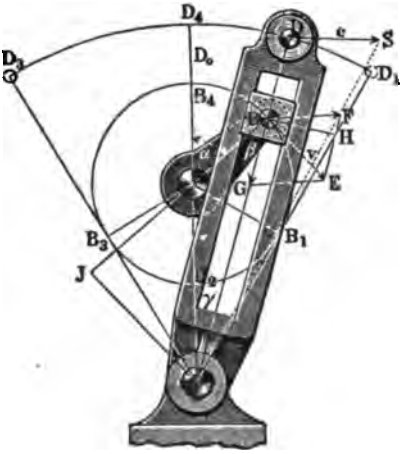
so that the general formula for the piston motion becomes

$$s = r\,(1 - \cos \alpha) \mp \frac{r^2}{2\,a} \sin^2 \alpha.$$

This expression agrees with that found in § 139 for the crank train, when we substitute in the latter for the length l of the connecting rod the distance a of the two bearings A and C. This also corresponds to the deduction of this oscillating train from the ordinary one, by rendering the connecting rod of the latter stationary. Consequently all conclusions with reference to the velocities of the ordinary crank train hold for the oscillating train.. This arrangement is a very compact one, and for this reason only it is occasionally chosen for steam-engines, for there are decided disadvantages connected with its use, due, for instance, to the oscillating motion of such a heavy body as the steam cylinder necessarily is.

On the other hand, the oscillatory motion of the piston is repeatedly employed in certain kinds of machinery, the inequalities of its motion being turned to advantage. For example, it is often applied in shaping machines and small planers, in which the work or tool receives the slow forward motion required by the cutting process, and the quick return motion needed to economize time. In all such machines the crank shaft is the driver, whose rotation gives to the part in question a regular swinging motion. Here we may always regard the rotation of the crank as a perfectly uniform one, for its connection with the driving gear of the shop unites it directly and indirectly with such great masses that the inequalities of resistance in the little planing machine can have practically no effect in producing variations of velocity. In this case it is of interest to examine the oscillatory motion in this train, under the supposition that the crank turns uniformly. Let $r = AB$, Fig. 552, be the length of the crank, $a = AC$ the distance between the axis, and $l = CD$ the length of the swinging bar, whose outer end D may be regarded as connected with the rod DS which moves the slide of the planing machine to and fro. Let the angle of rotation α of the crank be now estimated from the middle position CB_4, in which the slotted link coincides with the direction of the crank so that $B_4AB = \alpha$. Let the angles at the vertices B and C of the triangle ABC be represented by β and γ respectively. In the first place, we see that the link CD in its extreme positions touches the crank circle at B_1 and B_3, and these two positions would correspond to the dead points of the system if the link were the driver, but this would very rarely occur. The time needed by the oscillating link to pass from D_3 to D_1 is to the time needed for the return motion as the arc $B_3\ B_4\ B_1$ is to $B_1\ B_2\ B_3$, which arcs measure the corresponding rotations of the crank. The direction of motion of the slide is so taken that it bisects the rise $D_0\ D_4$ of the arc $D_3\ D_1$, and is perpendicular to the middle position $C\ D_4$. As this rise is small compared with the length of the connecting rod $D\ S$, the direction of the latter may be regarded as remaining parallel to itself, the slight inclination of the rod $D\ S$ to the stroke of the slide being neglected. Under this supposition the velocity c of the slide or of the end D of the link may be found as follows: If we resolve the constant velocity $v = B\ E$ of the crank pin into two components, one BG in the direction of the swinging link, and the other BF in the direction of the usually horizontal stroke of the slide, we get for the velocity c of the slide

FIG. 552.



$$c = \frac{D\ C}{B\ C}\ B\ F.$$

Now with $B\ H \perp B\ C$,

$$B\ F = \frac{B\ H}{\cos\gamma} = v\,\frac{\cos\beta}{\cos\gamma};$$

hence

$$c = \frac{D\ C}{B\ C\cos\gamma}\ v\cos\beta = \frac{l}{a + r\cos\alpha}\ v\cos\beta.$$

On the other hand, when CJ is perpendicular to AB we obtain for $\cos\beta$

$$\cos\beta = \frac{BJ}{BC} = \frac{r + a\cos\alpha}{\sqrt{a^2 + 2ar\cos\alpha + r^2}},$$

so that

$$c = v\,\frac{r + a\cos\alpha}{a + r\cos\alpha}\;\frac{}{\sqrt{a^2 + 2ar\cos\alpha + r^2}}.$$

From this we obtain for $\alpha = 0$,

$$c\ \text{min} = v\,\frac{l}{a + r} = v\,\frac{CD}{CB_4};$$

and for $\alpha = 180°$,

$$c\ \text{max} = -\,v\,\frac{l}{a - r} = -\,v\,\frac{CD}{CB_2};$$

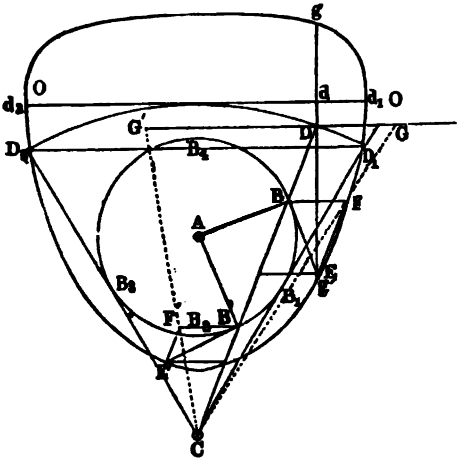
while for the crank positions B_1 and B_3, where $\beta = 90°$, we have $c = 0$.

We can best obtain an idea of the variation of the velocity c by a graphical representation such as is given in Fig. 553. To construct this figure, proceed as follows: If the ratio n of the duration of the forward stroke to that of the return stroke is given, divide the crank circle at B_1 and B_3 in this ratio, making

$$B_1B_2B_3 = \frac{1}{n}\,B_3B_4B_1.$$

Draw tangents at B_1 and B_3, and prolong them till they intersect at C; this point will be the axis of the link whose length CD may then be determined from the known stroke D_1D_3 of the slide. Now for any position CD of the link lay off at B and B' the velocity $v = BE = B'E'$ of the crank pin, and at B, B', and D, the direction of the stroke of the slide; also draw EF and $E'F'$ parallel to the link. If we now draw through C the rays CF and CF', we shall get at DG and DG' the velocities for the forward and return stroke of the slide for the assumed position of the swinging link. If we repeat this construction for a sufficient number of positions of the system, and lay off the velocities $D G = d g$ and $D G' = d g'$ at d on opposite sides of an axis $O O$ parallel to the stroke of the slide, we shall get the curve $d_1 g d_3 g' d_1$, representing the variations of velocity of the slide. The diagram, Fig. 553, shows that the motion of the slide during its forward stroke is very nearly uniform, and the velocity v of the crank must of course be so chosen that the average velocity during the forward stroke will have that value which experience shows to be most advantageous for the process of planing. We also see that the acceleration of the slide is very rapid, and the diagram enables us to judge how well the mechanism here considered is suited for obtaining the desired object.

FIG. 553.



§ **141. Crank with Variable Velocity.**—If, as we have assumed hitherto, the crank shaft turns with a uniform velocity, we know that the velocity c of the cross-head will vary with the position of the crank, between the limits 0 at the dead point and the crank-pin velocity v at the middle positions of the crank, the connecting rod being regarded as infinitely long. This assumption gives us for any crank position the distance of the piston from the outer dead point equal to

$$s = B_1 D = r (1 - \cos \alpha),$$

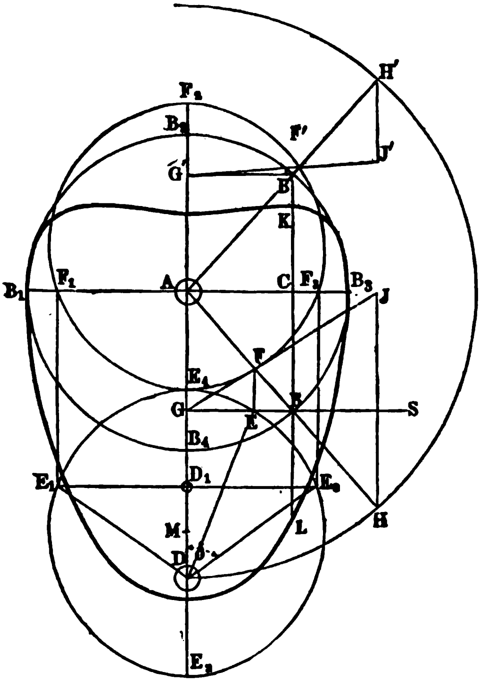
and from this the velocity of the piston,

$$c = v \sin \alpha,$$

v representing the velocity of the crank pin. We may therefore, as already mentioned, regard the crank circle as a diagram in which the ordinates from the crank pin perpendicular to the direction of the stroke give the velocity $v \sin \alpha$ of the cross-head for any crank angle α, provided the radius of the crank r is taken as the measure of the velocity of the crank pin. This will help us to understand how the variation in the velocity of the rod may be represented graphically when the driving crank-shaft turns with a variable velocity. Such cases repeatedly occur in different machines, and mention was made in § 50 of two cases of this kind, when non-circular wheels were discussed.

For giving a slow advance and quick return to the slides of planing machines, a crank $A\,B$, Fig. 554, is often used, which receives its variable motion from a uniformly rotating shaft D, through the medium of a pair of elliptical wheels E and F. The arrangement should be such that the crank will coincide with the direction $A\,B_4$ of the major axis of the ellipse F, the slide being moved to and fro through the distance $B_1\,B_3 = 2\,r$, by the connecting rod $B\,S$. If the rotation about A were uniform, we know from the remarks at the beginning of the article that the perpendicular $B\,C$ could be taken as the measure of velocity. But as these wheels are elliptical, this is not the case; the rotation of the shaft D, at the instant when the crank occupies the position $A\,B$, being increased in the ratio

FIG. 554.



of the radii $\frac{DE}{AF}$, for at this instant two points of the ellipse symmetrical at F and E to AD come in contact. To find the instantaneous motion c of the slide, it is therefore only necessary to increase the length BC in the ratio $\frac{DE}{AF}$, the following construction serving this purpose: Draw $BG \perp AD$; lay off on the prolongation of the crank $FH = ED$, which may be conveniently done by the help of a circle described about A through D; join G with F, and draw through H a line HJ parallel to AD till it intersects GF at J. We then have

$$HJ = \frac{FH}{AF} AG = \frac{DE}{AF} BC,$$

as the velocity of the slide for the crank position AB of the return stroke. It is evident that when the slide occupies the same position on the forward stroke, *i.e.*, when the crank is at AB', the velocity of the slide is given by $H'J'$, which may be found by drawing $B'G'$ perpendicular to AD, then uniting G' with F', and drawing through the intersection H' of the crank with the circle H a parallel $H'J'$ to AD. Now if we lay off from C on opposite sides of B_1B_3 the velocity $CK = J'H'$ for the forward stroke, and the velocity $CL = JH$ for the return stroke, the closed curve $B_1KB_3LB_1$ thus obtained gives the diagram for the velocities of the slide. From the similarity of this curve to that drawn in Fig. 553, for the train with the oscillating link, we see that the mechanism here considered may likewise serve as a quick return motion.

To find the ratio of the times occupied in making the forward and return stroke of the slide in this mechanism, it is only necessary to consider that while the crank shaft A makes a half revolution $F_1F_2F_3$ for the forward stroke, the driving shaft D turns through the arc $E_1E_2E_3$, F_1E_1 and F_3E_3 being parallel to AD; in like manner the arc $E_1E_4E_3$ corresponds to the return stroke. The times of advance and return are therefore to each other as the angles E_2DE_3 and E_4DE_3.

When the ratio of these times is given, and consequently the angle $\delta = E_4DE_3$, and when the distance $AD = 2a$ between the axes is taken equal to the major axis of the ellipse, we can easily find the eccentricity of the ellipse from its known property

$$AF_3 + DE_3 = D_1E_3 + DE_3 = 2a;$$

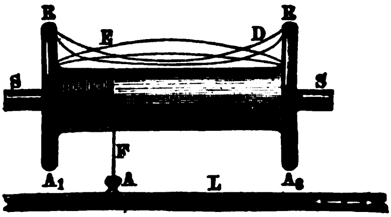
consequently, since D_1 is the other focus and $D_1D = 2e$, we have

$$2e \tan \delta + \frac{2e}{\cos \delta} = 2a, \quad \text{or} \quad e = a\frac{\cos \delta}{1 + \sin \delta},$$

which expression can be easily constructed.

It was mentioned in § 50 that non-circular wheels in combination with a crank are also occasionally used in spooling machines, where the spools must be wound as uniformly as possible. To understand this arrangement, let us suppose a cylindrical spool $S\,S$, Fig. 555, to turn with uniform velocity, thus winding the thread F. In order that the thread may be wound on the spool in regular, adjacent coils, it is led before winding through a glass eyelet A on a sliding bar L, holding all the eyelets of adjacent spools. To this sliding bar is communicated a reciprocating motion whose extent A_1A_3 is about equal to the length of the spool between its rims $R.R$. In this way the surface of the spool is covered by a layer of thread at every stroke of the guide L. The form of the surface of the layer depends principally on the kind of motion communicated to the guide. If this motion is uniform the layers are cylindrical, but when the motion of the guide is slower at the centre of the spool the surface will be convex, as at E; while in the opposite case, *i.e.*, when the guide moves more quickly at the centre of its stroke, the surface of the layers is concave, as at D. The latter motion would evidently be communicated if the guide L were driven by a crank with a constant angular velocity, for the sliding would be slower as the crank approached its dead points. As a consequence

FIG. 555.



the coils of thread will pile up on each other near the rims of the spools, and slide down or off. To avoid this evil we may move a crank AB, Fig. 556, which drives the guide, in such a way that it turns with a slower angular velocity at the middle position B_2 than at the dead points B_1 and B_3. This motion may be communicated to the crank from the uniformly rotating shaft D by means of the spirally shaped wheels R and R_1 (see § 50). We can obtain an idea of the motion of the guide of the thread by means of a diagram. If the crank turned uniformly and the crank were long, we know from what has preceded that the crank circle $B_1 B_2 B_3$ would give for every crank position AB an ordinate BH, which would measure the velocity of the guide, provided the crank radius AB were taken as the measure of the velocity of the crank pin. Since the wheels R and R_1 touch at F and G when the crank is at AB, it is only necessary to reduce the length BH in the ratio

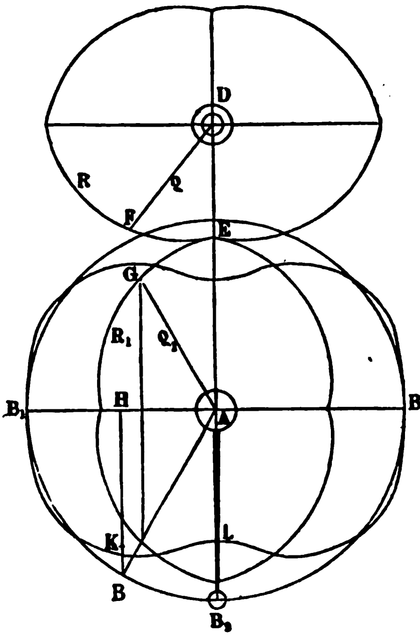
$$\frac{\rho}{\rho_1} = \frac{DF}{AG}$$

in order to obtain in

$$HK = \frac{\rho}{\rho_1} HB$$

the measure of the velocity of the guide of the thread. If this construction is repeated for a great number of crank positions,

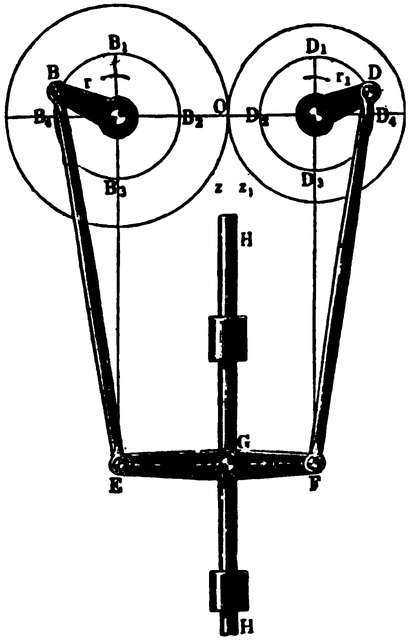
Fig. 556.



we get the curve $B_1 K L B_3$ as the velocity diagram of the guide. We see from this that a uniform motion cannot be obtained, the motion of the guide at the dead points also becoming equal to 0, and we must consequently regard this mechanism as a very imperfect means for diminishing the aforesaid evil in spool winding. We also see that the evil is not to be overcome by any use of the crank, for the velocity at the dead points will always be equal to 0. By using *cams* (see following chapter) the desired uniform motion of the guide may be attained.

§ **142. Roemer's Mechanism.**—An interesting mechanism invented by the astronomer *Roemer* consists of a combination of two crank trains $A B$ and $C D$, Fig. 557, whose connecting rods are attached to a beam $E F$ whose centre G is guided along the straight line $H H$. The motion of this point G will vary greatly according to the relations of the crank motions, and may be examined as follows: Let r and r_1 be the lengths of the cranks $A B$ and $C D$, and a and a_1 the lever arms of the beam whose whole length $E F = l$, and let two crank shafts be connected by two spur wheels having z and z_1 teeth. In the calculation we will assume, the length of the connecting rods as infinite. Let us suppose the crank $A B$ to occupy its middle position $A B_2$, *i.e.*, perpendicular to $H H$, and that the other crank $C D$ has passed its middle position $C D_2$ by an angle β. Now a rotation of the crank $A B$ through any angle $B_2 A B = \alpha$ will cause a motion of the point E approximately equal to $s = r \sin \alpha$.

FIG. 557.



In consequence of this motion of the point E the centre G will be moved in the direction HH an amount

$$s' = \frac{a_1}{l} s = \frac{a_1}{l} r \sin \alpha.$$

During this rotation α of the shaft A the shaft C turns through the angle

$$\alpha_1 = \frac{z}{z_1} \alpha.$$

Now if the crank CD is at an angular distance β from its middle position CD_2 when AB is at its middle position AB_2, the whole travel of the point F from its middle position is expressed by

$$s_1 = r_1 \sin (\alpha_1 + \beta) = r_1 \sin \left(\frac{z}{z_1} \alpha + \beta\right),$$

and in consequence of this motion the centre G of the beam is moved through .

$$s'' = \frac{a}{l} s_1 = \frac{a}{l} r_1 \sin \left(\frac{z}{z_1} \alpha + \beta\right).$$

The whole motion of the rod HG from the initial position, *i.e.*, the one corresponding to the middle position B_2 of the crank AB, and to that position of the crank CD at which it is at the angular distance β beyond the middle position CD_2, may be determined from

$$s' + s'' = \frac{a_1}{l} r \sin \alpha + \frac{a}{l} r_1 \sin \left(\frac{z}{z_1} \alpha + \beta\right).$$

If when the toothed wheels are equal ($z = z_1$) and the cranks are of the same length ($r = r_1$), we also make $\beta = 0$, *i.e.*, place both cranks simultaneously in their middle positions, we get for the motion of G,

$$s' + s'' = \frac{a_1 + a}{l} r \sin \alpha = r \sin \alpha,$$

and the whole stroke of the rod is given by $2r$ exactly as in a single crank train. Evidently this mechanism is then like the straight-line motion described in § 105, Fig. 400. But if for the same suppositions we take $\beta = 180°$, *i.e.*, if the crank CD were at D_4 when AB is at B_2, we should have

$$s' + s'' = \frac{a_1 - a}{l} r \sin \alpha ;$$

or for $a = a_1$ we should have $s' + s'' = 0$, *i.e.*, the rod HH would have no motion whatever, the action of the cranks being confined to oscillating the beam about its stationary centre G. Strictly speaking, this last remark holds only for infinite connecting rods; for finite rods the centre G of the beam would make slight oscillations, on account of the difference in the crank motion on the forward and return stroke.

It follows from these remarks that the stroke of the beam centre G depends principally on the relative position of the two cranks, the stroke being $2r$ for equal cranks and wheels when both cranks are simultaneously at their inner dead points B_3 and D_3, while on the other hand the stroke is 0 when one crank is at its outer dead point D_1 and the other crank at its inner dead point D_3, *i.e.*, when the angle of advance β in the first case equals 0 and in the second equals 180°. It is therefore easy to see that when the angle of advance β is between 0 and 180°, the stroke of the beam centre will be between $2r$ and 0. Now let us suppose that at a certain instant the cranks are at the inner dead points B_3 and D_3, and that the wheels have an unequal number of teeth. When the crank AB makes a complete revolution 2π, the other crank CD will turn through $\frac{z}{z_1} 2\pi$, $\frac{z}{z_1}$ representing the ratio of the number of teeth, the smallest whole numbers being taken for z and z_1 to express this ratio. One crank will therefore move in advance of the other by the angle

$$E = \left(1 - \frac{z}{z_1}\right) 2\pi = \frac{z_1 - z}{z_1} 2\pi.$$

During the second revolution of the crank AB this angle of advance becomes

$$2E = 2\frac{z_1 - z}{z_1} 2\pi,$$

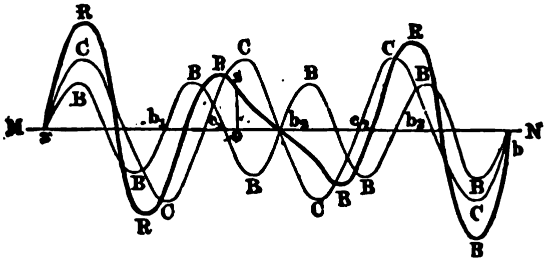
and after n revolutions it becomes

$$n \frac{z_1 - z}{z_1} 2\pi.$$

If we here make $n = z_1$, the advance becomes $(z_1 - z) 2\pi$, *i.e.*, a whole number of revolutions; in other words, the cranks are now both again at their inner dead points B_3 and D_3. It follows from this that during these z_1 revolutions of the crank A the beam centre G left its lowest position, and, after making a number of oscillations whose magnitude varied as the angle of advance varied, gradually returned to its lowest position. The amplitude of these separate oscillations gradually increases and decreases according to a law depending on the links of the cranks r and r_1 and the arms a and a_1 of the beam. If we suppose the centre G to be provided with a pencil which marks its oscillating motion on a strip of paper kept moving uniformly at right angles to HH, we shall get a diagram of the motion of the point G. But we can represent the motion by a drawing; for this purpose any distance AB of an axis of abscissas, Fig. 558, may be taken as the measure of the angular motion of the shaft A during a period of the motion, *i.e.*, the time during which the crank AB makes z_1 revolutions. In the figure we have assumed $\frac{z}{z_1} = \frac{3}{4}$, consequently ab is divided by b_1, b_2, and b_3 into four equal parts:

$$ab_1 = b_1 b_2 = b_2 b_3 = b_3 b = 2\pi.$$

FIG. 558.



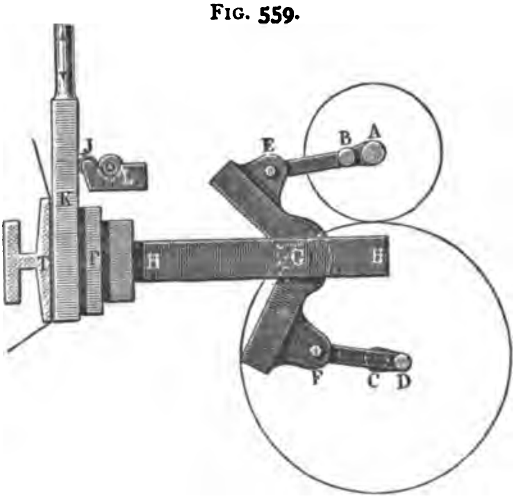
If at a sufficient number of points of this axis of abscissas we erect ordinates representing

$$s' = \frac{a_1}{l} r \sin \alpha,$$

the curve obtained, *BBB* . . . , is a sinoid whose ordinates on the opposite sides of *MN* represent that travel of the beam-centre on opposite sides of its middle position which is due to the crank *A B*.

In the same manner the sinoid *CCC* . . . may be laid off for the other crank, but it must be remembered that with the relation $\frac{z}{z_1} = \frac{3}{4}$ this crank makes only three revolutions, and consequently that the length ab must be divided at c_1 and c_2 into three equal parts, so that each part $a c_1 = c_1 c_2 = c_2 b = 2\pi$ corresponds to a whole revolution of the second crank. If we now add the ordinates of these curves, taking into account their positions on opposite sides of the line *MN*, we shall obtain the resulting curve *RR R* . . . as the travel diagram of the beam centre. Here the points designated by *R* represent the highest and lowest positions of the aforesaid centre, and the ordinate os, at any point o of the axis, shows how far the beam centre has travelled from its middle position when the cranks have rotated an amount represented by the abscissa ao and its respective divisions. The mechanism just described has been repeatedly applied in calico-printing for communicating a reciprocating motion to an engraved platen. The essential features of the mechanism as here applied are shown in Fig. 559. The two shafts *A* and *C*, connected by toothed wheels whose ratio is 1 : 2, carry the two cranks whose connecting rods are jointed to the piece *EF*. The centre *G* of the latter is jointed to the rod *HH*, which is

FIG. 559.



guided along a straight line and provided at the other end with the platen *P*. Opposite to this platen *P* the frame is arranged so as to form a bed *T*, over which the stuff to be printed is drawn periodically, in lengths equal to the height of the platen *P*. The plate *K* can move between this platen and its bed, receiving during its motion from the color-roller *J* the color contained in the color-tub *L*, and giving it off again in its lowest position to the approaching platen *P*. Now when the platen *P* recedes, the color-plate *K* is drawn up, enabling the platen *P* when it again comes forward to press the color received on to the stuff stretched over *T*. For this purpose it is necessary that the platen *P* have alternately a short stroke extending to the color-plate *K* and then a longer stroke to the bed *T*.

§ **143. Influence of the Masses.**—In the foregoing investigations the masses of the moved parts were not particularly considered. But as the masses have considerable influence on the motions of the crank train when the velocities are not very small, we will now make them a subject of special investigation. In the crank train there are principally three different masses, those rigidly connected to the crank shaft and having *rotary motion* only, those connected directly or indirectly with the cross-head and partaking of its *reciprocating rectilinear motion*, and finally those whose motion, like that of the connecting rod, is composed of *turning and sliding*.

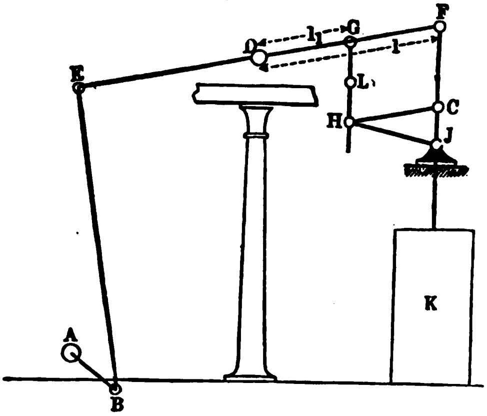
The *rotating* masses are the crank shaft, crank, and crank pin, and all the pieces rigidly attached to the shaft, as wheels, pulleys, etc. If the crank is fastened to the shaft of a water-wheel the mass of the latter will preponderate, while in a steam-engine it will usually be the fly-wheel that outweighs the rest. When the cranks are fastened to shafting, as is sometimes the case when they drive pumps, the rotating mass will be the whole line of shafting, and the wheels, pulleys, couplings, etc., rigidly attached to it. In what follows, m_1 always represents the rotating mass *reduced to a point at a distance from the axis of the crank equal to the length r of the crank.*

The reduction of the masses to the crank pin will not be difficult, for in every case the reduced mass of any machine-part can be obtained from

$$m = \frac{W}{r^2},$$

W representing the moment of inertia of the part referred to the axis of the crank (see Vol. I. § 283). The reciprocating masses of a crank train are the cross-head and its slide blocks, and the piston rod and piston. If another rod is directly connected with the piston rod, for instance, the rod of an air or water pump, the mass of this additional rod and its piston must be directly added to the mass of the cross-head. In engines in which there is a lever or beam inserted between the piston rod and the connecting rod, as in Fig. 560, the swinging mass of the beam must be reduced to the point *F*, at which the piston rod acts. Calling *W* the moment of inertia of the beam *E D F* referred to the axis *D*, the mass reduced to *F* becomes $\frac{W}{l^2}$. This mass and the mass of the link *F C* are to be directly added to the piston rod. In very exact calculations the mass of the radius bar *J H* would be reduced to the point *H* and added to the mass of the rod *L* and the link *G H*, the sum *m* thus obtained being reduced to $m\,\frac{l_1^2}{l^2}$, which may be regarded as acting at *F*. If we also wish to take into account the parallel bar *H C*, it will be necessary to find its moment of inertia with reference to the axis *D*, and reduce it to *F* by dividing it by l^2. In general, however, the small masses of the parallelogram may be neglected. In a similar

FIG. 560.

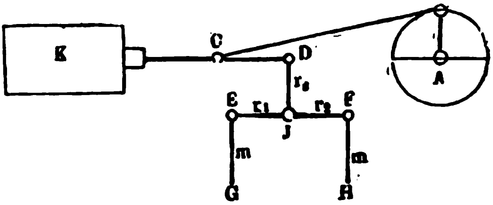


manner the masses may be reduced when the cross-head drives pumps by means of levers, as in horizontal engines, Fig. 561. Here let r_1, r_2, and r_3 represent the arms of the bell-crank lever *E F D*, whose moment of inertia referred to its axis *J* is represented by *W*, and let m' and m'' be the masses of the pump rods *E G* and *F H*. Then we must regard the reduced mass

$$\frac{W + m' r_1^2 + m'' r_2^2}{r_3^2}$$

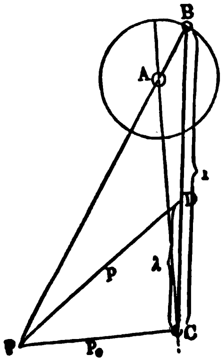
as taking part in the reciprocating motion of the cross-head. The connecting rod *C D* must be added unchanged to the mass of the cross-head, for the motion of every point of this rod is very nearly the same as that of the cross-head. We shall hereafter designate by m_2 the sum of all the masses thus reduced to the cross-head.

FIG. 561.



In most cases the mass of the connecting rod is so small that it can be neglected in comparison with the much larger rotating and reciprocating masses; nevertheless in exact calculations its influence may be approximately taken into account in the following manner: The instantaneous motion of the connecting rod *B C*, Fig. 562, for any crank position *A B*, can be regarded as an infinitely small rotation of the rod about the instantaneous centre *P*, the latter being always found at the intersection of the direction of the crank *B A* with the normal *C P* erected at *C* to the motion of the cross-head. Now if p_0 represents the instantaneous radius *P C* of the cross-head whose velocity is *c*, and *p* the instantaneous radius *P D* of any point *D* at the distance λ from *C*, the velocity of the latter will be

FIG. 562.



$\frac{p}{p_0} c$. The mass of the element at the point D of the connecting rod whose cross-section is f and whose heaviness is γ will be $m = f \gamma \delta \lambda$, where $\delta \lambda$ represents the length of an element. Consequently the work stored up in this elementary mass by virtue of its velocity is equal to

$$f \gamma \left(\frac{p}{p_0}\right)^2 \frac{c^2}{2} \delta \lambda;$$

hence the living force of the whole mass is

$$\int_0^l f \gamma \left(\frac{p}{p_0}\right)^2 \frac{c^2}{2} \delta \lambda.$$

To determine this value, let us suppose the cross-section f constant, *i.e.*, cylindrical, which is a permissible assumption, as the volume m_3 is small. Again, let us suppose the length l of the rod relatively to the length r of the crank to be sufficiently great so that in the expression for the velocity of the cross-head

$$c = v\left(\sin \alpha \mp \frac{r}{2l} \sin 2\alpha\right)$$

we may neglect the term $\frac{r}{2l} \sin 2\alpha$, a supposition the more allowable the nearer α is to a right angle, *i.e.*, the nearer $\sin 2\alpha$ approaches the value 0. Furthermore, if we regard the triangle PCD as right-angled at C by neglecting the angle γ on account of the smallness of the ratio $\frac{r}{l}$, we find for the instantaneous radius PD

$$p = \sqrt{p_0^2 + \lambda^2},$$

or since $p_0 = l \tan \alpha$, we have

$$\left(\frac{p}{p_0}\right)^2 = \frac{l^2 \tan^2 \alpha + \lambda^2}{l^2 \tan^2 \alpha} = 1 + \left(\frac{\lambda}{l}\right)^2 \cot^2 \alpha.$$

Substituting this value $\left(\frac{p}{p_0}\right)^2$ and $c = v \sin \alpha$, we get for the above integral

$$\frac{\gamma f}{2}\int_0^l \left(1+\frac{\lambda^2}{l^2}\cot^2\alpha\right) v^2 \sin^2\alpha \, \partial\lambda = \frac{\gamma f}{2} v^2 \sin^2\alpha \left(l + \frac{l}{3}\cot^2\alpha\right)$$

$$= \frac{m_3 v^2}{2}\left(\sin^2\alpha + \tfrac{1}{3}\cos^2\alpha\right) = \frac{m_3 v^2}{2}\left(\tfrac{2}{3}\sin^2\alpha + \tfrac{1}{3}\right).$$

Now as the living force of the mass m_1 due to the velocity v is expressed by

$$\frac{m_1 v^2}{2},$$

and the living force of the reciprocating mass m_2 due to the velocity $v \sin \alpha$ by

$$\frac{m_2 v^2 \sin^2\alpha}{2},$$

we see from the expression deduced above for the work stored up in the mass m_3 of the connecting rod, that *one third of this mass m_3 may be regarded as added to the rotating mass m_1, and the remaining two thirds as added to the reciprocating mass m_2.* In the following investigations we shall always assume such a distribution of the mass m_3 of the connecting rod that it will not be necessary to specially introduce m_3 into the calculation.

In order to obtain a general idea of how the masses affect the motion of the crank, we must first note that in every case, whether the crank shaft turns uniformly or not, the speed of the reciprocating parts during the first portion of each stroke increases from 0 at the dead point till it attains a certain maximum value c_{max}, whereupon its velocity gradually diminishes till it again becomes 0 when the crank is at its second dead point. Consequently, during the first part of each stroke, this mass m_2 absorbs for its acceleration a certain amount of work, the whole of which is again given up in the second part, for when it comes to rest at the end of the stroke it has lost all its living force. During every stroke, therefore, the reciprocating parts exert first a retarding and then an accelerating influence on the motion. Of course the work needed to accelerate the reciprocating masses must be expended by the driving piece, the steam piston in steam-engines and the crank shaft in pumps. The limit between these two opposite actions corresponds to the instant at which the cross-head reaches its greatest velocity c_{max}, for at this instant the acceleration passes into a retardation. This maximum piston speed was determined in § 139, under the supposition that the crank turned uniformly; but it is easy to see that the crank shaft never has an exactly uniform motion, for if it had it would be necessary that the driving force, including that exerted by the *retardation* of the reciprocating masses, should be at every instant equal to the resistance, the latter including the resistance to *acceleration* of these reciprocating masses. Generally this is not the case; it is much more common for the driving force to first exceed then fall short of the resistance, in consequence of which the now positive now negative excess of the former force over the latter causes an alternate acceleration and retardation of the crank shaft. The extent of these variations of velocity is governed principally by the magnitude of the rotating mass m_1. During the periods when the driving force is in excess this rotating mass is accelerated and absorbs work, which stored-up work is given off when the driving force becomes less than the resistance. Therefore the rotating mass, whose principal representative is the fly-wheel, also acts as a resisting and a driving force alternately. It is not difficult to see that the fluctuations of the velocities will be smaller, *i.e.*, the motion will be more uniform, the greater the rotating mass m_1, a subject which will be more fully investigated hereafter. Of course the reciprocating mass m_2 also takes part in these fluctuations of velocity; but in most cases, in comparison with the rotating mass m_1, it has a subordinate influence on the uniformity of motion of the crank shaft. The work alternately absorbed and given off by the masses during certain periods of motion is exactly the same in amount only when the machine has reached what we may call its *dynamic condition of permanency*, a condition rapidly reached by all machines, and one which will be assumed as existing in the following investigations. As long as the driving force of a machine exceeds its resistance, as is the case when the machine is *starting*, the excess of work of the driving force will be expended in accelerating the masses, and the condition of permanency will not be attained until the resistances accompanying increased velocity have reached an average amount equal to the average value of the driving force, which force usually becomes smaller with increase of speed. For reasons given above, the condition of permanence is accompanied by certain periodical variations of velocity which are regularly repeated so long as there is no change in the action of the driving force and of the resistance. When the machine is slowing down or stopping, *i.e.*, when the driving force is removed, the motion will last for some time by virtue of the living force of the masses. This *stopping* of the machine is characterized by a gradual retardation up to the point of complete rest, just as the starting is characterized by a continual acceleration from the condition of rest up to the normal velocity. It is self-evident that the work stored up in the masses during the starting of the machine is entirely given up again when the machine is brought to rest.

§ **144. The Crank with Constant Force on the Piston.**—In order to determine the motion of a crank when the influence of the masses is considered, we will first make the simple supposition that the force Q acting at the cross-head is constant,—a case corresponding to pumps and to engines working without expansion. Later on the investigation will be extended to the case of a variable force at the piston, such as obtains in engines working with expansion and in air-pumps. We will also suppose the force at the crank shaft to have a constant value, and that its value, at the end of a radius equal to the crank arm r, is represented by P. In the case of a steam-engine this resistance P would be the constant resistance of a mill or shop driven by the engine. When, as in pumps, the crank is the driver, P must be regarded as the constant driving force reduced to the crank radius exerted on the crank shaft by toothed wheels or belt pulleys. Let m_1 and m_2 represent respectively the rotating and the reciprocating masses, the mass of the connecting rod being distributed as mentioned above, one third being contained in the mass m_1 and the other two thirds in the mass m_2. No attention will be paid to the weights of the reciprocating masses, for we will assume either that the stroke of the cross-head is horizontal, or, in case the piston motion is vertical, that the weight of the reciprocating parts is balanced by a counterweight on the crank shaft. The amount and action of such a counterweight will be more fully discussed hereafter.

Supposing the engine to have attained its condition of permanency, it follows that for a crank train the period of the motion is represented by one revolution of the crank, for when the whole system is in the same position, say at a dead point, it always has exactly the same velocities and is under the same conditions. The work of the force Q on the piston during a forward and return stroke is exactly equal to the work of the force P at the shaft, and therefore in the present case, where Q is constant, we have

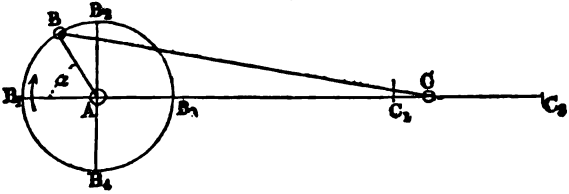
$$Q \times 2 \times 2r = P \times 2\pi r,$$

and from this the value

$$P = \frac{2}{\pi} Q = 0.6366\, Q,$$

provided we neglect all hurtful resistances, as friction. Let us retain the signification hitherto given to l, r, β, and γ, and let

FIG. 563.



v represent the velocity of the crank pin at any instant, v_1 its velocity at the outer dead point B_1, Fig. 563, v_0 the average velocity of the crank pin, and c the velocity of the cross-head in any position.

If we suppose the crank AB to have moved through the angle α from the exterior dead point $A B_1$, the cross-head will have travelled the distance

$$C_1 C = s = r(1 - \cos \alpha) - \frac{r^2 \sin^2 \alpha}{2 l},$$

and the difference between the quantities of work performed by the exterior forces P and Q will be expressed by

$$A = Q s - P r \alpha = Q r (1 - \cos \alpha) - Q \frac{r^2 \sin^2 \alpha}{2 l} - Q \frac{2}{\pi} r \alpha.$$

From what has preceded we know that this difference of work A during the rotation α has been absorbed or given off by the masses m_1 and m_2 in the form of living force, according as the expression for A has a positive or negative value. The work absorbed or given off by the masses may be ascertained as follows: In order that the mass m_1 with its velocity v_1 at the dead point might attain the velocity v, there was needed the work

$$m_1 \frac{v^2 - v_1^2}{2}.$$

On the other hand, in order that the reciprocating mass m_2 might start from rest and attain the velocity

$$c = v \left(\sin \alpha - \frac{r}{2 l} \sin 2 \alpha\right)$$

at C, there was needed the work

$$m_2 \frac{c^2}{2} = m_2 \left(\sin \alpha - \frac{r}{2 l} \sin 2 \alpha\right)^2 \frac{v^2}{2}.$$

Adding these expressions for the energy of the masses, and placing the sum equal to the work A of the exterior forces, we have

$$Q r \left(1 - \cos \alpha - \frac{r}{2 l} \sin^2 \alpha - \frac{2}{\pi} \alpha\right)$$
$$= m_1 \frac{v^2 - v_1^2}{2} + m_2 \left(\sin \alpha - \frac{r}{2 l} \sin 2 \alpha\right)^2 \frac{v^2}{2}.$$

From this we get the velocity v of the crank pin for any position of the crank at the angular distance α from the dead point,

$$v = \sqrt{\frac{m_1 v_1^2 + 2Qr\left(1 - \cos\alpha - \frac{r}{2l}\sin^2\alpha - \frac{2}{\pi}\alpha\right)}{m_1 + m_2\left(\sin\alpha - \frac{r}{2l}\sin^2\alpha\right)^2}}.$$

This formula was developed for the forward motion of the crank from C_1 and C_3, but it is also applicable to the return motion when the terms containing $\frac{r}{2l}$ are given opposite signs and the angle α is estimated from the inner dead point AB_3.

As $\frac{r}{2l}$ is small, seldom exceeding 0.1, we may neglect the terms containing it, and then the expression becomes

$$v = \sqrt{\frac{m_1 v_1^2 + 2Qr\left(1 - \cos\alpha - \frac{2}{\pi}\alpha\right)}{m_1 + m_2\sin^2\alpha}}.$$

This may also be written as follows:

$$v = v_1\sqrt{\frac{1 + \frac{2Qr}{m_1 v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\alpha\right)}{1 + \frac{m_2}{m_1}\sin^2\alpha}}.$$

This will enable us to examine the motion of the crank and ascertain its least and greatest velocity. For this purpose, it should be noticed that the reciprocating mass m_2 is usually small in comparison with m_1, the ratio $\frac{m_2}{m_1}$ being generally only a fraction of one per cent; consequently it will be sufficiently accurate if we neglect the term

$$\frac{m_2}{m_1}\sin^2\alpha$$

of the denominator. The above expression for v is then simplified to

$$v = v_1 \sqrt{1 + \frac{2\,Q\,r}{m_1\,v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\,\alpha\right)}$$

$$= v_1 \left(1 + \frac{Q\,r}{m_1\,v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\,\alpha\right)\right).$$

The angle corresponding to the least and greatest velocity of the crank may be readily found from

$$\frac{\delta v}{\delta \alpha} = 0; \quad \text{that is,} \quad \sin\alpha = \frac{2}{\pi} = 0.6366.$$

This gives two angles which are supplements of each other,

$$\alpha_1 = 39^\circ\ 32' \quad \text{and} \quad \alpha_2 = 140^\circ\ 28'.$$

In the present case, where the piston rod is the driver, the acute angle α_1 corresponds to the smallest and the obtuse angle α_2 to the greatest velocity, for the second differential

$$\frac{\delta^2 v}{\delta \alpha^2} = \cos\alpha$$

is positive in the first case and negative in the second. This could also be inferred from what was said in § 139 concerning circumferential force, for this force U is 0 at the beginning of the motion, and does not become equal to the resistance P at the crank till the crank has rotated a certain distance. It is also clear that the crank positions represented by α_1 and α_2 are those in which the circumferential force U becomes exactly equal to the resistance P, for in these positions of maximum and minimum velocity the acceleration of the crank shaft becomes 0. In like manner it follows, when the crank is the driver as in pumps, that the acute angle α_1 corresponds to the greatest and the obtuse angle α_2 to the least velocity, for the work A of the exterior forces takes the opposite sign, and the second differential becomes

$$\frac{\delta^2 v}{\delta \alpha^2} = -\cos\alpha,$$

negative for α_1 and positive for α_2.

These leading velocities may themselves be determined by substituting in the formula for v the above values for α_1 and α_2; hence for $\alpha_1 = 39^\circ\ 32'$,

$$v_{min} = v_1 \sqrt{1 - 0.4210 \frac{Qr}{m_1 v_1^2}} = v_1 \left(1 - 0.2105 \frac{Qr}{m_1 v_1^2}\right);$$

for $\alpha_2 = 140^\circ\ 28'$,

$$v_{max} = v_1 \left(1 + 0.2105 \frac{Qr}{m_1 v_1^2}\right).$$

When the crank drives, the same values are obtained for the velocities, but then, as has been already remarked, v_{min} belongs to α_2 and v_{max} to α_1. The formulas deduced for v become applicable to this case when the term which contains Q, and represents the work of the exterior P and Q is given an opposite sign; for this case therefore

$$v = \sqrt{\frac{m_1 v_1^2 - 2Qr\left(1 - \cos\alpha - \frac{r}{2l}\sin^2\alpha - \frac{2}{\pi}\alpha\right)}{m_1 + m_2\left(\sin\alpha - \frac{r}{2l}\sin^2\alpha\right)^2}}.$$

If in the general formula for v we substitute for α the angle π corresponding to a half revolution, we get for the inner dead point:

$$v = \sqrt{\frac{m_1 v_1^2 + 2Qr\left(1 - \cos 180^\circ - \frac{r}{2l}\sin^2 180^\circ - \frac{2}{\pi}\pi\right)}{m_1 + m_2\left(\sin 180^\circ - \frac{r}{2l}\sin 360^\circ\right)^2}} = v_1,$$

from which we conclude that the velocity of the *single* crank is the same at both dead points. In order that we may also determine the average velocity v_0, which for n revolutions may be written

$$v_0 = \frac{n \times 2\pi r}{60} = 0.1047\, nr,$$

we may start with the approximate formula

$$v = v_1\left(1 + \frac{Qr}{m_1 v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\alpha\right) - \frac{m_2}{2m_1}\sin^2\alpha\right),$$

and then transform this to

$$\frac{1}{v} = \frac{1}{v_1}\left(1 - \frac{Qr}{m_1 v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\alpha\right) + \frac{m_2}{2m_1}\,\frac{1 - \cos 2\alpha}{2}\right).$$

If δt represents an element of time, we have

$$v\,\delta t = r\,\delta\alpha:$$

hence

$$\delta t = \frac{r\,\delta\alpha}{v}$$

$$= \frac{r\,\delta\alpha}{v_1}\left(1 - \frac{Qr}{m_1 v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\alpha\right) + \frac{m_2}{2m_1}\,\frac{1 - \cos 2\alpha}{2}\right).$$

By integrating this expression between the limits π and 0, we find the time needed for a half revolution to be

$$t = \frac{r}{v_1}\left(\pi - \frac{Qr}{m_1 v_1^2}\left(\pi - \sin\pi - \frac{2\pi^2}{\pi\,2}\right) + \frac{m_2}{4m_1}\left(\pi - \frac{\sin 2\pi}{2}\right)\right)$$

$$= \frac{r\pi}{v_1}\left(1 + \frac{m_2}{4m_1}\right).$$

Consequently the average velocity is

$$v_0 = \frac{r\pi}{t} = \frac{v_1}{1 + \dfrac{m_2}{4m_1}} = v_1\left(1 - \frac{m_2}{4m_1}\right).$$

This value differs but little from v_1, and we may therefore with sufficient accuracy assume the average velocity v_0 to be the same as the velocity v_1 at the dead points, and may substitute v_0 instead of v_1 in all the formulas thus far developed.

REMARK. If it is not regarded as admissible to neglect m_2 in comparison with m_1, we can find more exact formulas for the leading velocities from the general formula

$$v = v_1 \sqrt{\frac{1 + \frac{2\,Q\,r}{m_1\,v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\alpha\right)}{1 + \frac{m_2}{m_1}\sin^2\alpha}}$$

as follows: Transforming the last formula, we have

$$v = v_1 \sqrt{\left(1 - \frac{m_2}{m_1}\sin^2\alpha\right)\left[1 + \frac{2\,Q\,r}{m_1\,v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\alpha\right)\right]}$$

$$= v_1\left(1 + \frac{Q\,r}{m_1\,v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\right) - \frac{m_2}{2\,m_1}\sin^2\alpha\right),$$

and from this by differentiation

$$\frac{\delta v}{\delta\alpha} = v_1\left(\frac{Q\,r}{m_1\,v_1^2}\left(\sin\alpha - \frac{2}{\pi}\right) - \frac{m_2}{2\,m_1}\sin 2\,\alpha\right) = 0.$$

Hence we have the equation

$$\sin\alpha = \frac{2}{\pi} + \frac{m_2\,v_1^2}{2\,Q\,r}\sin 2\,\alpha.$$

From this equation α_1 and α_2 may best be obtained by trial, starting with the values 39° 32′ and 140° 28′, and then correcting these values so as to satisfy the equation. For the values $\alpha_1 = 39° \, 32'$ and $\alpha_2 = 140° \, 28'$, we obtain for the velocities the expressions

$$v_{min} = v_1\left(1 - 0.2105\,\frac{Q\,r}{m_1\,v_1^2} - 0.2026\,\frac{m_2}{m_1}\right)$$

and

$$v_{max} = v_1\left(1 + 0.2105\,\frac{Q\,r}{m_1\,v_1^2} - 0.2026\,\frac{m_2}{m_1}\right).$$

§ 145. Influence of the Length of the Connecting Rod.—Hitherto we have always assumed that the length l of the connecting rod relative to the crank radius r was so large that terms containing the factor $2\,l$ could be neglected. Strictly speaking, therefore, the relations found apply only to the crank train with slotted piston rod, for which l may be regarded as infinite. In practice l is often made five times the length of the crank, so that

$$\frac{r}{2\,l} = 0.1.$$

In consequence of the finite length of the connecting rod, the maximum and minimum velocities of the crank have somewhat different velocities, and these velocities occur at different crank positions than those calculated above. In order to ascertain the influence of the length of the connecting rod on these values, let us go back to the general formula for velocity in § 144,

$$v = \sqrt{\frac{m_1 v_1^2 + 2 Q r\left(1 - \cos\alpha \mp \frac{r \sin^2\alpha}{2l} - \frac{2}{\pi}\alpha\right)}{m_1 + m_2\left(\sin\alpha \mp \frac{r}{2l}\sin 2\alpha\right)}},$$

where the upper signs refer to forward stroke and the lower to return stroke. As m_2 is small in comparison with m_1, we may write the above expression as follows:

$$v = v_1\sqrt{1 + \frac{2Qr}{m_1 v_1^2}\left(1 - \cos\alpha \mp \frac{r \sin^2\alpha}{2l} - \frac{2}{\pi}\alpha\right)}$$

$$= v_1\left(1 + \frac{Qr}{m_1 v_1^2}\left(1 - \cos\alpha - \frac{2}{\pi}\alpha \mp \frac{r\sin^2\alpha}{2l}\right)\right).$$

From this we obtain the condition of maximum and minimum velocities,

$$\frac{\delta v}{\delta\alpha} = \sin\alpha - \frac{2}{\pi} \mp \frac{r}{2l}\sin 2\alpha = 0,$$

or

$$\sin\alpha = \frac{2}{\pi} \pm \frac{r}{2l}\sin 2\alpha.$$

This equation gives for every value of $\frac{r}{l}$ two pairs of angles α_1 and α_2 for the forward motion when the upper sign is used, and (α_1) and (α_2) for the return motion when the lower sign is used. In steam-engines α_1 and (α_1) correspond to the least and α_2 and (α_2) to the greatest velocities, the velocities being obtained by substituting the corresponding values of α in the formula for v.

It is easy to see that these four angles α_1, (α_1), α_2, and (α_2)

are so related that the acute angle α_1 is the supplement of the obtuse angle (α_2), and likewise that α_2 and (α_1) are supplements of each other, for if the equation

$$\sin \alpha = \frac{2}{\pi} + \frac{r}{2l} \sin 2\alpha$$

is satisfied by the angle α_1, the equation

$$\sin \alpha = \frac{2}{\pi} - \frac{r}{2l} \sin 2\alpha$$

must also satisfy the angle $(\pi - \alpha_1)$.

Another remarkable property is found in connection with the four velocities, v_{max} and v_{min} for the forward motion, and (v_{max}) and (v_{min}) for the return motion. This property is that v_{max} exceeds the average velocity v_0 or v_1 of the dead points by exactly the amount that (v_{min}) is less than these velocities, and the same relation exists between (v_{max}) and v_{min}. For

$$v_{max} = v_1\left(1 + \frac{Qr}{m_1 v_1^2}\left(1 - \cos\alpha_2 - \frac{2}{\pi}\alpha_2 - \frac{r\sin^2\alpha_2}{2l}\right)\right)$$
$$= v_1\left(1 + \frac{Qr}{m_1 v_1^2}k\right)$$

when the quantity in the inner parenthesis is represented by k. Now we also have

$$v_{min} = v_1\left(1 + \frac{Qr}{m_1 v_1^2}\left(1 - \cos(\pi - \alpha_2) - \frac{2}{\pi}(\pi - \alpha_2) + \frac{r}{2l}\frac{\sin^2(\pi - \alpha_2)}{2l}\right)\right) = v_1\left(1 + \frac{Qr}{m_1 v_1^2}(k)\right).$$

But it is at once evident that the two values

$$k = 1 - \cos\alpha_2 - \frac{2}{\pi}\alpha_2 - \frac{r\sin^2\alpha_2}{l}$$

and

$$(k) = 1 - \cos(\pi - \alpha_2) - \frac{2}{\pi}(\pi - \alpha_2) + \frac{r\sin^2(\pi - \alpha_2)}{2l}$$

are equal in amount, but of opposite sign.

The values α_1 and (α_1) differ, and so do α_2 and (α_2); and we also find that v_{max} differs from (v_{max}) and v_{min} from (v_{min}) so long as $\frac{r}{l}$ has a finite value. It follows from this that the motion of the crank train on the forward stroke is not exactly like that on the return stroke. It is only when $l = \infty$, as in the crank train with slotted piston rod, that $\alpha_1 = (\alpha_1)$ and $\alpha_2 = (\alpha_2)$; and here the maximum velocities for the forward and return motion are exactly alike, and the minimum velocities are also alike.

When the crank is the driver the maximum velocities become the minimum velocities, and *vice versa*. For this case it is only necessary in the preceding formulas to prefix an opposite sign to the term which contains Q.

If we assume in the calculation the very common ratio $\frac{r}{l} = \frac{1}{5}$, we obtain for the forward stroke, from

$$\sin \alpha = \frac{2}{\pi} \alpha + 0.1 \sin 2\alpha,$$

the angles α_1 and α_2; then the leading velocities, thus:

$$\alpha_1 = 47° \; 25'; \quad v_{min} = v_0 \left(1 - 0.2577 \frac{Qr}{m_1 v_0^2}\right);$$

$$\alpha_2 = 146° \; 45'; \quad v_{max} = v_0 \left(1 + 0.1757 \frac{Qr}{m_1 v_0^2}\right).$$

On the other hand, for the return motion we get from

$$\sin \alpha = \frac{2}{\pi} \alpha - 0.1 \sin 2\alpha$$

the values

$$(\alpha_1) = 33° \; 15'; \quad (v_{min}) = v_0 \left(1 - 0.1757 \frac{Qr}{m_1 v_0^2}\right);$$

$$(\alpha_2) = 132° \; 35'; \quad (v_{max}) = v_0 \left(1 + 0.2577 \frac{Qr}{m_1 v_0^2}\right).$$

If we wish to ascertain the crank positions for which the velocity of the crank is exactly equal to the average velocity v_1 or v_0, it is only necessary to place

$$1 - \cos \alpha - \frac{2}{\pi} \alpha \mp 0.1 \sin^2 \alpha = 0,$$

and then get for the forward motion, by using the upper sign,

$$\alpha_0 = 106° \; 30',$$

and for the return motion

$$(\alpha_0) = 73° \; 30' = \pi - \alpha_0.$$

For the crank train with slotted piston rod $\frac{r}{l} = 0$, we of course find from

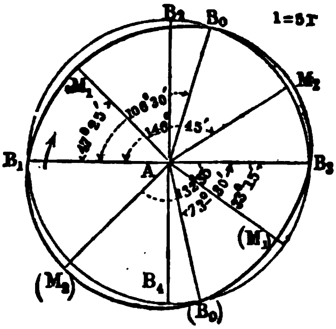
$$1 - \cos \alpha - \frac{2}{\pi} \alpha = 0$$

the values

$$\alpha_0 = (\alpha_0) = \frac{\pi}{2}.$$

We can obtain an idea of the variation of the velocity of the crank by a diagram if we regard the crank radius $A B$, Fig. 564, as the measure of the average velocity v_0, and lay off on each crank position from the centre A the velocity v corresponding to that position. The deviations of the closed curve $B_1 M_1 B_0 M_2 B_3 (M_1) B_0 (M_2) B_1$ thus obtained from the crank circle show the fluctuations of velocity relatively to the average velocity v_0. We see from the diagram that the velocity of the crank in its middle position B_2, forward stroke, is less than the average velocity v_0 by the same amount that its velocity in the middle position B_4, return stroke, exceeds the average value v_0. These velocities at B_2

FIG. 564.



and B_4 may be obtained from the formula v by substituting $\alpha = \frac{\pi}{2}$, thus getting

$$v = v_0\left(1 \mp \frac{r}{2l}\frac{Qr}{m_1 v_0^2}\right).$$

For the crank train with slotted piston rod these velocities become equal to the average one, v_0, and the velocity curve passes through the ends B_1, B_2, B_3, and B_4 of all the quadrants. From what has been said we know that the velocity diagram of Fig. 564 will also answer for the case in which the crank shaft drives, provided we suppose the crank to turn in the opposite direction B_1, B_4, B_3. . .

The motion of the crank will be the more unsteady the greater the ratio of the difference between the absolutely greatest and least velocities, $(v_{max}) - v_{min}$, to the average velocity v_0. This ratio

$$\frac{v_{max} - v_{min}}{v_0} = \delta$$

has been called the *coefficient of unsteadiness.* This ratio may also be expressed by

$$2\frac{(v_{max}) - v_0}{v_0};$$

and from what has preceded we know that

$$\delta = 0.5154\frac{Qr}{m_1 v_0^2} \quad \text{when} \quad \frac{r}{l} = \frac{1}{5},$$

and

$$\delta = 0.4210\frac{Qr}{m_1 v_0^2} \quad \text{when} \quad \frac{r}{l} = 0.$$

In general, the unsteadiness diminishes with the ratio $\frac{r}{l}$; and in order to obtain as uniform a motion as possible we must use a long connecting rod and a sufficiently large rotating mass m_1. We also see that, for a given force Q and mass m_1, the uniformity of the motion will be greater the greater the average circumferential speed v_0. For fuller information on this point see *Fly-Wheels.*

The following table contains the angles, the values of the minimum and maximum velocities, and the coefficients of unsteadiness for the usual ratios of $\frac{r}{l}$. Opposite the designation α_0 the table also gives the angles at which the crank velocity becomes equal to the average velocity, and opposite the symbol $v_{\frac{\pi}{2}}$ the table gives the velocities of the crank in its middle positions.

TABLE OF CRANK VELOCITIES.

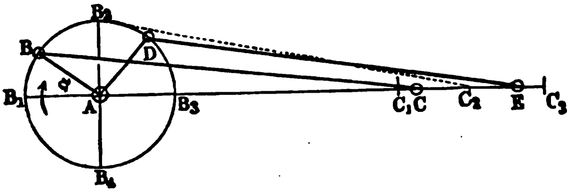
$$C = \frac{Qr}{m_1 v_0^2}.$$

$l =$	$4r$	$5r$	$6r$	∞
$v_{\min}$	$(1 - 0.2718\,C)\,v_0$	$(1 - 0.2577\,C)\,v_0$	$(1 - 0.2489\,C)\,v_0$	$(1 - 0.2105\,C)\,v_0$
α_1	$49^\circ\ 29'$	$47^\circ\ 25'$	$46^\circ\ 3'$	$39^\circ\ 32'$
$v_{\max}$	$(1 + 0.1686\,C)\,v_0$	$(1 + 0.1757\,C)\,v_0$	$(1 + 0.1807\,C)\,v_0$	$(1 + 0.2105\,C)\,v_0$
α_2	$148^\circ\ 14'$	$146^\circ\ 45'$	$145^\circ\ 58'$	$140^\circ\ 28'$
$(v_{\min})$	$(1 - 0.1686\,C)\,v_0$	$(1 - 0.1757\,C)\,v_0$	$(1 - 0.1807\,C)\,v_0$	$(1 - 0.2105\,C)\,v_0$
(α_1)	$31^\circ\ 46'$	$33^\circ\ 15'$	$34^\circ\ 2'$	$39^\circ\ 32'$
$(v_{\max})$	$(1 + 0.2718\,C)\,v_0$	$(1 + 0.2577\,C)\,v_0$	$(1 + 0.2489\,C)\,v_0$	$(1 + 0.2105\,C)\,v_0$
(α_2)	$130^\circ\ 31'$	$132^\circ\ 35'$	$133^\circ\ 57'$	$140^\circ\ 28'$
α_0	$108^\circ\ 30'$	$106^\circ\ 30'$	$102^\circ\ 50'$	90°
$v_{\frac{\pi}{2}}$	$(1 \mp 0.125\,C)\,v_0$	$(1 \mp 0.1\,C)\,v_0$	$(1 \mp 0.0833\,C)\,v_0$	v_0
δ	$0.5436\,C$	$0.5154\,C$	$0.4978\,C$	$0.4210\,C$

§ **146. The Double Crank.**—Two cranks fastened to the same shaft at right angles to each other are often used in steam-engines, particularly when the latter are to be frequently reversed. In such an arrangement there are, properly speaking, no dead points in the system, for the dead points of one crank are between those of the other, and consequently the pressure on the pistons can drive the crank shaft in any crank position, which is not the case when a single crank is on its dead points. It is for this reason that double cranks are almost always used in steam-engines, which, like locomotive and marine engines, must be capable of starting forward or backward easily, in every position. Occasionally the double crank is used in pumps and similar machines driven by the crank shaft.

In order to develop the theory of the double crank we will again assume that the crank shaft is driven by the forces on the piston. Let P again represent the constant resistance acting on the crank shaft at a distance r equal to the radius of the equal cranks, let Q be the constant pressure on each of the two pistons, m_1 the rotating mass reduced to the crank pin, and m_2 the reciprocating mass connected with *each* cross-head. The mass of the connecting rod must be distributed as explained in § 143, $\frac{1}{3}$ going to the crank pin, and $\frac{2}{3}$ to the cross-heads, these parts being included in m_1 and m_2. We will suppose the cranks to be set, as is usually done, at right angles, $BAD = 90°$, Fig. 565. When the engine has attained its condition of permanency, we may equate the quantities of work $P \times 2\pi r$ and $2 \times Q 4 r$, getting

FIG. 565.



$$P = \frac{4}{\pi} Q.$$

If we again suppose the crank to have moved through the angle α from the outer dead point AB_1, so that the crank AB has moved from B_1 to B and the crank AD from B_2 to D, the cross-head C will have travelled the distance

$$s_1 = C_1 C = r(1 - \cos\alpha) \mp \frac{r^2 \sin^2 \alpha}{2l},$$

and the cross-head E has moved from C_2 to E through the distance

$$s_2 = C_1 E - C_1 C_2 = r\left(1 - \cos(90° + \alpha)\right) \mp \frac{r^2 \sin^2(90° + \alpha)}{2l}$$

$$- \left(r\,(1 - cos\,90^\circ) \mp \frac{r^2\,sin^2\,90^\circ}{2l}\right) = r\,sin\,\alpha \pm \frac{r^2}{2\,l}\,(1 - cos^2\,\alpha)$$

$$= r\,(sin\,\alpha \pm \frac{r}{2\,l}\,sin^2\,\alpha).$$

The work performed by the two rods is given by $A_1 + A_2 = Q\,s_1 + Q\,s_2$

$$= Q\,r\left(1 + sin\,\alpha - cos\,a \mp \frac{r\,sin^2\,\alpha}{2\,l} \pm \frac{r\,sin^2\,a}{2}\right)$$

when the upper or the lower sign must be taken, according as the crank under discussion is on the forward or return stroke. During this motion the crank shaft has performed the work

$$P\,r\,\alpha = \frac{4}{\pi}\,Q\,r\,\alpha = A_3,$$

so that the excess of work

$$A_1 + A_2 - A_3 = Q\,r\left(1 + sin\,\alpha - cos\,\alpha \mp \frac{r\,sin^2\,\alpha}{2l} \pm \frac{r\,sin^2\,\alpha}{2l} - \frac{4}{\pi}\,\alpha\right)$$

has been absorbed or given off as living force by the masses. The work stored up by the mass m_1 of the crank shaft while passing from the velocity v_1 at B_1 to the velocity v at B is again

$$L_1 = m_1\,\frac{v^2 - v_1^2}{2},$$

and the work or energy absorbed by the mass m_2 of the crosshead C while increasing in speed from 0 at the dead point to

$$c = v\left(sin\,\alpha \mp \frac{r}{2\,l}\,sin\,2\,\alpha\right),$$

at the point B, becomes

$$L_2 = m_2\left(sin\,\alpha \mp \frac{r}{2\,l}\,sin\,2\,\alpha\right)^2\,\frac{v^2}{2}.$$

On the other hand, the mass m_2 of the second cross-head E has passed from the velocity

$$v_1\left(\sin 90^\circ \mp \frac{r}{2l}\sin 180^\circ\right) = v_1,$$

which it had at C_2 at the beginning of the motion, to its present velocity,

$$v\left(\sin(90^\circ + \alpha) \mp \frac{r}{2l}\sin 2(90^\circ + \alpha)\right) = v\left(\cos\alpha \pm \frac{r}{2l}\sin 2\alpha\right),$$

which corresponds to an increase of living force

$$L_3 = m_2\left(\cos\alpha \pm \frac{r}{2l}\sin 2\alpha\right)^2\frac{v^2}{2} - m_2\frac{v_1^2}{2}.$$

If in the expressions for L_2 and L_3 we neglect all higher powers of $\frac{r}{2l}$, we shall have

$$L_2 = m_2\left(\sin^2\alpha \mp \frac{r}{l}\sin\alpha\sin 2\alpha\right)\frac{v^2}{2}$$

and

$$L_3 = m_2\left(\cos^2\alpha \mp \frac{r}{l}\cos\alpha\sin 2\alpha\right)\frac{v^2}{2} - m_2\frac{v_1^2}{2},$$

from which follows

$$L_1 + L_2 + L_3 = m_1\frac{v^2 - v_1^2}{2} + m_2\frac{v^2 - v_1^2}{2}$$
$$+ m_2\frac{r}{l}(\mp\sin\alpha \pm \cos\alpha)\sin 2\alpha\frac{v^2}{2}.$$

As m_2 is small in comparison with m_1 and r in comparison with l, we can drop the last term of this expression, and then get

$$L_1 + L_2 + L_3 = (m_1 + m_2)\frac{v^2 - v_1^2}{2}.$$

If we place this increase of living force equal to the work of $A_1 + A_2 + A_3$ of the exterior forces, we have

$$(m_1 + m_2)\frac{v^2 - v_1^2}{2}$$

$$= Qr\left(1 + \sin\alpha - \cos\alpha \mp \frac{r\sin^2\alpha}{2l} \pm \frac{r\sin^2\alpha}{2l} - \frac{4}{\pi}\alpha\right),$$

which finally gives us for the velocity v of the crank pin

$$v = \sqrt{v_1^2 + \frac{2\,Qr}{m_1 + m_2}\left(1 + \sin\alpha - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha \pm \frac{r}{2l}\sin^2\alpha - \frac{4}{\pi}\alpha\right)}.$$

By neglecting the terms containing $\frac{r}{2l}$, *i.e.*, for infinitely long connecting rods and for large masses, we get the expression

$$v = v_1\left(1 + \frac{Qr}{(m_1 + m_2)\,v_1^2}\left(1 + \sin\alpha - \cos\alpha - \frac{4}{\pi}\alpha\right)\right).$$

Therefore, for this case of infinitely long connecting rods we obtain, by differentiation, the maximum and minimum velocities from

$$\cos\alpha + \sin\alpha - \frac{4}{\pi} = 0,$$

or

$$\left(\frac{4}{\pi}\right)^2 = (\sin\alpha + \cos\alpha)^2 = 1 + \sin 2\alpha;$$

i.e.,

$$\sin 2\alpha = \left(\frac{4}{\pi}\right)^2 - 1 = 0.6211.$$

This equation gives two angles that are complements of each other:

$$\alpha_1 = 19^\circ\ 12' \quad \text{and} \quad \alpha_2 = 70^\circ\ 48'.$$

From what was said before, we know that the angle $\alpha_1 = 19^\circ\ 12'$ corresponds to the least and the angle $\alpha_2 = 70^\circ 48'$ to the greatest velocity, if, as is here assumed, the piston rod drives. When pumps are driven by the crank shaft just the opposite occurs. The leading velocities themselves may be obtained by substituting the values for α_1 and α_2 just found, thus getting

$$v_{min} = v_1 \left(1 - 0.0422 \frac{Q r}{(m_1 + m_2) v_1^2}\right)$$

and

$$v_{max} = v_1 \left(1 + 0.0422 \frac{Q r}{(m_1 + m_2) v_1^2}\right).$$

Moreover, it is plain that the relations here found hold for each of the four quadrants, provided the angle α is kept between the limits 0 and 90°, and is continually estimated from the initial points of the quadrants B_1, B_2, B_3, and B_4, in the direction of the motion, for then the work $A_1 + A_2$ of the two cross-heads is always expressed by

$$Q r (1 + sin\ \alpha - cos\ \alpha),$$

as given above. In like manner the work absorbed by the masses can always be expressed with sufficient accuracy by

$$L = (m_1 + m_2) \frac{v_1^2 - v^2}{2},$$

where v_1 represents the velocity at the initial point of the quadrant in question.

We see from this that under the supposition $\frac{r}{l} = 0$ the motion of the crank consists of four exactly equal periods, each of which corresponds to a quadrant, and that there is in each quadrant a maximum and a minimum velocity equal to that calculated above. It follows from the above formula for v that when

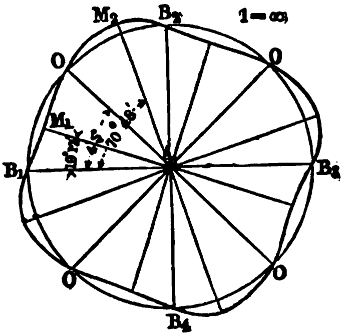
$$\alpha = \frac{\pi}{2} = 90°,$$

also when

$$\alpha = \frac{\pi}{4} = 45°,$$

the velocity $v = v_1$, and consequently we have at the limits of the quadrants and at their middle points the same velocity v_1 as at the dead points. Here also we can take the velocity v_1 at these points equal to the average velocity v_0, particularly as the maximum velocity v_{max} exceeds the velocity v_1 by exactly the amount that v_{min} is less than v_1. Consequently, if we suppose the crank velocities laid off on the crank positions, we shall get the wavy curve $B_1 O B_2 \ldots$, Fig. 566, showing the regular fluctuations of the velocity. Here the angles $B_1 A M_1 = 19° \, 12'$ and $B_1 A M_2 = 70° \, 48'$ were calculated as above.

FIG. 566.



§ 147. **Influence of the Length of Connecting Rods.**—The velocity relations of the crank shaft are quite different when the rods have a finite length, and this case will now be investigated under the supposition that $\frac{r}{l}$ has a value lying between $\frac{1}{4}$ and $\frac{1}{6}$. This supposition requires us to return to the general equation

$$A = A_1 + A_2 - A_3 = L_1 + L_2 + L_3.$$

It was shown above that

$$L = L_1 + L_2 + L_3 = (m_1 + m_2)\frac{v^2 - v_1^2}{2}$$

was approximately true for all the quadrants, provided v_1 represents the crank velocity at the initial points of the quadrants, *i.e.*, at B_1, B_2, B_3, and B_4 respectively. The work A, on the other hand, differs for the different quadrants.

For the motion from B_1 to B_2 in the first quadrant the work

$$A = A_1 + A_2 - A_3$$

is expressed by

$$Qr\left(1+\sin\alpha-\cos\alpha-\frac{4}{\pi}\alpha\right),$$

for the two terms $\frac{r}{2l}\sin^2\alpha$ cancel each other. Consequently the same formula for v is obtained as in the case of infinitely long connecting rods, namely,

$$v=v_1\left(1+\frac{Qr}{(m_1+m_2)v_1^2}\left(1+\sin\alpha-\cos\alpha-\frac{4}{\pi}\alpha\right)\right).$$

The determinations of the preceding article therefore apply here also, for instance, when $\alpha=0$, 45° and 90°, we have $v=v_1$, and for the angles $\alpha_1=19°\ 12'$ and $\alpha_2=70°\ 48'$ we have the corresponding minimum and maximum velocities given by the values

$$v_1\left(1\mp 0.0422\frac{Qr}{(m_1+m_2)v_1^2}\right).$$

For the second quadrant we have found the initial velocity at B_2 the same as at B_1, namely, v_1; on the other hand, the work of the forces is given by

$$Qr\left(1+\sin\alpha-\cos\alpha+\frac{r}{l}\sin^2\alpha-\frac{4}{\pi}\alpha\right),$$

when the angle α is estimated from the initial point B_2 of the *second* quadrant, so that we now get

$$v=v_1\left(1+\frac{Qr}{(m_1+m_2)v_1^2}\left(1+\sin\alpha-\cos\alpha+\frac{r}{l}\sin^2\alpha-\frac{4}{\pi}\alpha\right)\right).$$

Differentiating this expression, we find the maximum and minimum velocities by obtaining the corresponding values of α_1 and α_2 from

$$\cos\alpha + \sin\alpha + \frac{r}{l}\sin 2\alpha - \frac{4}{\pi} = 0,$$

or

$$(\cos\alpha + \sin\alpha)^2 = \left(\frac{4}{\pi} - \frac{r}{l}\sin 2\alpha\right)^2.$$

Consequently

$$1 + \sin 2\alpha = \left(\frac{4}{\pi}\right)^2 - \frac{8}{\pi}\frac{r}{l}\sin 2\alpha + \frac{r^2}{l^2}\sin^2 2\alpha,$$

or

$$\sin 2\alpha = \frac{\left(\frac{4}{\pi}\right)^2 - 1}{1 + \frac{8}{\pi}\frac{r}{l} - \frac{r^2}{l^2}\sin 2\alpha}.$$

From this equation we obtain the following angles and the corresponding leading velocities:

for $\frac{r}{l} = \frac{1}{4}$,

$$\alpha_1 = 11^\circ\ 20\tfrac{1}{2}'; \; v_{min} = v_1\left(1 - 0.02618\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right);$$

$$\alpha_2 = 78^\circ\ 39\tfrac{1}{2}'; \; v_{max} = v_1\left(1 + 0.27618\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right);$$

for $\frac{r}{l} = \frac{1}{5}$,

$$\alpha_1 = 12^\circ\ 17\tfrac{1}{2}'; \; v_{min} = v_1\left(1 - 0.02827\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right);$$

$$\alpha_2 = 77^\circ\ 42\tfrac{1}{2}'; \; v_{max} = v_1\left(1 + 0.22827\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right);$$

for $\frac{r}{l} = \frac{1}{6}$,

$$\alpha_1 = 13^\circ\ 3'; \quad v_{min} = v_1\left(1 - 0.02987\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right),$$

$$\alpha_2 = 76^\circ\ 57'; \quad v_{max} = v_1\left(1 + 0.19155\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right).$$

Moreover, if we substitute

$$\alpha = \frac{\pi}{2} = 90^\circ,$$

we obtain for the velocity at the inner dead point B_3,

$$v_2 = v_1\left(1 + \frac{r}{l}\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right).$$

Consequently one difference between the double and the single crank is, that while in the former the velocities at the inner and outer dead points are unequal, in the latter they are equal.

For the motion of the crank shaft in the third quadrant the work of the forces becomes again

$$A = Q\,r\left(1 + \sin\alpha - \cos\alpha - \frac{4}{\pi}\,\alpha\right),$$

as in the first quadrant, the difference between the two quadrants of motion consisting in this, motion began in the first quadrant at the outer dead point B_1 with the velocity v_1, while in the third quadrant it begins at the inner dead point B_3 with the velocity

$$v_2 = v_1\left(1 + \frac{r}{l}\,\frac{Qr}{(m_1 + m_2)\,v_1^2}\right).$$

In the third quadrant therefore, as in the first, the velocity for the angle of rotation α is found from

$$v = v_2\left(1 + \frac{Q\,r}{(m_1 + m_2)\,v_2^2}\left(1 + \sin\alpha - \cos\alpha - \frac{4}{\pi}\,\alpha\right)\right).$$

We also get, as in the first quadrant, for the minimum and maximum velocities the values

$$\alpha_1 = 19^\circ\ 12'; \quad v_{min} = v_2\left(1 - 0.0422\frac{Q\,r}{(m_1 + m_2)\,v_2^2}\right)$$
$$\alpha_2 = 70^\circ\ 48'; \quad v_{max} = v_2\left(1 + 0.0422\frac{Q\,r}{(m_1 + m_2)\,v_2^2}\right).$$

We also find equal velocities for $\alpha = 0$, 45° and 90°, namely, for each

$$v_2 = v_1\left(1 + \frac{r}{l}\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right).$$

For $\frac{r}{l} = \frac{1}{5}$,

$$v_2 = v_1\left(1 + 0.2\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right);$$

hence

$$v_{max} = v_1\left(1 + 0.2\frac{Q\,r}{(m_1 + m_2)v_1^2}\right)\left(1 + 0.0422\frac{Q\,r}{(m_1 + m_2)\,v_2^2}\right)$$
$$= v_1\left(1 + 0.242\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right) \text{ approximately}$$

and

$$v_{min} = v_1\left(1 + 0.2\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right)\left(1 - 0.0422\frac{Q\,r}{(m_1 + m_2)\,v_2^2}\right)$$
$$= v_1\left(1 + 0.158\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right) \text{ approximately.}$$

In the fourth quadrant the motion begins at B_4 with the same velocity v_2 as in the third; and here, since the work of the forces P and Q is given by

$$A = Q\,r\left(1 + \sin\alpha - \cos\alpha - \frac{r}{l}\sin^2\alpha - \frac{4}{\pi}\alpha\right),$$

we have for this last quadrant of motion

$$v = v_2\left(1 + \frac{Q\,r}{(m_1 + m_2)\,v_2^2}\left(1 + \sin\alpha - \cos\alpha - \frac{r}{l}\sin^2\alpha - \frac{4}{\pi}\alpha\right)\right).$$

If we again place

$$\frac{\delta v}{\delta \alpha} = 0,$$

we get

$$\cos \alpha + \sin \alpha - \frac{r}{l} \sin 2\alpha - \frac{4}{\pi} = 0,$$

or, as above,

$$1 + \sin 2\alpha = \left(\frac{4}{\pi}\right)^2 + \frac{8}{\pi}\frac{r}{l} \sin 2\alpha + \frac{r^2}{l^2} \sin^2 2\alpha,$$

i.e.,

$$\sin 2\alpha = \frac{\left(\frac{4}{\pi}\right)^2 - 1}{1 - \frac{8}{\pi}\frac{r}{l} - \frac{r^2}{l^2} \sin 2\alpha}.$$

If in this equation we substitute for $\frac{r}{l}$ the values $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$, we shall find that the right member is constantly greater than unity, from which we must conclude that with these proportions there is no minimum velocity in the fourth quadrant, but that there is a gradual diminution of the crank velocity from its value at B_4,

$$v_2 = v_1 \left(1 + \frac{r}{l} \frac{Q r}{(m_1 + m_2) v_1^2}\right),$$

till it reaches the original value v_1 at B_1. That this latter value v_1 does actually exist at the end of the motion in the fourth quadrant, is shown by substituting

$$\alpha = \frac{\pi}{2} = 90^\circ$$

in the last formula for v, whereupon we obtain

$$v = v_2 \left(1 - \frac{r}{l}\right) \frac{Q r}{(m_1 + m_2) v_2^2};$$

and when the value of v_2 is substituted,

$$v = v_1\left(1 + \frac{r}{l}\,\frac{Q\,r}{(m_1 + m_2)\,v_1^2}\right)\left(1 - \frac{r}{l}\,\frac{Q\,r}{(m_1 + m_2)\,v_2^2}\right) = v_1.$$

In this connection we may remark that under certain circumstances there will also be a maximum and minimum velocity in the fourth quadrant, namely, when the ratio $\frac{r}{l}$ is still smaller than was assumed above; for when we assumed that $\frac{r}{l} = 0$, we found a maximum and minimum in the fourth as in the other quadrants. The limiting value of $\frac{r}{l}$ for this case is found from the condition $sin\ 2\,\alpha = 1$, or

$$\left(\frac{4}{\pi}\right)^2 - 1 = 0.6211 = 1 - \frac{8}{\pi}\frac{r}{l},$$

when we neglect the small quantity $\left(\frac{r}{l}\right)^2 sin\ 2\,\alpha$. From this follows that $\frac{r}{l}$ must be smaller than 0.148, or about $\frac{1}{7}$, if there is to be a maximum or minimum in the fourth quadrant. For example, if we assume $\frac{r}{l} = \frac{1}{8}$, the corresponding angles may be found from

$$sin\,2\,\alpha = \frac{0.6211}{1 - 0.3183} = 0.9111,$$

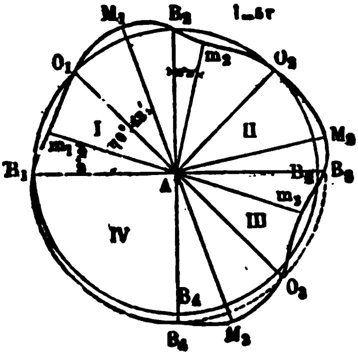
which gives $2\,\alpha = 65°\ 40'$ and $114°\ 20'$; therefore $\alpha_1 = 32°\ 50'$ and $\alpha_2 = 57°\ 10'$.

Finally, the results found are represented by a diagram, Fig. 567, the length of a radius from *A* to the curve representing the velocity of the crank shaft when the crank *A B* has the position of this radius. The diagram is drawn for a ratic

$\frac{r}{l} = \frac{1}{5}$, and, after what has been said, needs no further explanation except that m_1, m_2 . . . represent the points of minimum and M_1, M_2, M_3 the points of maximum velocity. The circle described through B'_3 corresponds to the velocity v_2 at B_3, and this circle is cut by the velocity curve when $\alpha = 45°$, *i.e.*, diametrically opposite to O_1. The position of the intersection O_2 may be found when we make $v = v_1$ in the second quadrant, *i.e.*, from

$$1 + sin\,\alpha - cos\,\alpha + \frac{r}{l}\,sin^2\,\alpha - \frac{4}{\pi}\,\alpha = 0.$$

FIG. 567.

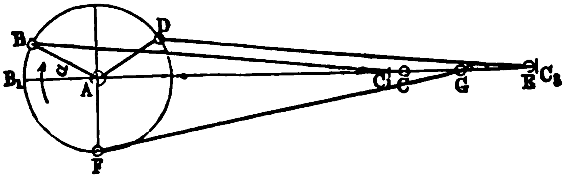


According to what has preceded, the largest of these maximum velocities is in the third quadrant at M_3 and the smallest minimum velocity at m_1 in the first quadrant. Here, therefore, the coefficient of unsteadiness becomes

$$\delta = (0.242 + 0.042)\frac{Q\,r}{(m_1 + m_2)\,v_1^2} = 0.284\,\frac{Q\,r}{(m_1 + m_2)v_1^2}.$$

§ **148. The Triple Crank.**—It occasionally though rarely happens that a shaft is provided with three equal cranks so set that each makes an angle of 120° with its neighbor. The shaft may be driven by three steam pistons, or it may communicate reciprocating motion to three pump pistons. If we regard the initial position of the crank shaft as that in which the crank

FIG. 568.



A B occupies the dead point B_1, Fig. 568, then it is clear that a rotation of 120° will suffice to bring the system into a similar position, for each crank will have taken the position of the preceding one, the connection with its cross-head being exactly the same as that of its predecessor. A complete revolution of the shaft will therefore consist of three periods exactly alike, and it suffices to investigate the motion for a rotation of 120°, but each of these rotations can be separated into two equal parts, each having 60° of rotation, and differing from the other part only in this, that in the first part two cranks *B* and *D* are engaged in forward motion and the third crank *F* in return motion, while in the second half of the 120° one crank *B* is engaged in forward motion and the other two *D* and *F* in return motion. Keeping the symbols employed before, the appropriate equations can be easily deduced as in the preceding cases, provided we take into account that the travel of a cross-head for an angle of rotation α is

$$s = r\left(1 - \cos\alpha - \frac{r}{2l}\sin^2\alpha\right) \text{ for forward stroke,}$$

and

$$s = r\left(1 - \cos\alpha + \frac{r}{2l}\sin^2\alpha\right) \text{ for return stroke.}$$

Accordingly, the work of the exterior forces *P* and *Q* during the rotation α of the crank shaft for the first sixth of the motion is expressed by

$$\begin{aligned} A = &- P r \alpha + Q r\left(1 - \cos\alpha - \frac{r}{2l}\sin^2\alpha\right) \\ &+ Q r\Big[1 - \cos(120° + \alpha) - \frac{r}{2l}\sin^2(120° + \alpha) \\ &\qquad\qquad - \left(1 - \cos 120° - \frac{r}{2l}\sin^2 120°\right)\Big] \\ &+ Q r\Big[1 - \cos(60° + \alpha) + \frac{r}{2l}\sin^2(60° + \alpha) \\ &\qquad\qquad - \left(1 - \cos 60° + \frac{r}{2l}\sin^2 60°\right)\Big]; \end{aligned}$$

from which follows, after slight transformation,

$$A = -Pr\alpha + Qr\left[1 - 2\cos(60^\circ + \alpha) + \frac{r}{l}\sin\alpha\sin(60^\circ - \alpha)\right].$$

In like manner we obtain for the second half of the period

$$A = Pr\alpha + Qr\left[1 - 2\cos(60^\circ + \alpha) - \frac{r}{l}\sin\alpha\sin(60^\circ - \alpha)\right].$$

For a complete revolution of the shaft during the condition of permanency we have

$$P \times 2\pi r = 3 \times Q \times 4r,$$

and from this

$$P = \frac{6}{\pi}Q,$$

and hence the work performed is

$$A = Qr\left[1 - 2\cos(60^\circ + \alpha) \pm \frac{r}{l}\sin\alpha\sin(60^\circ - \alpha) - \frac{6}{\pi}\alpha\right].$$

This work is converted into the living force of the masses m_1 and $3\,m_2$. The rotating mass m_1 changes from the velocity v_1 at the dead point β_1 of the crank into the velocity v, and in so doing stores up the work

$$m_1\frac{v^2 - v_1^2}{2}.$$

The mass m_2 of the cross-head C changes from the velocity 0 to

$$c = v\left(\sin\alpha - \frac{r}{2l}\sin 2\alpha\right).$$

If here, as before, we neglect terms containing

$$\frac{r}{l}\frac{m_2}{m_1},$$

we have for the work stored up by the mass

$$m_2 = \frac{v^2\sin^2\alpha}{2}.$$

In like manner the work absorbed or given up by the masses of the cross-heads E and G may be determined, the initial velocities $v_1 \sin 60°$ becoming $v \sin (60° - \alpha)$ and $v \sin (60° + \alpha)$ respectively. Consequently the total work taken up by the masses is

$$L = m_1 \frac{v^2 - v_1^2}{2} + m^2 \frac{v^2}{2} \left[\sin^2 \alpha + \sin^2 (60° - \alpha) + \sin^2 (60° + \alpha)\right]$$

$$- m^2 \frac{v_1^2}{2} \left(\sin^2 120° + \sin^2 60°\right)$$

$$= m_1 \frac{v^2 - v_1^2}{2} + m_2 \frac{v^2}{2} \left(\sin^2 \alpha + \tfrac{3}{2} \cos^2 \alpha + \tfrac{1}{2} \sin^2 \alpha\right) - \tfrac{3}{2} m_2 \frac{v_1^2}{2}$$

$$= \left(m_1 + \tfrac{3}{2} m_2\right) \frac{v^2 - v_1^2}{2}.$$

If we place this value of L equal to the work A of the exterior forces, we find as in preceding cases the velocity

$$v = \sqrt{v_1^2 + \frac{2 Q r \left[1 - 2 \cos (60° + \alpha) \pm \frac{r}{l} \sin \alpha \sin (60° - \alpha) - \frac{6}{\pi} \alpha\right]}{m_1 + \frac{3}{2} m_2}}$$

$$= v_1 \left[1 - \frac{Q r}{(m_1 + \frac{3}{2} m_2) v_1^2} \left(\frac{6}{\pi} \alpha + 2 \cos (60° + \alpha) \mp \frac{r}{l} \sin \alpha (\sin (60° - a) - 1)\right)\right],$$

when the upper sign refers to the first sextant and the lower sign to the second sextant of the motion.

By differentiation we obtain the equation

$$\frac{6}{\pi} - 2 \sin (60° + \alpha) \mp \frac{r}{l} \left[\sin (60° - \alpha) \cos \alpha - \sin \alpha \cos (60° - \alpha)\right] = 0,$$

or

$$\sin (60 + \alpha) = \frac{3}{\pi} \mp \frac{r}{2 l} \sin (60° - 2 \alpha),$$

from which we may determine the leading velocities.

For the ratio $\frac{r}{l} = \frac{1}{5}$ we obtain for the first sextant:

$$\alpha_1 = 0° 20'; \quad v_{min} = v_1 \left(1 - 0.000 \frac{Q r}{(m_1 + \frac{3}{2} m_2) v_1^2}\right)$$

and

$$\alpha_2 = 39° 18'; \quad v_{max} = v_1 \left(1 + 0.0580 \frac{Q r}{(m_1 + \frac{3}{2} m_2) v_1^2}\right).$$

On the other hand, with the lower sign we have for the second sextant:

$$\alpha_1 = 20^\circ\, 42'; \quad v_{min} = v_1 \left(1 - 0.0580 \frac{Q\, r}{(m_1 + \frac{3}{2} m_2)\, v_1^2}\right)$$

and

$$\alpha_2 = 59^\circ\, 40'; \quad v_{max} = v_1 \left(1 + 0.000 \frac{Q\, r}{(m_1 + \frac{3}{2} m_2)\, v_1^2}\right).$$

This gives for the degree of unsteadiness of the motion:

$$\delta = 2 \times 0.0580 \frac{Q\, r}{(m_1 + \frac{3}{2} m_2)\, v_1^2} = 0.116 \frac{Q\, r}{(m_1 + \frac{3}{2} m_2)\, v_1^2}.$$

When $\alpha = 60^\circ$ we have for both sextants, $v = v_1$.

For infinitely long connecting-rods we obtain, from

$$\frac{6}{\pi} = 2 \sin\left(60^\circ + \alpha\right),$$

for both sextants:

$$\alpha_1 = 12^\circ\, 44'; \quad v_{min} = v_1 \left(1 - 0.0181 \frac{Q\, r}{(m_1 + \frac{3}{2} m_2)\, v_1^2}\right)$$

$$\alpha_2 = 47^\circ\, 16'; \quad v_{max} = v_1 \left(1 + 0.0181 \frac{Q\, r}{(m_1 + \frac{3}{2} m_2)\, v_1^2}\right),$$

hence a coefficient of unsteadiness,

$$\delta = 0.0362 \frac{Q\, r}{(m_1 + \frac{3}{2} m_2)\, v_1^2}.$$

In the latter case we also have for

$$\alpha = 30^\circ = \frac{\pi}{6}$$

the velocity $v = v_1$, but where the ratio $\frac{r}{l} = \frac{1}{5}$, the crank positions for which the velocity $v = v_1$ within the limits of the sextant must be found from

$$\frac{6}{\pi}\, \alpha + 2 \cos\, (60^\circ + \alpha) \mp 0.2 \sin \alpha \sin\, (60^\circ - \alpha) - 1 = 0);$$

the preceding determinations of v_{min} and v_{max} show that these positions coincide very closely with the crank positions for v_{min} in the first sextant and v_{max} in the second sextant.

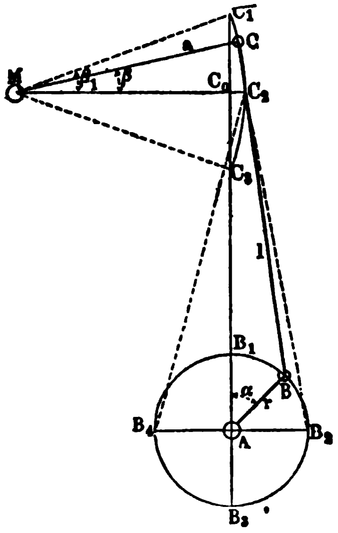
As the period here examined repeats itself three times during a complete rotation of the crank, there are altogether six maximum and six minimum velocities, and there are twelve positions in which the velocity has the value v_1, the dead points of each of the cranks being included in the number. There ought to be no difficulty in representing this motion by a velocity diagram like that given for the single and double crank.

When the crank shaft drives, the above results apply, except that the maximum and minimum velocities exchange places in each sextant.

It is evident, from the denominator $m_1 + \frac{3}{2} m_2$ of the above expression for the velocity, that the reciprocating masses $3\ m_2$ have only half as much influence on the motion of the crank as an equally large mass attached to and rotating with the crank pin. The same remark holds for the double crank and is approximately true for the single one.

§ **149. The Crank and the Beam.**—In large engines it is often the case that the crank is not directly connected with the piston rod, but indirectly by a double lever or beam. Here the end of the connecting rod moves in an arc $C_1 C_2 C_3$, Fig. 569, instead of moving in a straight line. Consequently this is a case of the general quadric crank train (§ 135). The laws of its motion are determined as follows: Let r again represent the crank radius AB, let l be the length BC of the connecting rod, a the lever arm of the beam, β any angle of deviation $C M C_2$ from the middle position $M C_2$, and B_1 the greatest angle of deviation for the extreme position $M C_1$. We will first suppose an arrangement in which the crank shaft A is situated on the direction of the chord $C_1 C_3$ of the arc of oscillation, and let γ represent the inclination of the connecting rod to this usually

FIG. 569.



vertical chord. It is clear that with this arrangement the two dead points B_1 and B_3 of the crank will be diametrically opposite, so that exactly a half revolution will be needed for the up-stroke as well as for the down-stroke of the beam.

To find a relation between the crank angle α and the travel s of the end C of the beam at which the constant force Q may be regarded as acting, we have the following equations for the horizontal and vertical projection of the polygon $ABCMC_1A$:

$$a \cos \beta + l \sin \gamma = r \sin \alpha + a \cos \beta_1$$

and

$$r \cos \alpha + l \cos \gamma - a \sin \beta + a \sin \beta_1 = l + r;$$

or, since $a \sin \beta_1 = r$,

$$r \cos \alpha + l \cos \gamma = l + a \sin \beta.$$

From these two equations we obtain

$$l^2 (\sin^2 \gamma + \cos^2 \gamma) = l^2 = [r \sin \alpha - a (\cos \beta - \cos \beta_1)]^2 + (l - r \cos \alpha + a \sin \beta)^2.$$

From this follows

$$a \sin \beta = r \cos \alpha - l + \sqrt{l^2 - [r \sin \alpha - a (\cos \beta - \cos \beta_1)]^2};$$

or, approximately,

$$a \sin \beta = r \cos \alpha - \frac{[r \sin \alpha - a (\cos \beta - \cos \beta_1)]^2}{2 l}.$$

If in this we substitute

$$\cos \beta - \cos \beta_1 = 1 - \tfrac{1}{2} \sin^2 \beta - (1 - \tfrac{1}{2} \sin^2 \beta_1) = \tfrac{1}{2} (\sin^2 \beta_1 - \sin^2 \beta),$$

we get

$$a \sin \beta = r \cos \alpha - \frac{[r \sin \alpha - \frac{1}{2} a (\sin^2 \beta_1 - \sin^2 \beta)]^2}{2 l}.$$

It will be sufficiently exact to substitute in the right member

$$\sin \beta = \frac{r \cos \alpha}{a};$$

and since we also have

$$\sin \beta_1 = \frac{r}{a},$$

it follows that

it follows that

$$a \sin \beta = r \cos \alpha - \frac{1}{2\,l}\left[r \sin \alpha - \frac{r^2}{2\,a}(1 - \cos^2 \alpha)\right]^2$$

$$= r \cos \alpha - \frac{r^2 \sin^2 \alpha}{2\,l}\left(1 - \frac{r \sin \alpha}{2\,a}\right)^2,$$

or

$$\sin \beta = \frac{r \cos \alpha}{a} - \frac{r^2 \sin^2 \alpha}{2\,a\,l}\left(1 - \frac{r \sin \alpha}{2\,a}\right)^2.$$

As $\frac{r}{a}$ is seldom greater than $\frac{1}{8}$ and often less, we may neglect the higher powers of $\frac{r}{a}$ and then obtain the approximation

$$\sin \beta = \frac{r \cos \alpha}{a} - \frac{r^2 \sin^2 \alpha}{2\,a\,l} + \frac{r^3 \sin^3 \alpha}{2\,a^2\,l}.$$

The arc β can be determined from the series

$$\beta = \sin \beta + \tfrac{1}{6} \sin^3 \beta + \ldots$$

and this gives

$$\beta = \frac{r \cos \alpha}{a} - \frac{r^2 \sin^2 \alpha}{2\,a\,l} + \frac{r^3 \sin^3 \alpha}{2\,a^2\,l} + \frac{r^3 \cos^3 \alpha}{6\,a^3},$$

and for $\alpha = 0$,

$$\beta_1 = \frac{r}{a} + \frac{r^3}{6\,a^3}.$$

From this we finally get the travel $C_1 C = s = a\,(\beta_1 - \beta)$ of the end of the beam when the crank turns through the angle $B_1 A B = \alpha$:

$$s = r\left[1 - \cos \alpha + \frac{r}{2\,l} \sin^2 \alpha - \frac{r^2}{2\,a\,l} \sin^3 \alpha + \frac{r^2}{6\,a^2}(1 - \cos^3 \alpha)\right].$$

This formula applies to the forward stroke of the crank from B_1 to B_3; in like manner, for the return stroke, *i.e.*, for the upward motion of the beam from C_3 to C_1,

$$(s) = r\left[1 - \cos \alpha - \frac{r}{2\,l} \sin^2 \alpha - \frac{r^2}{2\,a\,l} \sin^3 \alpha + \frac{r^2}{6\,a^2}(1 - \cos^3 \alpha)\right].$$

For $\alpha = 0$, we have of course

$$s = (s) = 0,$$

and for $\alpha = 180°$,

$$s = (s) = 2\,r\left(1 + \frac{r^2}{6\,a^2}\right) = 2\,a \times arc\ sin\ \frac{r}{a} = 2\,a\,\beta_1.$$

On the other hand, for $\alpha = 90°$, that is, when the crank pin is at B_2 and B_4, we have

$$s = r\left(1 + \frac{r}{2\,l} - \frac{r^2}{2\,a\,l} + \frac{r^2}{6\,a^2}\right)$$

and

$$(s) = r\left(1 - \frac{r}{2\,l} - \frac{r^2}{2\,a\,l} + \frac{r^2}{6\,a^2}\right).$$

Now if a constant force Q acts tangentially to the path of the end C of the beam overcoming the resistance P at the crank pin, the relation between the forces will be given by the equation.

$$2\,Q \times 2\,r\left(1 + \frac{r^2}{6\,a^2}\right) = P \times 2\,\pi\,r,$$

or

$$P = Q\,\frac{2}{\pi}\left(1 + \frac{r^2}{6\,a^2}\right).$$

The work stored up in accelerating the masses while the crank turns through α may accordingly be found from

$$L = Q\,s - P\,r\,\alpha = Q\,r\left[1 - cos\ \alpha \pm \frac{r}{2\,l}\,sin^2\ \alpha - \frac{r^2}{2\,a\,l}\,sin^3\ \alpha \right.$$
$$\left. + \frac{r^2}{6\,a^2}\,(1 - cos^3\ \alpha) - \frac{2}{\pi}\,\alpha\left(1 + \frac{r^2}{6\,a^2}\right)\right].$$

As in former determinations, the angle α corresponding to the maximum and minimum velocities of the crank is found from the condition $\frac{\delta L}{\delta \alpha} = 0$, *i.e.*, from the equation

$$sin\ \alpha \pm \frac{r}{2\,l}\,sin\ 2\,\alpha - \frac{3\,r^2}{2\,a\,l}\,sin^2\ \alpha\ cos\ \alpha$$
$$+ \frac{3\,r^2}{6\,a^2}\,cos^2\ \alpha\ sin\ \alpha - \frac{2}{\pi}\left(1 + \frac{r^2}{6\,a^2}\right) = 0,$$

or

$$sin\ \alpha = \frac{2}{\pi}\left(1 + \frac{r^2}{6\,a^2}\right) - \frac{r}{2\,l}\,sin\ 2\,\alpha\left(\pm 1 - \frac{3\,r}{2\,a}\,sin\ \alpha + \frac{r\,l}{2\,a^2}\,cos\ \alpha\right).$$

If m_1 again represents the mass reduced to the crank pin, and if the reciprocating mass m_2 reduced to the point C is neglected, we have as before

$$v = v_1 \left(1 + \frac{L}{m_1 v_1^2}\right);$$

and by substituting in this the values of L corresponding to the values of α, we obtain the maximum and minimum velocities.

For the ordinary case, *i.e.*, $\frac{r}{l} = \frac{1}{5}$ and $\frac{r}{a} = \frac{1}{3}$, we get

$$\sin \alpha = \frac{2}{\pi}\frac{55}{54} - 0.1 \sin 2\alpha \left(\pm 1 - \tfrac{1}{2} \sin \alpha + \tfrac{5}{18} \cos \alpha\right),$$

and

$$L = Q r \left(1 - \cos \alpha \pm 0.1 \sin^2 \alpha - \tfrac{1}{30} \sin^3 \alpha \right.$$
$$\left. + \tfrac{1}{54}(1 - \cos^3 \alpha) - \frac{55}{54}\frac{2}{\pi} \alpha\right).$$

From the first formula are found the two angles for the down-stroke of the beam :

$$\alpha_1 = 34^\circ\ 4' \quad \text{and} \quad \alpha_2 = 136^\circ\ 5',$$

and for the up-stroke

$$(\alpha_1) = 49^\circ\ 35' \quad \text{and} \quad (\alpha_2) = 148^\circ\ 56'.$$

Introducing these values into L, we obtain for the down-stroke

$$v_{min} = v_1 \left(1 - 0.1804 \frac{Q r}{m_1 v_1^2}\right) \quad \text{for } \alpha_1 = 34^\circ\ 4';$$

$$v_{max} = v_1 \left(1 + 0.2428 \frac{Q r}{m_1 v_1^2}\right) \quad \text{for } \alpha_2 = 136^\circ\ 5';$$

and for the up-stroke

$$(v_{min}) = v_1 \left(1 - 0.2687 \frac{Q r}{m_1 v_1^2}\right) \quad \text{for } (\alpha_1) = 49^\circ\ 35';$$

$$(v_{max}) = v_1 \left(1 + 0.1701 \frac{Q r}{m_1 v_1^2}\right) \quad \text{for } (\alpha_2) = 148^\circ\ 56';$$

Consequently the degree of unsteadiness is given by

$$\delta = (0.2428 + 0.2687) \frac{Q r}{m_1 v_1^2} = 0.5115 \frac{Q r}{m_1 v_1^2},$$

which value differs so slightly from the value $\delta = 0.5154$ (see

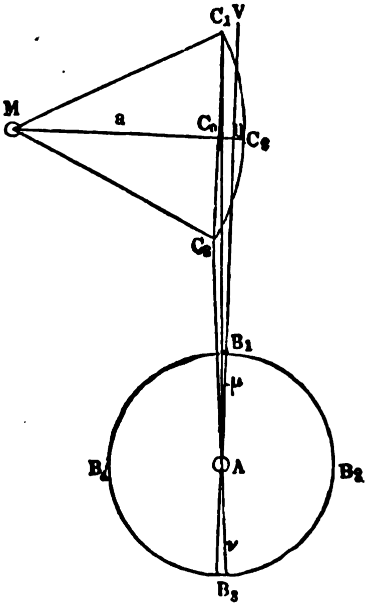
§ 145), found for the direct motion and for the same ratio $\frac{r}{l} = \frac{1}{5}$, that in this respect there is no practical difference between the two cases.

The usual constructions vary slightly from that assumed in the calculation, the crank shaft *A* being situated not in the chord $C_1 C_3$, but in a line parallel to this chord and bisecting the rise $C_0 C_2$ of the arc of the beam. This arrangement is chosen in order that the connecting rod *BC* may deviate equally to each side of the stroke. If we represent the rise $C_0 C_2$ by e, it is clear that for the arrangement of Fig. 569 the two greatest deviations γ of the connecting rod on opposite sides of the chord will be approximately given by

$$sin\ \gamma_1 = \frac{r - e}{l} \quad \text{and} \quad sin\ \gamma_2 = \frac{r + e}{l}.$$

If these two deviations from the vertical were to be made equal, it is obvious that the crank shaft *A* would have to be placed on a tangent to the arc $C_1 C_2 C_3$ at C_2 and parallel to the chord, but usually the shaft *A* is placed at a distance from the chord equal to only half the rise, $\frac{e}{2}$, in order to diminish the evil attending a distant location, for we see from Fig. 570, in which the shaft *A* is located on the parallel *VOA* bisecting the rise at *O*, that the two dead points B_1 and B_3 are no longer diametrically opposite, the arc $B_1 B_2 B_3$ being smaller than $B_3 B_4 B_1$. In consequence of this inequality in the rotation of the crank on the up- and down-stroke of the beam, a certain unsteadiness in the running of the machine must of course arise, which becomes more marked the farther

FIG. 570.



the axis A is from the chord $C_1 C_3$. If we assume this distance $C_0 O = \frac{1}{2} C_0 C_2 = \frac{1}{2} e$, as is ordinarily done, the inequality occasioned by it will be very small, as the following examination shows. If at the dead points the deviations of the connecting rod from the vertical are represented by μ and ν, *i.e.*, $C_1 A O = \mu$ and $C_3 A O = \nu$, we have as a very close approximation

$$\mu = \frac{C_0 O}{C_1 A} = \frac{e}{2(l+r)} \quad \text{and} \quad \nu = \frac{C_0 O}{C_3 A} = \frac{e}{2(l-r)}.$$

As the half swing β_1 of the beam is small, we can assume the rise to be

$$e = \frac{C_1 C_0^2}{2a} = \frac{r^2}{2a},$$

and then get for the angles μ and ν,

$$\mu = \frac{r^2}{4a(l+r)} \quad \text{and} \quad \nu = \frac{r^2}{4a(l-r)}.$$

For instance, when $\frac{r}{a} = \frac{1}{5}$ and $\frac{r}{l} = \frac{1}{5}$,

$$\mu = \tfrac{1}{72}; \quad \text{hence} \quad \mu^\circ = \frac{180^\circ \mu}{\pi} = 0^\circ\, 48';$$

and

$$\nu = \tfrac{1}{48}; \quad \text{hence} \quad \nu^\circ = \frac{180^\circ \nu}{\pi} = 1^\circ\, 12'.$$

Consequently the distance apart of the dead points will be

$$180^\circ \pm (\nu - \mu) = 180^\circ\, 24' \quad \text{and} \quad 179^\circ\, 36'.$$

The travel of the end of the beam for these small rotations, μ and ν, of the crank is, therefore,

$$\sigma_1 = r(1 - \cos\mu) = r(1 - \cos 0^\circ\, 48') = 0.00010\, r,$$

and

$$\sigma_2 = r(1 - \cos\nu) = r(1 - \cos 1^\circ\, 12') = 0.00022\, r.$$

This travel or inequality of motion produced by the one-sided location of the crank shaft is so small that it can be neglected.

§ 150. Crank with Variable Force on the Piston.—Up to this point we have assumed in all investigations a constant force Q acting on the cross-head, an assumption approximately true for pumps and for steam-engines working without expansion. But if the steam is cut off before the stroke is completed, so that it may work with expansion, the result will be that as soon as expansion begins the pressure on the piston will diminish; consequently in an engine working with expansion we must distinguish between two periods of motion within each stroke, during the first part of which, known as the period of *admission*, the force of the piston is constant, while during the second or *expansion* period it is variable, as was more fully explained in Vol. II. In blowing engines, where during each stroke a certain volume of air is forced into a space in which there is greater pressure, there are two such different periods of motion, the first characterized by a *gradually diminishing* resistance known as the period of *compression*, while in the second period there is a *constant* resistance on the piston. It makes no difference in the theory of the crank whether the piston drives, as in steam-engines, or resists, as in pumps, except that the maximum and minimum velocities change places, a retardation occurring in one case and an acceleration in the other under otherwise equal circumstances. As the expansion of steam is of great importance in practice, we will investigate the crank motion for an engine working with expansion, and will assume a ratio of expansion ϵ, *i.e.*, that the steam admitted into the cylinder can expand to ϵ times its volume. If $2r$ again represents the whole stroke, $\frac{2r}{\epsilon}$ will represent the piston travel during admission or while the piston force is constant, and $\frac{\epsilon - 1}{\epsilon} 2r$ the travel during expansion or while the pressure is diminishing. The law according to which the steam pressure diminishes during expansion was also more fully discussed in Vol. II. In the following investigation we will assume that the expansion takes place according to *Mariotte's* law. To be sure, this law is only an approximate one, but the use of more exact laws and formulas, such as are given by the mechanical theory of heat, would unnecessarily render the investigation more difficult. Let Q again represent the steam pressure on the piston before expansion, and let R represent the counter pressure on the back of the piston due to the atmospheric pressure or the pressure in the condenser. This counter pressure R must here be specially taken into account, because it is constant during the whole stroke of the piston. In the preceding investigations of the crank motion with constant force on the piston this special introduction of the counter pressure R could be omitted so long as the force Q on the piston represented the difference of pressure on the two sides of the piston. To determine the circumferential force P on the crank pin, we must again suppose the engine to have attained its condition of permanency, and may then place the work $P \pi r$ of the force P during a half revolution equal to the work of the force on the piston during a single stroke. The latter quantity of work is composed of the work A_1 before the beginning of expansion and the work A_2 during the expansion, diminished by the work A_3 of the counter pressure. Therefore when $s_1 = 2r$ represents the whole stroke, and $s' = \frac{2r}{\epsilon}$ the travel before cut-off, we have, according to Vol. I. § 388, for the work of one stroke,

$$A_1 + A_2 - A_3 = Q s' \left(1 + hyp\ log \frac{s_1}{s'}\right) - R s_1$$

$$= Q \frac{2r}{\epsilon} \left(1 + hyp\ log\ \epsilon\right) - R\, 2r.$$

Placing this equal to $P \pi r$, we have

$$P = \frac{2}{\pi \epsilon} (1 + hyp\ log\ \epsilon)\, Q - \frac{2}{\pi} R.$$

The angle α_ϵ, through which the crank turns when moving from the inner dead point to where expansion begins, may be found from

$$s' = \frac{2r}{\epsilon} = r \left(1 - cos\ \alpha_\epsilon \mp \frac{r}{2l} sin^2\ \alpha_\epsilon\right),$$

or

$$\cos \alpha_e = 1 - \frac{2}{\epsilon} \mp \frac{r}{2l} \sin^2 \alpha_e .$$

For the motion *before* expansion we again have

$$L = m_1 \frac{v^2 - v_1^2}{2} = (Q - R)\, r \left(1 - \cos \alpha \mp \frac{r}{2l} \sin^2 \alpha\right) - P r \alpha,$$

from which we obtain

$$v = \sqrt{v_1^2 + 2 \frac{Q - R}{m_1} r \left(1 - \cos \alpha \mp \frac{r}{2l} \sin^2 \alpha\right) - 2 \frac{P r}{m_1}},$$

or, approximately,

$$v = v_1 \left\{ 1 + \frac{r}{m_1 v_1^2} \left[(Q - R) \left(1 - \cos \alpha \mp \frac{r}{2l} \sin^2 \alpha\right) - P \alpha \right] \right\}.$$

To find the angle α_1 corresponding to the minimum velocity, we again place

$$\frac{\delta v}{\delta \alpha} = 0,$$

and get

$$(Q - R) \left(\sin \alpha \mp \frac{r}{2l} \sin 2\alpha\right) - P = 0,$$

or

$$\sin \alpha = \frac{P}{Q - R} \pm \frac{r}{2l} \sin 2\alpha = \frac{P}{Q\left(1 - \frac{R}{Q}\right)} \pm \frac{r}{2l} \sin 2\alpha.$$

If we substitute in this the ratio determined above,

$$\frac{P}{Q} = \frac{2}{\pi} \left(\frac{1 + hyp \log \epsilon}{\epsilon} - \frac{R}{Q} \right),$$

we obtain

$$\sin \alpha_1 = \frac{2}{\pi \left(1 - \frac{R}{Q}\right)} \left(\frac{1 + hyp \log \epsilon}{\epsilon} - \frac{R}{Q} \right) \pm \frac{r}{2l} \sin 2\alpha_1 .$$

For any crank angle α greater than α_e, *i.e.*, for any angle within the period of expansion, the work of the piston can be found as follows: During admission the steam acted through a distance

$$s' = \frac{2r}{\epsilon};$$

after admission, and at the instant considered, the volume has increased in the ratio

$$r\left(1 - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha\right) \text{ to } \frac{2r}{\epsilon};$$

consequently we have for the ratio of expansion at this instan

$$\frac{\epsilon}{2}\left(1 - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha\right),$$

and from this the *total* work of the steam on the front surface of the piston is

$$A_1 = Q\frac{2r}{\epsilon}\left[1 + hyp\ log\ \frac{\epsilon}{2}\left(1 - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha\right)\right].$$

During this period the counter pressure R has performed the work

$$R \cdot r\left(1 - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha\right);$$

hence the total work expended in accelerating the masses during the rotation of the crank through α is

$$L = Q\frac{2r}{\epsilon}\left[1 + hyp\ log\ \frac{\epsilon}{2}\left(1 - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha\right)\right]$$
$$- Rr\left(1 - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha\right) - Pr\alpha.$$

Then in the same manner as before, we find from

$$L = m_1\frac{v^2 - v_1^2}{2},$$

for v the expression

$$v = v_1\left(1 + \frac{L}{m_1 v_1^2}\right).$$

To determine the angle α for the maximum velocity, we again place $\frac{\delta v}{\delta \alpha} = 0$, *i.e.*,

$$Q\frac{2}{\epsilon}\,\frac{\sin\alpha \mp \frac{r}{2l}\sin 2\alpha}{1 - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha} - R\left(\sin\alpha \mp \frac{r}{2l}\sin 2\alpha\right) = P,$$

or

$$\left(\sin\alpha \mp \frac{r}{2l}\sin 2\alpha\right)\left(\frac{2}{\epsilon}\,\frac{Q}{1 - \cos\alpha \mp \frac{r}{2l}\sin^2\alpha} - R\right) = P.$$

This can also be written

$$\frac{\sin\alpha\left(1 \mp \frac{r}{l}\cos\alpha\right)}{2\sin^2\frac{\alpha}{2}\left(1 \mp \frac{r}{l}\cos^2\frac{\alpha}{2}\right)}\left[\frac{2}{\epsilon}Q - 2R\sin^2\frac{\alpha}{2}\left(1 \mp \frac{r}{l}\cos^2\frac{\alpha}{2}\right)\right] = P,$$

or

$$\cot\frac{\alpha}{2} = \frac{P}{\frac{2}{\epsilon}Q\,\frac{1 \mp \frac{r}{l}\cos\alpha}{1 \mp \frac{r}{l}\cos^2\frac{\alpha}{2}} - 2R\sin^2\frac{\alpha}{2}\left(1 \mp \frac{r}{l}\cos\alpha\right)}$$

$$= \frac{P}{\frac{2}{\epsilon}Q\left(1 \pm \frac{r}{l}\sin^2\frac{\alpha}{2}\right) - 2R\sin^2\frac{\alpha}{2}\left(1 \mp \frac{r}{l}\cos\alpha\right)}.$$

If we substitute in this the value of P,

$$\frac{2}{\pi}\left(\frac{1 + hyp\log\epsilon}{\epsilon} - \frac{R}{Q}\right)Q,$$

we finally obtain

$$cot\,\frac{\alpha}{2} = \frac{1 + hyp\ lag\ \epsilon - \dfrac{\epsilon R}{Q}}{\pi\left[1 \pm \dfrac{r}{l}\,sin^2\dfrac{\alpha}{2} - \dfrac{\epsilon R}{Q}\,sin^2\dfrac{\alpha}{2}\left(1 \mp \dfrac{r}{l}\,cos\ \alpha\right)\right]}.$$

If we assume the most advantageous ratio of expansion ϵ, *i.e.*, one with which the pressure $\frac{Q}{\epsilon}$ at the end of the expansion is just equal to the counter pressure R, the above expressions for the angles α_1 and α_2 corresponding to minimum and maximum velocities become simpler; thus for the minimum velocity (before expansion),

$$sin\ \alpha_1 = \frac{2\ hyp\ log\ \epsilon}{\pi(\epsilon - 1)} \pm \frac{r}{2l}\,sin\ 2\,\alpha,$$

and for the maximum velocity,

$$cot\,\frac{\alpha_2}{2} = \frac{hyp\ log\ \epsilon}{\pi\ cos^2\dfrac{\alpha}{2}\left(1 \pm \dfrac{2r}{l}\,sin^2\dfrac{\alpha}{2}\right)}.$$

For this case the circumferential force P becomes

$$P = \frac{2}{\pi}\left(\frac{1 + hyp\ log\ \epsilon}{\epsilon} - \frac{1}{\epsilon}\right)Q = \frac{2}{\pi}\,\frac{Q}{\epsilon}\,hyp\ log\ \epsilon = \frac{2}{\pi}\,R\ hyp\ log\ \epsilon$$

consequently the leading velocities are :

$$v_{min} = v_1\left\{1 + \left[(\epsilon - 1)\left(1 - cos\ \alpha \mp \frac{r}{2l}\,sin^2\alpha\right)\right.\right.$$
$$\left.\left. - \frac{2}{\pi}\,\alpha\ hyp\ log\ \epsilon\right]\frac{Q r}{\epsilon m_1 v_1'^2}\right\}$$

and

$$v_{max} = v_1\left\{1 + \left[1 + cos\ \alpha \pm \frac{r}{2l}\,sin^2\alpha + \right.\right.$$
$$\left.\left. 2\ hyp\ log\left(1 - cos\ \alpha \mp \frac{r}{2l}\,sin^2\alpha\right)\frac{\epsilon}{2} - \frac{2}{\pi}\,\alpha\,hyp\ log\ \epsilon\right]\frac{Q r}{\epsilon m_1 v_1'^2}\right\}.$$

If we again assume $\frac{r}{l} = \frac{1}{5}$ and $\epsilon = 2$, we obtain from

$$sin\ \alpha_1 = \frac{2\ hyp\ log\ 2}{\pi} \pm 0.1\ sin\ 2\ \alpha_1$$

the angles

$$\alpha_1 = 32^\circ\ 5\tfrac{1}{2}' \text{ for forward stroke,}$$

and

$$(\alpha_1) = 21^\circ\ 50' \text{ for return stroke.}$$

By introducing these values into the equations for L and v, we get

$$v_{min} = \left(1 - 0.0613 \frac{Q r}{m_1 v_1^2}\right) v_1$$

and

$$(v_{min}) = \left(1 - 0.0667 \frac{Q r}{m_1 v_1^2}\right) v_1 .$$

On the other hand, for the angle α_2 corresponding to the maximum velocities, we have

$$cot\ \frac{\alpha}{2} = \frac{hyp\ log\ 2}{\pi\ cos^2 \frac{\alpha}{2}\left(1 \pm 0.4\ sin^2 \frac{\alpha}{2}\right)},$$

which equation is satisfied by the angles

$$\alpha_2 = 116^\circ\ 32' \text{ for the forward stroke,}$$
$$(\alpha_2) = 104^\circ\ 12' \text{ for the return stroke.}$$

The corresponding maximum velocities are

$$v_{max} = \left(1 + 0.1803 \frac{Q r}{m_1 v_1^2}\right) v_1$$

and

$$(v_{max}) = \left(1 + 0.2212 \frac{Q r}{m_1 v_1^2}\right) v_1 .$$

Therefore the degree of unsteadiness becomes

$$\delta = (0.2212 + 0.0667) \frac{Q r}{m_1 v_1^2} = 0.2879 \frac{Q r}{m_1 v_1^2} ;$$

or since

$$P = \frac{2\,Q}{\pi\,\epsilon}\,hyp\ log\ \epsilon = 0.22063\,Q,$$

we have

$$\delta = \frac{0.2879}{0.2206}\frac{Pr}{m_1 v_1^2} = 1.305\,\frac{Pr}{m_1 v_1^2}.$$

The following table gives the coefficients of unsteadiness for different ratios of expansion when $l = 5\,r$.

Ratio of expansion $\epsilon =$	2	3	4	5	6	
Coefficient of unsteadiness $\delta =$	1.305	1.374	1.421	1.453	1.477	$\frac{Pr}{m_1 v_1^2}$.

§ **151. Accelerating Force.**—The transmission of the force Q on the piston of a steam engine or pump, to the crank pin is considerably influenced by the action of the reciprocating masses m_2, as will be seen in the following investigation. If we suppose the crank pin to turn with a perfectly or approximately uniform velocity v, the reciprocating mass m_2 will move with a very variable velocity during every half revolution. Thus the velocity c of the cross-head is 0 at the dead point of the crank, and it then increases till its speed becomes a maximum at or near the middle of its stroke, according as the connecting rod is infinite or finite. Thereupon the velocity of the reciprocating mass gradually diminishes till it again becomes 0 at the other dead point. This is repeated at every succeeding half revolution, consequently during the first part of each stroke a certain force is needed to accelerate the cross-head, the work performed by the accelerating force being stored as living force in the reciprocating mass. When the latter attains its greatest velocity the acceleration is 0, and now changes into a retardation. Therefore during the second part of the motion the work stored up in the mass m_2 is given up, its amount being exactly equal to that expended in accelerating it during the first part of the stroke, for at the end of the stroke the velocity of the cross-head is 0, as at the beginning. In all cases, therefore, whether the piston or the crank drives, the reciprocating mass m_2 resists the motion during the first part of the stroke and assists it during the second part. In pumps where the shaft drives, the work needed for accelerating the mass m_2 must of course be expended by the shaft, the resistance which it overcomes being greater during the period of acceleration than during the period of retardation in which the living force of the reciprocating mass assists the motion. On the other hand, in steam engines, acceleration of the mass m_2 is directly effected by the steam pressure Q acting on the piston, only the excess of Q over the force needed for acceleration being transmitted through the connecting rod to the crank pin. In order to determine what force is transmitted from the connecting rod to the cross-head, or *vice versa*, it is necessary to determine the pressure needed for this acceleration of the mass m_2, which pressure we will call the accelerating force and designate it by M. It is also clear from what has preceded that this action of the masses does not involve any loss or gain of work.

The acceleration of a mass whose velocity is c may be found from the general expression $\frac{\delta c}{\delta t}$ (Vol. I, § 20), where t is the time. Now, since the velocity of the cross-head is

$$c = \left(\sin \alpha \mp \frac{r}{2l} \sin 2\alpha\right) v,$$

the accelerating force of a unit of mass, when the circumferential velocity v is constant, must be

$$p = \frac{\delta c}{\delta t} = \left(\cos \alpha \mp \frac{r}{l} \cos 2\alpha\right) v \frac{\delta \alpha}{\delta t}.$$

So we have

$$r\,\delta\alpha = v\,\delta t, \quad \text{or} \quad \frac{\delta \alpha}{\delta t} = \frac{v}{r}$$

when the crank pin has a constant velocity v, it follows that

$$p = \left(\cos \alpha \mp \frac{r}{l} \cos 2\alpha\right) \frac{v^2}{r}$$

is the accelerating force per unit of mass, and

$$M = m_2 p = \left(\cos \alpha \mp \frac{r}{l} \cos 2\alpha\right) \frac{m_2 v^2}{r},$$

the accelerating force for the mass m_2.

When the connecting rod is infinitely long the acceleration becomes

$$M = \cos \alpha \frac{m_2 v^2}{r},$$

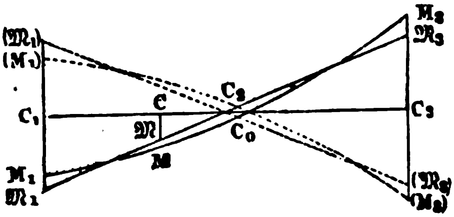
hence it is zero when $\alpha = 90°$ and equal to

$$\mp \frac{m_2 v^2}{r}$$

at the dead points when $\alpha = 0$ and $\alpha = 180°$, *i.e.*, it is exactly equal to what the *centrifugal* acceleration of this same mass m_2 would be were it fastened to the crank pin. If we suppose $C_1 C_3$, Fig. 571, to be an axis of abscissas whose length $C_1 C_3 = 2r$, and lay off on this axis from C_1 and C_3 the piston travel $s = C_1 C$ as abscissa and the corresponding acceleration p as ordinate $C\mathfrak{M}_1$, and repeat this construction for different points of the forward and return stroke, we shall obtain for the curve of acceleration two straight lines passing through the centre C_2, of which one $\mathfrak{M}_1 \mathfrak{M}_3$ belongs to the forward stroke and the other dotted line $(\mathfrak{M}_1)(\mathfrak{M}_3)$ to the return stroke, the ordinates at the end being

$$C_1 \mathfrak{M}_1 = C_3 \mathfrak{M}_3 = \frac{m_2 v^2}{r}.$$

Fig. 571.



For the travel of the cross-head, when it has reached any position C, is

$$s = C_1 C = r(1 - \cos \alpha);$$

hence

$$C_2 C = r \cos \alpha:$$

and since

$$C\mathfrak{M} = \frac{m_2 v^2}{r} \cos \alpha,$$

it follows that

$$\frac{C_2 C}{C\mathfrak{M}} = \text{constant},$$

which is the equation of condition of a straight line.

On the other hand, when the connecting rod is of finite length the accelerations at the dead points C_1 and C_3 are

$$\frac{m_2 v^2}{r}\left(1 \mp \frac{r}{l}\right),$$

for instance, when $l = 5r$, they are

$$0.8\,\frac{m_2 v^2}{r} \quad \text{and} \quad 1.2\,\frac{m_2 v^2}{r}.$$

The accelerations are now bounded by two curved lines $M_1 M_3$ and $(M_3)(M_1)$ which intersect the axis $C_1 C_3$ at a point C_0, somewhat back of the centre C_2, corresponding to a point at which the velocity of the cross-head is a maximum, and for which the condition

$$\cos \alpha = \pm \frac{r}{l} \cos 2\alpha$$

was established in § 139. These two curves of acceleration are symmetrical with respect to the axis $C_1 C_3$, and can be easily constructed for a given ratio $\frac{r}{l}$. This may best be done by calculating the ordinates CM for a number of piston positions, the trigonometric functions $\cos \alpha$ and $\cos 2\alpha$ for the corresponding crank positions being taken from trigonometric tables.

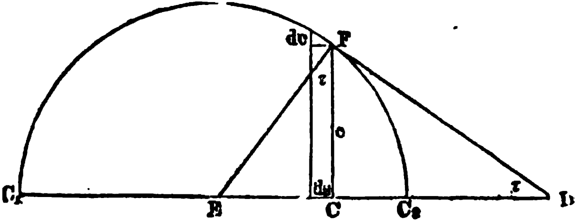
We may here mention that this acceleration curve can be directly constructed from the velocity curve $C_1 F C_3$, Fig. 572, found in § 139, its ordinate CF representing the velocity v of the cross head. For example, at any position C for which the velocity of the cross-head is given by $c = CF$, the acceleration $p = \frac{\delta c}{\delta t}$ is simply the sub-normal CE of the point F, and for the following reason: If τ represents the angle made by the curve at F with the axis $C_1 C_3$, we have $\delta c = \delta s \times \tan \tau$, or

$$\frac{\delta c}{\delta t} = \frac{\delta s}{\delta t} \tan \tau = c \tan \tau;$$

i.e., according to the figure

$$\frac{\delta c}{\delta t} = p = CE.$$

FIG. 572.



But this method of finding the acceleration curve is not advisable, for the reason that it is generally difficult to draw the tangents with sufficient accuracy, and, in the present case, the inaccuracies caused are augmented by the unavoidable inaccuracies of the velocity curve itself. It is better therefore to construct the curve of acceleration in the manner indicated above.

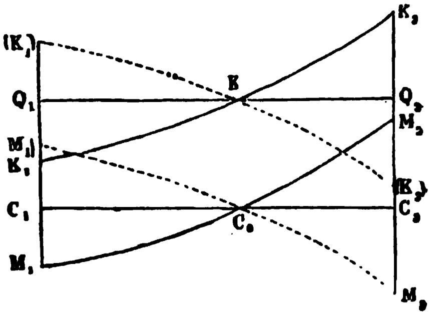
§ **152. Piston Pressure.**—It is now easy to determine the pressure transmitted at every instant of the motion, from the cross-head, through the connecting rod to the crank pin, this pressure being simply the resultant of the force Q acting directly on the piston and of the accelerating force M.* Graphical method may here be employed with advantage. Let $C_1 C_3 = 2r$, Fig. 573, be an axis of abscissas, and from every point of it lay off ordinates equal to the force Q acting on the

---

*See *Radinger*, "Ueber Dampfmaschinen mit hoher Kolbengeschwindigkeit," Zeitschrift des oesterr. Ing. u. Archit. Vereins, 1869.

surface of the piston; a diagram $C_1 Q_1 Q_3 C_3$ is thus obtained. If we first suppose that the force Q on the piston is constant, as in pumps and in engines working without expansion, the line $Q_1 Q_3$ limiting the ordinates will be straight and parallel to the axis $C_1 C_3$. Also draw on this diagram the curve for the accelerating force M. In the steam engine, where the force M, needed to accelerate the cross-head, is exerted directly by the steam pressure on the piston, only the excess of this steam pressure Q over the accelerating force M can be transmitted through the connecting rod to the crank pin. Therefore if we algebraically add the ordinates of the two curves for Q and M, the resulting line $K_1 K_3$ will limit the ordinates representing the piston pressure that is transmitted to the crank, the line $K_1 K_3$ corresponding to the forward motion and (K_1) (K_3) the return motion. It is clear from this that toward the end of the stroke the force transmitted through the connecting rod to the crank pin may considerably exceed the piston force Q, a fact to be remembered when determining the dimensions of the crank pin and connecting rod. When the crank is the driver, as in pumps, the resistances to acceleration in the first part of the stroke act in the same direction as the piston resistance Q, and therefore the curve (K_1) $(K$ corresponds to the forward stroke, and $K_1 K_3$ to the retur stroke.

FIG. 573.

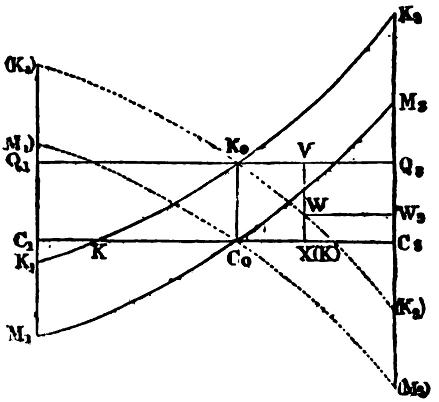


Now if, in consequence of a large value of m_2 or of crank velocity v, the initial accelerating force $m_2 \frac{v^2}{r} = C_1 M_1$ is greater than the actual steam pressure $C_1 Q_1$ on the piston, the resultant pressure will be represented by a curve $K_1 K_3$, Fig. 574, which cuts the axis $C_1 C_3$ at a certain point K. We see from this that here the cross-head at the beginning $C_1 K$ of its motion is not driven by the force of the piston alone, but that it is necessary for the crank to also assist in driving it, exerting a force which at the dead point is equal to $C_1 K_1$ and does not become zero till the position K is reached, after which the piston force again drives. It is therefore evident that a reversal of pressure will occur at K, the connecting rod being pushed by the crank during the travel $C_1 K$ of the cross-head, and drawn by the piston from K on. In like manner on the return stroke the connecting rod will be pulled along by the crank from C_3 to (K), and when the cross-head has passed K the rod will be pushed along by the steam pressure and subjected to compression. In consequence of this reversal of pressure at K and (K), certain shocks will occur at those joints of the crank train where the wear has caused some play or lost motion, *i.e.*, at the bearings of the connecting rod and crank shaft. This causes the engine to run in a noisy, vibratory manner, the shocks becoming greater the more the initial accelerating force

$$\frac{m_2 v^2}{2}\left(1 + \frac{r}{l}\right)$$

exceeds the steam pressure Q. It follows that the average velocity v of the crank must not be allowed to exceed a certain limit, and that this greatest permissible velocity* increases with the steam pressure Q. At the dead points such a reversal of pressure is of course unavoidable.

FIG. 574.



When the crank shaft drives and the accelerating force at the dead points is greater than the piston force, the reversal of pressure at the points K and (K) has a peculiar effect. During the forward stroke the crank must overcome at the beginning of the motion a resistance $C_1(K_1) = Q + M$, when $Q = C_1Q_1$ represents the resistance offered to the piston by the column of water to be lifted. This total resistance gradually becomes smaller and at the piston position C_0, corresponding to the maximum piston velocity, is exactly equal to Q, *i.e.*, equal to the weight of a column of water whose height is equal to the lift and whose cross-section is equal to the area of the piston. For further motion past C_0, the moved mass m_2 is retarded and drives the piston, so that the crank has to overcome a constantly diminishing resistance. This condition of motion would last to the end of the stroke as in a steam engine, if the whole mass m_2 were rigidly attached to the cross-head. But in pumps this is not the case, for the moving mass m_2 is here composed of the masses of the machine parts, piston and piston rod, directly connected with the cross-head, and of the mass of water in the suction pipe where the water must follow the motion of the piston. Now when retardation occurs at C_0 this mass of water will also assist the motion of the piston by virtue of its living force and thus diminish the piston resistance for the crank, but this pressure of the water, following the piston, can only reach a certain amount. For when this pressure becomes large enough to open the delivery valve in the piston or in a special valve chamber, this valve will open, and from this instant to the end of the stroke a certain quantity of water will flow through and be discharged. During this latter period the water in the suction pipe will push the piston with a constant force equal to that which it exerted at the instant when the delivery valve opened. In this way the strange fact that pumps will sometimes deliver per stroke a quantity of water *greater* than the cylinder space traversed by the piston, may be explained. For example, if the force necessary to open the delivery valve were represented in the figure by VW, the water would be discharged at the instant when the piston passes the position X. The resistance to motion offered by the piston from this point to the end of the stroke would be constantly equal to XW. A premature discharge of water takes place whose amount can be determined from the work represented by the area $W W_3 (K_3) W$. The work performed by the crank shaft during this stroke is given by the area $C_1 (K_1) K_0 W W_3 C_3 C_1$.

---

* See *Radinger*, "Dampfmaschinen mit hoher Kolbengeschwindigkeit."

§ 153. Graphical Investigation of the Crank Motion.—The graphical investigation of the crank motion is not difficult when we have determined, in the manner described above, the force needed to accelerate the reciprocating masses. In Fig. 575, let the rectangle $C_1 Q_1 Q_3 C_3$ again represent the diagram of steam pressures for an engine working without expansion; let us suppose the curves M for the accelerating force and the curves K for the resultant steam pressure obtained by uniting the ordinates of Q and M to be also drawn, and let the dotted lines of the figure refer to the return motion. Now also draw, as in § 139, the curve U, whose ordinates represent the circumferential pressure exerted by the connecting rod on the crank pin. To determine this pressure U for any crank position C there was established in § 139 the proportion $K : U = A B : A F$, where K represents the piston pressure. Hence if we suppose the piston pressure $C K$ corresponding to C to be laid off on the crank equal to $A k$, and the line $k u$ drawn parallel to the connecting rod $B C$, we shall obtain $A u$ as a measure of the intensity of the circumferential force when the piston is at C. Laying off this length $A u = C U$ and repeating this construction for a great number of points, we obtain the diagram $C_1 U U' C_3$ for the circumferential or tangential pressures. Finally, if we draw a line $P_1 P_3$ parallel to the axis $C_1 C_3$ at a distance

$$C_1 P_1 = \frac{2}{\pi} C_1 Q_1$$

from the latter, the constant ordinates thus obtained will evidently represent the average resistance of the crank shaft at a distance from the axis equal to the crank radius r. This straight line $P_1 P_3$ intersects the curve U of circumferential pressures for the forward motion, in two points O_1 and O_2, and the curve (U) for the return motion in two other points (O_1) and (O_2). Therefore at the piston positions corresponding to these four intersections, the circumferential pressure U at the crank pin is exactly equal to the resistance P, whence

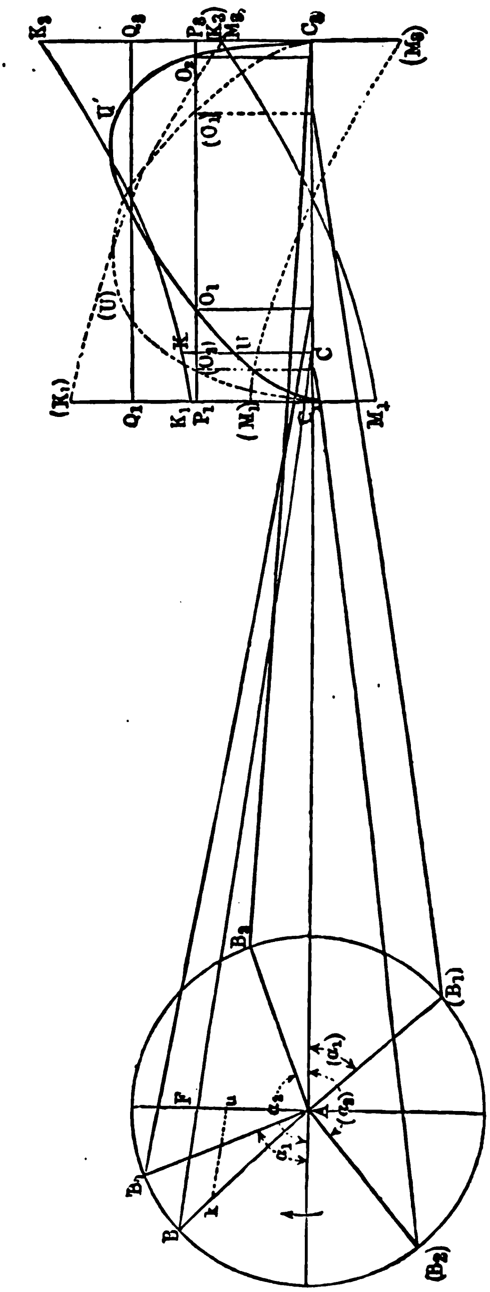
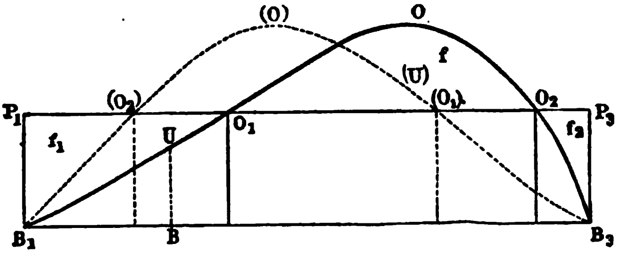

K3
Q3
P3
(K3)
M3
C0
(M0)
O2
U'
(O2)
(U)
O1
K
(U1)
U
C
(K1)
Q1
K1
P1
(N1)
C1
M1
B2
(B1)
F
u
α2
(α1)
(α2)
α1
B1
B
k
(B2)

FIG. 575.

follows that at these intersections the motion is neither accelerated nor retarded. But points whose acceleration is O correspond to a maximum or minimum velocity, and we at once see that the points O_1 and (O_1) correspond to the least and O_2 and (O_2) to the greatest velocities. If we find the crank positions corresponding to the piston positions O, we shall find the angles α_1, α_2, (α_1) and (α_2), through which the crank has turned from the dead points, when these leading velocities occur.

We can determine the values of these maximum and minimum velocities graphically, but then we must refer the circumferential pressures U to a semi-circumference of the crank circle instead of the stroke of the piston. Let $B_1 B_3$, Fig. 576, equal πr, and let any ordinate $B U$ represent the circumferential pressure on the crank pin at the instant that the latter has described the circular path whose length equals the abscissa $B_1 B$. We now see that the area included between the axes $B_1 B_3$ and the curve U of the circumferential pressures represents the work transmitted by the piston to the crank shaft during a half revolution. Consequently the two areas F and (F) enclosed by the curve U for forward motion and the curve (U) for return motion must be equal, for we have assumed that the piston forces are equal, therefore the work performed during the forward and the return stroke must be equal.

FIG. 576.



If here also we draw the straight line $P_1 P_3$ parallel to the axis $B_1 B_3$ at the distance

$$B_1 P_1 = \frac{2}{\pi} Q$$

from the latter, the area of the rectangle $B_1 P_3$ will also equal the work $P. r \pi$ performed during a half revolution. Consequently this area will be equal to each of the areas F and (F) enclosed by the curves U and (U) respectively. The straight line $P_1 P_3$ also cuts the curve U in four points which belong to the same piston and crank positions as the corresponding points O in Fig. 575. Moreover it follows because the areas F and (F) are each equal to the rectangle $B_1 P_3$, that the area $O_1 O O_2 = f$ lying above $P_1 P_2$ is exactly equal to the sum of two triangular areas $B_1 P_1 O_1 = f_1$ and $O_2 P_3 B_3 = f_2$ lying below $P_1 P_3$.

If v_1 again represents the velocity of the crank at the dead point B_1, and L the work of the exterior forces expended in accelerating the masses while the crank has moved from the dead point B_1 to any position B, we have as before for the velocity v at this point

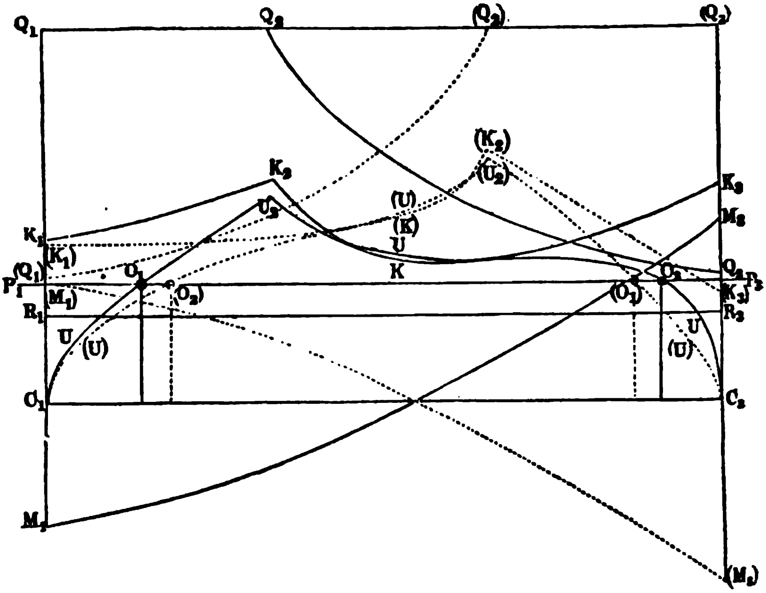
$$v = v_1\left(1 + \frac{L}{m_1 v_1^2}\right).$$

It is clear that the quantity L contained in this formula can be taken from the diagram, for the work expended in accelerating the masses is represented by the area included between the straight line $P_1 P_3$ and the curve U of circumferential pressures, the area being estimated from the dead point B_1 to the particular crank position B. The areas lying on opposite sides of $P_1 P_3$ must be given opposite signs. For instance, to determine the minimum velocity at O_1, we must place L equal to the work represented by the area $B_1 P_1 O_1 = f_1$, and must give L a negative sign, while the velocity v_{max} at O_2 is determined by placing L equal to the work represented by the difference $O_1 O O_2 - B_1 P_1 O_1 = f - f_1$. The areas may be determined by *Simpson's* rule or by means of a planimeter.

In like manner the crank motion of engines working with expansion may be investigated graphically. This diagram will differ from the preceding in the line Q of steam pressures, only that portion $Q_1 Q_2$, Fig. 577, of this line, which corresponds to the period of admission, being parallel to the axis. The remaining portion $Q_2 Q_3$ of the line is determined from the law of expansion which is assumed to represent the action of the steam. The graphical investigation is not rendered more difficult if a more exact law (see Vol. II) than *Mariotte's* is assumed, but in such a case the analytical investigation given in § 150 becomes very complicated. Indeed the diagram of steam pressures can be drawn without assuming any particular law, and without rendering the investigation more difficult, by simply laying off the results obtained by the indicator. The simplicity of this graphical procedure and the ease with which the results may be inspected, make it more popular with practical engineers than analytical methods of calculation.

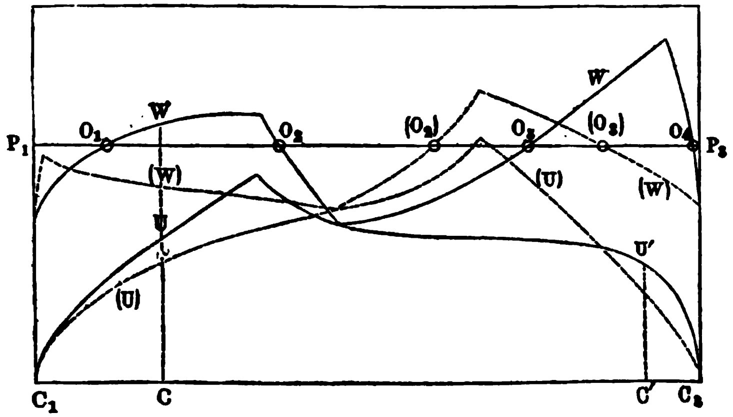
Fig. 577 is the diagram for an engine working with threefold expansion, and as the same letters as before are used for the lines, a special explanation is unnecessary. The line $R_1 R_3$ represents the constant counter pressure on the back of the piston, and the line of effective piston pressure K must therefore be regarded as resulting from the diagrams for Q, R, and M. Here also the dotted lines Q, M, K, and U refer to the return stroke. Finally, Fig. 578 is the diagram for a two-

FIG. 577.



cylinder engine working with expansion. Here the line of circumferential pressure $U\,(U)$ is drawn for *one* crank in the same manner as in Fig. 577. Now in order to ascertain the resultant circumferential pressure W, due to the action of both pistons, it is only necessary to make the ordinate $C\,W$, for any piston position C, equal to the sum of the two circumferential pressures $C\,U$ and $C'\,U'$ of the two cranks. The position C' of the second piston is easily found from the angle included by the cranks. In this manner is found the line $W\,(W)$ for the resultant circumferential pressure. In the figure

FIG. 578.



this line is cut at six points O by the straight line $P_1\,P_3$ representing the resistance P. These intersections lie in the first three quadrants, which means that with the proportions here chosen there are no maximum or minimum velocities in the fourth quadrant. A similar result has been already found in § 147.

§ **154. Friction of the Crank Train.**—The principal hurtful resistances in the crank train are the frictional resistances at the points where two machine parts are connected; in ordinary cases the friction of the air is so small that it can be neglected. Therefore the frictional resistances taken into account are those occurring in the crank-shaft bearings, at the crank pin, at the cross-head pin, and between the slide blocks of the cross-head and its guides.

The friction of the crank shaft in each of its bearings is simply ϕR, where R is the pressure against the bearing, and the work consumed by this journal friction during every revolution is expressed by $\phi R \pi d_1$, where d_1 represents the diameter of the particular journal of the shaft. Reduced to the crank pin, the friction of the shaft is expressed by

$$w_1 = \phi R \frac{d_1}{2r}.$$

The reactions R at the bearings of the shaft in each case may be found from the known principles of Statics. In determining these pressures against the bearings we must take into account the effective piston pressure K transmitted to the crank, the weights G of the crank shaft, fly wheel, belt pulleys, etc., and the thrust of the toothed wheels or the pull of the belts accompanying the transmission of the rotary motion. As the direction of the piston pressure varies, the reactions at the bearings will vary in direction during every revolution. This is specially true of the main bearing, which is placed very close to the crank and principally sustains the piston pressure. The magnitude of the pressure on the bearings is also variable during every revolution because the piston force varies. Consequently, to approximately determine the friction of the crank shaft, we must take an average value for the piston force, which value can best be determined from the curve K of effective pressures on the piston, given in Figs. 575 to 577.

At the crank pin the friction is caused by a force acting along the connecting rod, and the path through which the friction is overcome during every revolution of the crank pin is equal to the circumference ϕd_2 of the crank pin. The force along the connecting rod was found above to be $\frac{K}{\cos \gamma}$, where γ represents the inclination of the connecting rod to the direction of the stroke. For this angle γ there was found the relation

$$\frac{r \sin \alpha}{l} = \sin \gamma,$$

or

$$\cos\gamma = \sqrt{1 - \left(\frac{r}{l}\sin\alpha\right)^2}.$$

The angle of inclination γ, and hence $\frac{1}{\cos\gamma}$, are greatest for

$$\delta\left(\frac{r}{l}\sin\alpha\right) = 0,$$

i.e., for $\alpha = 90°$, which corresponds to the middle position of the crank. For this angle

$$\cos\gamma = \sqrt{1 - \left(\frac{r}{l}\right)^2};$$

hence the force in the direction of the connecting rod

$$S = \frac{K}{\cos\gamma} = \frac{K}{\sqrt{1 - \left(\frac{r}{l}\right)^2}} = K\left(1 + \frac{1}{2}\frac{r^2}{l^2}\right).$$

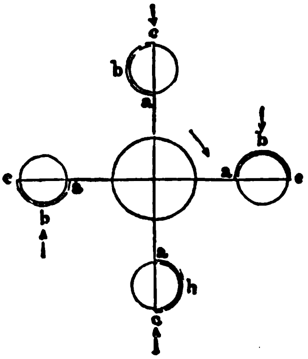
For the ordinary ratio $\frac{r}{l} = \frac{1}{5}$, the force along the rod becomes $S = 1.02\,K$ when the angle of deviation γ is greatest. It follows from this that it is sufficiently accurate to assume the piston force K to be the force that generates the friction. If K represents the average value of the piston force, the work consumed by the friction at the crank pin during a revolution of the crank is $\phi K \, . \, \pi d_2$. If we reduce this friction to the crank shaft and suppose it to act at the end of a lever arm equal to the crank length r, this resistance becomes

$$w_2 = \phi K \frac{d_2}{2r}.$$

The weight of the connecting rod, cross-head, and piston rod in vertical engines influences the crank pin friction when the crank is single-acting and the piston force is directed downward, as in lift pumps. But in double-acting cranks, and in single-acting cranks with piston force directed upward, the pressure on the crank pin is diminished by the weight of the parts on the up-stroke of the piston by exactly the amount that it is increased on the down-stroke. In horizontal engines a part of the weight of the connecting rod always acts on the crank pin. The same observations can be made with reference to the influence of the weight of the crank shaft on the friction of its bearings.

In this connection we may remark that the pressure of the piston-force K on the crank pin is confined to a semi-circumference $a\,b\,c$, Fig. 579, for a given direction of rotation, in consequence of which the pins of cranks running constantly in the same direction wear into an oval form in course of time. On the other hand, the brasses surrounding the pin are subject to wear at all points of the circumference.* This is just the opposite of what takes place in the ordinary bearings of rotating shafts where the journal is worn uniformly and the brasses ovally. If we assume an opposite direction of rotation of the crank, the piston force will act on the other half of the pin; therefore in machines where the direction of rotation is frequently reversed, as in hoisting engines, there is uniform wear on the crank pin.

FIG. 579.



The frictional resistance at the cross-head pin is likewise caused by the pressure S of the connecting rod, for which we may again take the piston pressure K. In most cases this resistance may be neglected, for the distance through which it is overcome is very small. Thus, during every revolution of the crank the greatest angle of deviation of the connecting rod to each side of the stroke is $\gamma = arc\ sin\ \frac{r}{l}$, and as this angle is small we may assume $\gamma = \frac{r}{l}$, and then the path through which friction is overcome is given by $4 \frac{r}{l} \frac{d_3}{2}$, where d_3 is the diameter of the pin. The frictional resistance of this pin reduced to the crank pin therefore becomes

---

* See *Radinger*, Zeitschr. des oesterr. Ing. u. Archit. Vereins, 1869.

$$\phi K \frac{4 r d_3}{2 l \times 2 \pi r} = w_3 = \phi K \frac{d_3}{\pi l}.$$

Although this friction is small, yet its varying direction, taken in connection with the variable force K, tends to loosen the pin; therefore it is important that the pin be well united with the cross-head in order to avoid the repeated small shocks and unsteady motion accompanying the loosening.

Between the slide blocks of the cross-head and the guides the sliding friction $Q \tan \gamma$ occurs, which is called forth by the lateral pressure of the more or less oblique position of the connecting rod. This lateral pressure is equal to 0 at the dead points, and becomes greatest for a certain middle position which principally depends on the variation of K. If the piston force is constant this maximum lateral pressure will occur when the angle of deviation γ is greatest, *i.e.*, when the crank angle $\alpha = 90°$. But the piston force is not generally constant even in engines working without expansion and in pumps, on account of the inertia of the reciprocating masses. Therefore an exact determination of the work of the resistance w_4 at the guides during a revolution requires integration of the expression

$$\int_{\alpha = 0}^{\alpha = 2\pi} \phi K \tan \gamma \, \delta s.$$

If we substitute in this

$$\tan \gamma = \frac{r \sin \alpha}{\sqrt{l^2 - r^2 \sin^2 \alpha}} = \frac{1}{\sqrt{\frac{l^2}{r^2 \sin^2 \alpha} - 1}},$$

and also, according to § 139,

$$\delta s = \left(r \sin \alpha \mp \frac{r}{2 l} \sin 2 \alpha \right) \delta \alpha,$$

we shall get for the work of friction at the guides

$$\phi \int_0^{2\pi} K \frac{r \sin \alpha \mp \frac{r}{2l} \sin 2\alpha}{\sqrt{\frac{l^2}{r^2 \sin^2 \alpha} - 1}} \delta \alpha.$$

The calculations connected with this expression are quite difficult. In order to determine an approximate expression, let us assume

$$\tan \gamma = \sin \gamma = \frac{r}{l} \sin \alpha, \quad \text{and} \quad \delta s = r \sin \alpha,$$

and an average value for the piston pressure K; we then get for the work of friction at the guides during a revolution

$$\int_0^{2\pi} \phi K \frac{r}{l} \sin \alpha \cdot r \sin \alpha \delta \alpha = \phi K \frac{r^2}{l} \int_0^{2\pi} \sin^2 \alpha \delta \alpha$$

$$= \phi K \frac{r^2}{l} \int_0^{2\pi} \frac{1 - \cos 2\alpha}{2} \delta \alpha = \phi K \frac{r^2}{l} \pi;$$

hence, for the frictional resistance reduced to the crank pin,

$$w_4 = \phi K \frac{r}{2l}.$$

Moreover, it is easy to see that the lateral pressure of the connecting rod for a given direction of rotation of the crank will always, during both the forward and return strokes, act in one direction and press against only one of the slide bars, the other bar being pressed upon only when the rotation is reversed. This explains why the cross-heads of engines which always turn in the same direction are guided at only one side.

Collecting the results found above and taking K as the average piston force, we have for the work of all the forces

$$P 2 \pi r = K 4 r \pm (w_1 + w_2 + w_3 + w_4) 2 \pi r,$$

or

$$P = \frac{2}{\pi} K \pm (w_1 + w_2 + w_3 + w_4),$$

where the upper sign refers to pumps and the lower sign to engines. If we substitute in this the values for the resistances w found above, we have

$$P = \frac{2}{\pi} K \pm \phi \left[R \frac{d_1}{2r} + K \left(\frac{d_2}{2r} + \frac{d_3}{\pi l} + \frac{r}{2l} \right) \right].$$

REMARK. If this approximate determination of the friction of the guides does not suffice, the general integral

$$\int_0^{2\pi} \phi K \tan \gamma \, \delta s$$

can most easily be found graphically, as follows: In Fig. 577, lay off, at every piston position, the value $\phi K \tan \gamma$ as ordinate; the curve w_4 thus obtained passes through the points C_1 and C_2, w_4 referring to the forward and (w_4) to the return stroke. The two areas enclosed between these curves and the axis $C_1 C_2$ represent the work consumed by friction at the guides during the forward and the return stroke respectively. These resistances w_4, reduced to the crank pin, are the altitudes of rectangles whose base is πr and whose contents equal the areas just mentioned.

It is of interest to know approximately the efficiency of the crank train. As the term R representing the friction on the shaft in the formula above developed for P depends principally on the weight G of the fly-wheel shaft and on its driving force, this formula represents a greater resistance than is due to the crank train alone. In order to approximately obtain the resistances belonging to this crank mechanism, and thus to estimate the efficiency of the crank train, we can assume this portion of the friction of the shaft as equal to $\phi K'$, an assumption which in most cases is very near the truth, as the crank-shaft bearing is usually close to the crank and principally resists the piston pressure. Under this assumption the force P acting at the crank pin will be

$$P = K' \left[\frac{2}{\pi} - \phi \left(\frac{d_1 + d_2}{2r} + \frac{d_3}{\pi l} + \frac{r}{2l} \right) \right]$$

when the piston drives; and since

$$P_0 = \frac{2}{\pi} K$$

when friction is neglected, the efficiency is approximately given by

$$\eta = \frac{P}{P_0} = \frac{\frac{2}{\pi} - \phi \left(\frac{d_1 + d_2}{2r} + \frac{d_3}{\pi l} + \frac{r}{2l} \right)}{\frac{2}{\pi}} = 1 - \frac{\phi \pi}{2} \left(\frac{d_1 + d_2}{2r} + \frac{d_3}{\pi l} + \frac{r}{2l} \right).$$

EXAMPLE. If in a steam engine the crank length $r = 0.5$ meter [19.69 ins.], the length of connecting rod $l = 2.5$ meters [98.43 ins.], the diameter of shaft 0.20 meter [7.87 ins.], the diameter of the crank pin 0.12 meter [4.72 ins.], and

the diameter of the cross-head pin 0.08 meter [3.15 ins.], what is the efficiency of the crank train when the coefficient of friction is 0.08?

In this case

$$\eta = 1 - 0.08 \times 1.57 \left(\frac{0.320}{2 \times 0.5} + \frac{0.08}{3.14 \times 2.5} + \frac{0.5}{5}\right)$$

$$= 1 - 0.126\,(0.32 + 0.01 + 0.10) = 1 - 0.054 = 0.946.$$

Consequently, when the friction caused by the weight of the fly wheel is neglected, the hurtful resistances of the crank train cause a loss of about five and one half per cent of the power.

§ 155. Crank Constructions.—The crank arm is usually a separate piece, bored so as to accurately fit the end of the crank shaft. The hand cranks used for windlasses are made of wrought iron, because their thickness is small in comparison with their length (0.3–.45 meter [12–18 ins.]), and are fastened to the crank shaft *A* by means of the nut *F*, Fig. 580. The crank pin is here a long rod *CD*, 0.3–.48 meter [12–18 ins.] long, riveted to the crank arm *C*, and provided with a handle for the workman, which is simply a wooden or metallic sleeve turning loosely on *CD*. Occasionally the handle and the crank arm are made in one piece *BCD*, Fig. 581, and the sleeve is omitted, but this omission is not to be recommended. If a sufficiently large fly wheel is placed at the end of the crank shaft the handle may be screwed into an arm of the wheel at the proper distance from the centre, as is often done with straw cutters. The crank pin used for turning the shaft of the fly

FIG. 580.

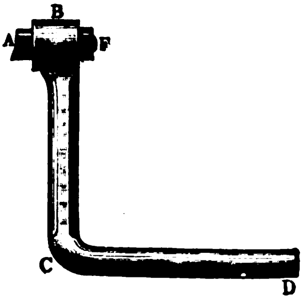
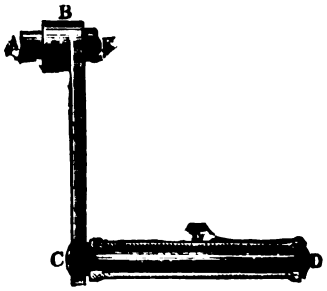
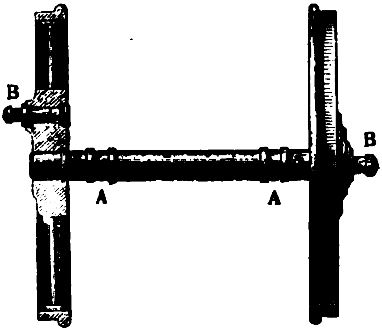


FIG. 581.



wheel in sewing machines is also often fastened to an arm of the wheel. The cranks of large steam engines, on the other hand, are usually made of cast iron on account of the greater ease of construction; though in particularly substantial constructions, where the greater cost is of less importance, wrought iron is used. In locomotives with outside cylinders the hubs of the driving wheels are strengthened so that they can receive the crank pins. In Fig. 582, *B*, *B* are the crank pins set at right angles.

FIG. 582.



The two cranks for steam engines here given are drawn according to *Redtenbacher's* proportions, Fig. 583 representing a cast-iron and Fig. 584 a wrought-iron crank. Great care must be taken in fastening the crank to the shaft *A*; usually keys and screws are not considered sufficient, and then the cranks are ***shrunk*** on to the shafts. When this is done the bore of the crank is made a little smaller than the diameter of the shaft, and then the crank is heated till its bore is sufficiently enlarged to permit of its being slipped on to the shaft. When the crank cools, its boss shrinks and presses against the shaft

FIG. 583.

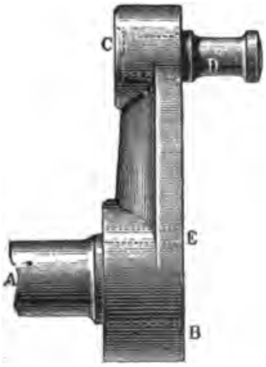
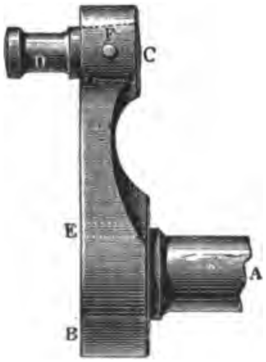


FIG. 584.

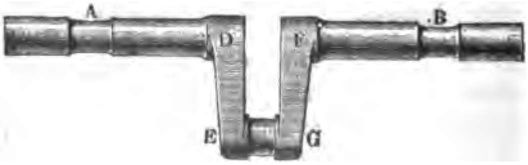


with such force that the friction thus caused prevents all loosening. As this subjects the material of the boss to a certain tension before the crank transmits any force, the difference between the diameter of the bore and that of the shaft must be very small. An exact determination of this permissible difference and of the tension which it causes in the boss cannot be made with certainty. However, we cannot assume that cast-iron cranks are heated more than about 200° C., for the strength of cast iron diminishes rapidly when it is heated. Assuming that the crank is heated to this degree, the cylindrical bore of diameter d would increase to $d(1 + 200 \times 0.000011) = 1.002d$; in other words, the diameter of the shaft may at most be $\frac{1}{500}$ larger than the bore of the boss. The fastening of the crank pin D to the crank is often effected by a key or pin F, Fig. 584, or by riveting as at C, Fig. 583; a nut is seldom used for this purpose because of the danger of its loosening under the influence of the continual variation of pressure in the connecting rod.

In order to vary the stroke of the crank as in planing and slotting machines, the crank arm is provided with a prismatic radial slit in any part of which a block carrying the crank pin may be fastened by a set-screw. In this case, instead of the crank arm we may use a disk keyed fast to the shaft, and provided, at different distances from the centre, with holes to receive the pin.

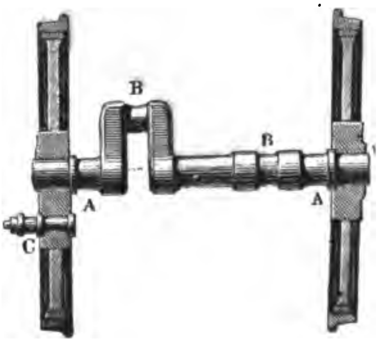
The crank constructions here described are only applicable when the crank can be attached to the free end of the shaft, *i.e.*, when the latter extends to only one side of the crank plane. On the other hand, when, as in marine engines and in locomotives with inside cylinders, the shaft extends to both sides of the crank plane, it is necessary to form the crank by a bend in

FIG. 585.



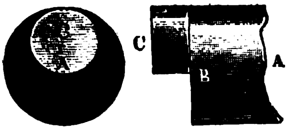
the shaft, as shown in Fig. 585. These double cranks are always of wrought iron, and usually forged in one piece with the shaft; it is only when the shafts are very large, as in marine engines, that the cranks DE and FG are separately fastened to the shaft and then united by a common crank pin EG. The progress made in the production of heavy forgings has rendered the use of cast iron for double cranks and their shaft almost obsolete, for cast-iron shafts offer but little security against shocks. For two-cylinder engines two sets of double cranks, placed at right angles are sometimes used, Fig. 586 representing such an arrangement applied to a locomotive. Here B, B are the pins of the double cranks, to which the connecting rods of the cylinder are attached, while the two pins in the hubs of the driving wheels, of which only one is seen in the figure at C, are connected to two coupling rods for the purpose of turning a second driving axle (see § 137).

FIG. 586.



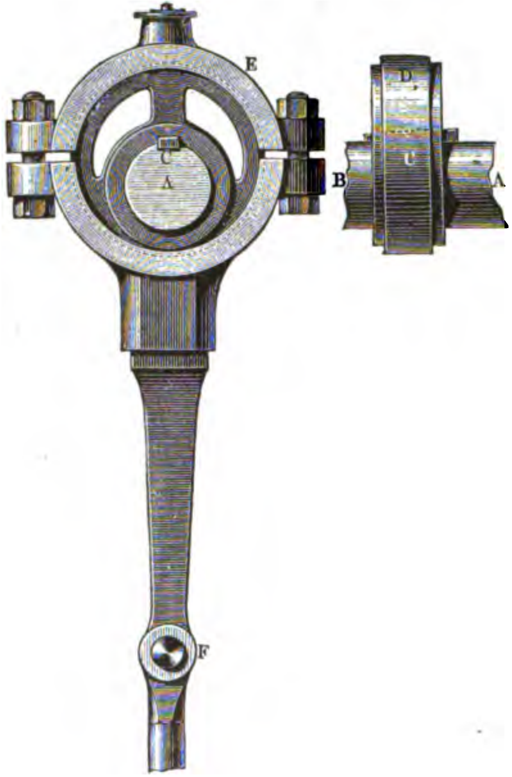
§ **156. Eccentrics.**—When a crank has a small radius r, the crank arm may be omitted, the construction taking the shape shown in Fig. 587, where the end of the crank shaft AB is turned down so as to form an eccentric pin C, whose eccentricity AC represents the desired length of crank. If in this case we wish to prolong the shaft AB to both sides of the pin without diminishing the diameter of the shaft, it may easily be done by increasing the pin C, and we then get the *eccentric*, a construction shown in Fig. 588. Here the crank is a sheave or disk, CD, whose cylindrical surface is directly enclosed by the strap E of the connecting rod EF. The size of the sheave and of the strap permits the shaft AB to extend in both directions, *i.e.*, such eccentrics can be conveniently placed at any point of a shaft. This last prop-

FIG. 587.



erty has caused eccentrics to be very widely used, especially for moving the valves of steam engines. That the motions of the eccentrics are like those of the ordinary crank is evident from the manner in which they were deduced, the eccentric train being nothing but a crank train. It follows from this that by fastening the various links of a train, such as sheave, eccentric rod and cross-head, we may obtain new mechanisms which will correspond to the mechanisms similarly deduced from the ordinary crank train. Further information concerning this formation of mechanisms can be found in *Reuleaux's Kinematics*, where the process of evolving the eccentric from the ordinary crank is designated as ***pin expansion***, for the crank pin *C* was increased in diameter till it included the shaft *A B*. In the work referred to there are many other interesting examples of mechanisms obtained by expansion of elements. For instance, when the eccentric rod *C F* is short, the sheave *C D* may be enlarged still more till it includes the pin *F*, or *vice versa*, the pin *F* may be enlarged till it includes the eccentric *C D*. If the disk of the eccentric is made stationary we get *Whitworth's* quick-return motion. Here the ring *E* enclosing the stationary sheave *A C* is provided with a

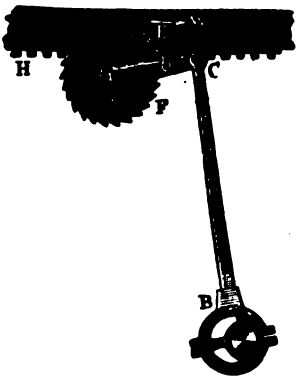
FIG. 588.



toothed circumference which is set in rotation by a pinion. This causes a pin fastened to the toothed ring to move its sliding bearing along a slot in a lever turning about the stationary pin *A*. The rotary motion thus communicated to the slotted lever has a variable velocity, and in principle this mechanism agrees with that more fully described in § 138 (see Fig. 539).

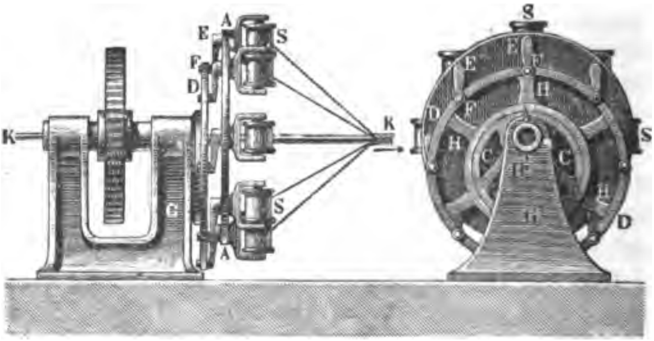
The eccentric may also replace the crank in the quadric train; in this form the mechanism is often used to produce automatic sliding or feeding in certain machinery; for instance, in lathes, planers, etc., also in gang saws for giving intermittent motion to the carriage while the saw is cutting. This kind of motion is illustrated in Fig. 589. *A* is a driving shaft which moves the saw frame up and down by means of a crank and at the same time, by means of the eccentric *B* and the connecting rod *BC*, communicates a swinging motion to the lever *CD* about the fixed shaft *D*. This lever moves downward with the saw frame and, by means of the pawl *E* and the ratchet wheel *F*, turns the pinion *G* so far that the rack *H* and carriage *W* are shoved forward a distance equal to the cut of the saw. This mechanism is repeatedly used with the eccentric sheave stationary; for instance, for keeping the paddles of the wheels of steamships parallel, also in wire-rope machinery for laying up wires into ropes and for covering telegraph cables with wire. Fig. 590 shows the machine used for the latter purpose. The cable *K*, which is to be covered, passes through the hollow shaft of a large spool-disk *A*, which carries on its surface the uniformly distributed spools *S* on which are the wires for covering the cable. Now if the cable is drawn slowly in the direction of the arrow, and at the same time the spool-disk rotates, the wires on the latter will cover the cable with uniform helical coils. If during this motion the spool holders were rigidly attached to

FIG. 589.



the disk A, the spools would also take part in the rotation, and their wires would be subjected to torsion, as was more fully explained in § 117. In order to avoid this twisting of the wires, each spool holder is turned backward on its pin as much as the rotation of the disk A turns it forward, the mechanism mentioned above being used for this purpose. Thus each spool holder is placed on a pin E that turns in and passes through a hole in the disk A parallel to the axis; back of the disk all the bolts are provided with equally long and parallel cranks $E F$. The pins F of these cranks take hold of a ring $D D$ whose size is equal to that of the circle which carries the holes for the pins E, the ring $D D$ being placed eccentric to the shaft of the disk A an amount $A B = E F = r$. If we suppose a disk C with centre B eccentric to A to be fastened to the frame G, and surround this disk by a loose ring connected by the arms H with the rings D receiving the pins F, it is clear that we have the mechanism with parallel cranks already described, and that all the spools will keep their direction in space while being led around in a circle. The spools S are not, properly speaking, turned about the axis A, but are subjected to a circular translation, for all their points describe parallel and equal paths. Therefore the material of the wires J is not subjected to torsion.

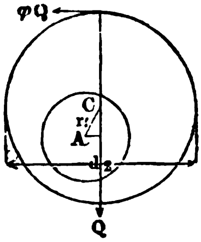
FIG. 590.



In general the use of the eccentric is accompanied by a large diameter of the sheave and, as the friction at the circumference increases with the diameter, the eccentric train cannot be regarded as economical of power; the loss of work is particularly great when the stroke is large, because then the diameter of the sheave is considerable. The eccentric is principally used, therefore, with small strokes, *i.e.*, with small eccentricities, and when the forces to be transmitted are small. Moreover the shaft is always the driver; probably the case would never occur in practice, where the eccentric would be driven by the rod. The reason of this is that the frictional resistances are so considerable that the eccentric shafts could not be moved by the pressure of the rod till the eccentric had turned through a certain angle from its position at the dead point. To determine this angle, let d_1 be the diameter of the shaft in its bearings, d_2 that of the eccentric sheave, and r the crank arm or eccentricity. As the latter is small in comparison with the length l of the connecting rod, the ratio $\frac{r}{l}$ and the deviation γ, of the connecting rod from the direction of the stroke, may be neglected. As a consequence, the friction at the guides of a rod and at the pin at the end of the rod are nearly equal to 0, and therefore the friction generated at the shaft and at the circumference of the eccentric are the only resistances of importance. Now if Q again represents the force along the eccentric rod, its turning moment will be $Q r \sin \alpha$ when the eccentric has turned through the angle α from the dead point, Fig. 591. On the other hand, the moment of the journal of friction with respect to the centre of the shaft will be $\phi Q \frac{d_1}{2}$ and the moment of the eccentric friction for the same point will be approximately given by $\phi Q \frac{d_2}{2}$.* Therefore, in order that Q may cause motion in the mechanism, the condition

$$Q r \sin \alpha \overset{=}{>} \phi Q \left(\frac{d_1 + d_2}{2}\right)$$

FIG. 591.



---

* This moment is expressed more exactly by $\phi Q \left(\frac{d_2}{2} \pm r \cos \alpha\right)$.

must be fulfilled, or α must at least have a value given by

$$\sin \alpha = \phi \frac{d_1 + d_2}{2r}.$$

Now d_2 is always greater than $2r + d_1$, for if the thickness of the nave at the weakest place is only $\frac{1}{4} d_1$, we have

$$d_2 = 2(r + \tfrac{3}{4} d_1) = 2r + \tfrac{3}{2} d_1.$$

If we substitute this smallest value of d_2 in the above equation, we shall get the smallest angle α for which motion is possible when the eccentric rod drives:

$$\sin \alpha = \phi \frac{2r + \frac{5}{2} d_1}{2r} = \phi \left(1 + \frac{5}{4} \frac{d_1}{r}\right).$$

For instance, if we assume $r = \frac{d_1}{2}$, *i.e.*, the centre of the eccentric on the surface of the shaft, and also assume $\phi = 0.08$, we have

$$\sin \alpha = 0.08 (1 + \tfrac{5}{2}) = 0.28; \quad \alpha = 16° \, 20'.$$

Consequently the eccentric rod cannot drive the mechanism till the eccentricity has turned through 16° 20′ on either side of the dead point, *i.e.*, it cannot produce motion during an angle of rotation 32° 40′ at each of the dead points, and while passing through these angular spaces the rotation of the shaft must be effected by the living force of the rotating masses. The circumstances are still more unfavorable when the eccentricity r is less than $\frac{d_1}{2}$, and evidently the eccentric rod cannot produce any motion whatever at any position of the eccentric when, supposing $d_2 = 2r + \frac{3}{2} d_1$, the following equation is satisfied:

$$\phi (1 + \tfrac{5}{4}) \frac{d_1}{r} = 1;$$

or when, assuming $\phi = 0.08$, $\frac{d_1}{r} = 9.2$.

§ 157. Dimensions of the Crank.—The crank pin is subjected to bending stress by the piston force K, and must therefore be calculated by the formulas deduced in § 3 for end journals. The diameter d_2 of the crank pin is found from

$$d_2 = 2.26 \sqrt{K \frac{\lambda}{k}} = \alpha \sqrt{K}$$

when the ratio $\frac{l}{d} = \lambda$ and the maximum permissible stress k are given. Here K represents the maximum pressure transmitted to the crank pin, and this pressure in high-speed engines is largest toward the end of the piston stroke, on account of the inertia of the masses. This is specially true in cases where the expansion is not very great. *Radinger* very properly calls attention to the fact that when engines *run away*, it is usually the crank pin that breaks because the pressure K becomes very large toward the end of the stroke, in consequence of the greatly increased pressure of the more swiftly moving masses. Under these circumstances, this pressure K may considerably exceed the steam pressure Q. There is, however, a certain advantage in this behavior of the crank pin, for it breakage does occur from unforeseen contingencies, it is better that some easily renewed part should break than that a large and costly piece like the crank or its shaft should be destroyed. There are some designers who therefore purposely make an easily replaced piece, like the crank or cross-head pin, somewhat weaker than the larger pieces which can only be replaced by a considerable expenditure of time and money.

The ratio $\lambda = \frac{l}{d}$ is taken small for the crank pin, in order that the distance from the centre of the pin to the shaft bearing may be as small as possible; this ratio is seldom greater than 1.5, and is often as low as 1. Moreover, a short length of pin is permissible here because of the variation in the direction of the pressure, every point of the bearing being subjected to pressure during a revolution. As a consequence the wear of the bearing is not as great, even with a larger pressure, as in the bearings of shafting, which are constantly kept in contact at the same place. This variation in the direction of the pressure also favors the distribution of the lubricant between the pin and its bearing. If we assume a ratio $\lambda = \frac{l}{d_2} = 1.5$, we shall have, according to § 3, for the diameter of a cast-steel crank pin,

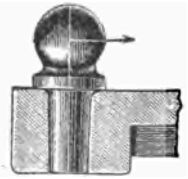
$$d_2 = 0.88 \sqrt{K}, \quad [d_2 = 0.023 \sqrt{K},]$$

and for a wrought-iron crank pin,

$$d_2 = 1.13 \sqrt{K}. \quad [d_2 = 0.03 \sqrt{K}.]$$

Sometimes with rapidly running cranks, and where there is danger that the shaft bearings will get out of line in consequence of shocks, as in the gang saws of saw mills, a spherical shape is given to the crank pin, Fig. 592, in order that the connecting rod may have a certain freedom of motion in a direction perpendicular to its plane of oscillation.

FIG. 592.



The crank shaft is subjected to a twisting stress by the piston pressure K, and to a bending stress by the reactions at the bearings. When the torsional stress is considered we must use the greatest twisting moment; this corresponds to the maximum value of the circumferential force U, the latter being found from the diagrams Fig. 574 to Fig. 577. In calculating the diameter, we must, according to § 14, employ the formula

$$d_1 = \alpha \sqrt[3]{Pa},$$

and must not make use of the formula

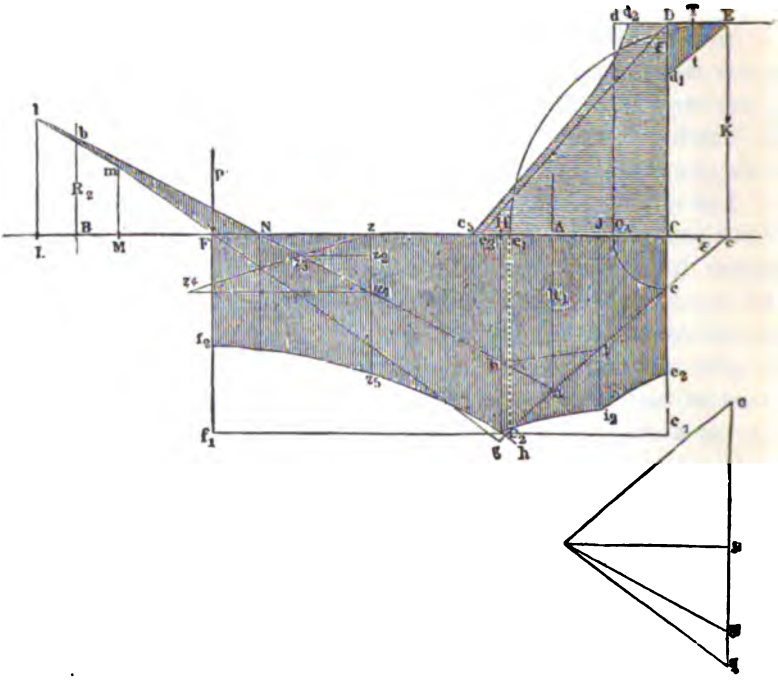
$$d_1 = \alpha_1 \sqrt[3]{\frac{N}{u}},$$

which presupposes a constant circumferential force. As the crank shafts are usually short, no attention need be paid to the torsional stiffness, *i.e.*, angle of twist. On the other hand, the strength of the crank shafts is considerably affected by the bending stresses. These are caused by the piston pressure K, the often great weight G of the fly wheel, and the thrust of wheels engaged in transmitting rotation. These stresses depend on the relative positions of crank, wheels and bearings, and it would take too much space and time to establish general formulas for them. At any rate, the object can be attained more conveniently and with sufficient accuracy by the methods of Graphical Statics. The diagrams obtained give at the same time the stress at every point of the crank arm and pin. The crank is also subjected to compound stress, the pressure K bending the arm as if it were a bracket fixed at one end and, by its one-sided action, twisting it about its longitudinal axis. After what was said in the Appendix to Vol. I, concerning the elements of Graphical Statics, there ought to be no difficulty in drawing these diagrams. Nevertheless we will apply these principles to two of the most commonly occurring examples.

Let A and B, Fig. 593, be the centres of the shaft bearings, CD the crank, and E the centre of the crank pin at which the piston force K may be supposed to act. Moreover, suppose the motion to be transmitted by a toothed wheel at F; let this wheel also act as a fly wheel, and let P be the resultant of the weight and the pressure of this wheel. Let K represent the greatest piston pressure, and let us assume that this pressure acts as a circumferential force, *i.e.*, we will assume the crank to be at right angles to the piston rod. These assumptions correspond to the most unfavorable twisting and bending stress on the shaft, and in reality the stresses are usually more favorable, for the maximum piston force K will not generally act at the same time that the crank is in the assumed position.

In order to determine the reactions R_1 and R_2 at the bearings A and B from the forces K and P, we first draw the force polygon $Oo\mathfrak{K}\mathfrak{P}$, making $o\mathfrak{K} = K$ and $\mathfrak{K}\mathfrak{P} = P$, and assuming pole distance $O\mathfrak{K}$ as the base of the moments. If we now draw through e and F the lines $eg \parallel Oo$ and $Fg \parallel O\mathfrak{P}$, the intersection g will give us the point of application of the resultant of K and P. By connecting the intersections a and b of the lines $g e$ and $g F$ with the vertical reactions at A and B, we shall get the closing line $a b$ of the equilibrium polygon. By drawing the line O 𝕽 of the force polygon parallel to $a b$, we find the reaction R_1 of the bearing at A to be equal to 𝕽 o and the reaction R_2 at B equal to 𝕻 𝕽. The equilibrium polygon for the crank shaft is therefore $F e a b F$, or if we take into account the length of the journals $H J$ and $L M$, this polygon becomes $F e i h l m F$. The bending moment at any point of the shaft will therefore be given by the ordinate at this point, provided the pole distance O 𝕶 be regarded as the base of the moments (see Vol. I, Appendix, § 44). The shaft is also subjected to a twisting moment represented by $K r$. This moment may be very easily found from the equilibrium polygon by laying off $e e_1$ equal to the crank arm r or $e E$, and the ordinate $e_1 e_2$ will give the desired moment. As this twisting mo-

FIG. 593.



ment has the same value $e_1 e_2$ for all points of the crank shaft from C to F, we shall have, for the diagram of the twisting moments, the rectangle $F C c_1 f_1$. This area of the twisting moments M_2 must be combined with that for the bending moments M_1, according to the formula given in Vol. I, § 284, Fifth Ger. Ed.:

$$M = \tfrac{3}{8} M_1 + \tfrac{5}{8} \sqrt{M_1^2 + M_2^2};$$

and for this purpose we may employ the general construction already given in § 16, for shafts. For instance, divide any ordinate $z z_1$, of the diagram for bending moments, at z_2 so that $z z_2 = \tfrac{3}{8} z z_1$, and then draw $z_1 z_4$ perpendicular to $z z_1$ and equal to the twisting moment $C c_1 = F f_1$, and connect z with z_4. Now draw $z_2 z_3 \parallel z_1 z_4$ and make $z_2 z_5 = z_3 z_4$; the ordinate $z z_5$ will then give the resultant moment M. If this construction is repeated for a number of points, we shall get the area shaded in the figure and representing the combined bending and twisting moments. Any ordinate of this area $C c_2 i_2 h_2 f_2 F m l N C$ will give the corresponding resultant moment M from which the dimensions of the shaft at the particular cross-section may be obtained by means of the formula

$$M = k \frac{W}{e}.$$

The crank arm DC is subjected to a bending stress and at the same time to a twisting stress about the axis DC by the force K (which we must suppose to be acting at E perpendicular to the plane of the paper). The area of the bending moments is represented by the triangle $C D c_3$, distance $C c_3$ being laid off equal to the length $e_1 e_2$ already found for the moment $K r$. Here it is only necessary to lay off the angle $C D c_3$ equal to the angle $C e a$, both angles being represented in the figure by ϵ. The moment that twists the crank arm about its longitudinal axis CD is given by $K . DE$, or by the ordinate $C c$. If this ordinate $C c$ is taken as the altitude of a rectangle $C D d c_4$ whose base is $C D$, this rectangle will represent the twisting moments of the crank arm. By combining the twisting and bending moments in the same manner as for the shaft, we get the resulting area of moments $C D d_2 c_5 C$ for the crank arm.

The crank pin is only subjected to the bending moment $K . D E$, and its area of moments is given by the triangle $C e c$ or $D E d_1$, and the cylindrical pin has everywhere the diameter corresponding to the point T, being determined from the moment $T t$ in accordance with the formula

$$T t = M = k \frac{W}{e}.$$

Here T is the point where the pin joins the crank arm.

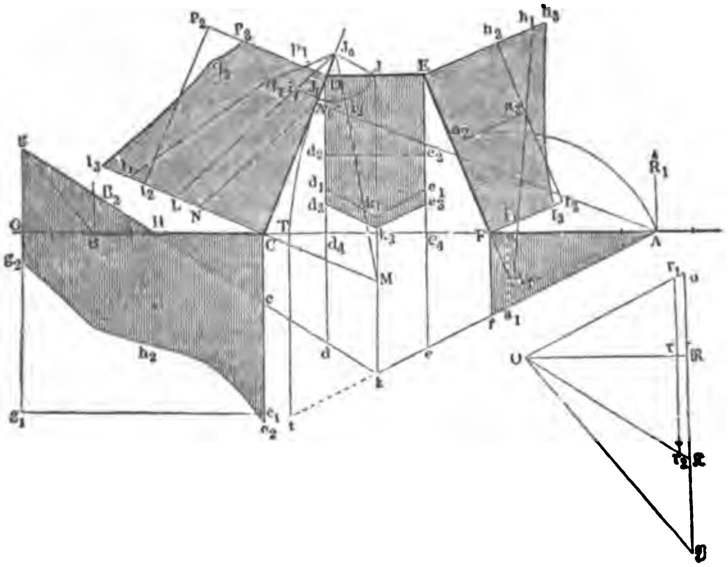
We will now investigate the double crank shaft $B C D E F A$, Fig. 594; this shaft is supported by the bearings A and B, and at the pin J it is acted upon by the force K, and at G by the resultant P of the weight and pressure of the wheels. The equilibrium polygon $A k g B A$ may be found from the force polygon $o \mathfrak{K} \mathfrak{P} \mathfrak{R} o$, where the pole O is taken on the horizontal through $\mathfrak{R}$ in order that the closing line of the equilibrium polygon between the bearings may be horizontal. The reactions at the bearings A and B are therefore given by $R_1 = \mathfrak{R} o$ and $R_2 = \mathfrak{P} \mathfrak{R}$. The portion $A F$ of the shaft is only subject to bending, so that its area of moments is represented by the triangle $A F f$. On the other hand, the portion $C G$ of the shaft is subjected not only to the bending represented by the area of moments $C c H g B C$, but also to a twisting moment by the force K acting with the arm r. To find this moment $K r$, it is only necessary to draw $\mathfrak{r}_1 \mathfrak{r}_2$ parallel to the direction of the force $o \mathfrak{K}$ at a distance $O \mathfrak{r} = r$ from the pole O; the piece $\mathfrak{r}_1 \mathfrak{r}_2$ included between the rays $O o$ and $O \mathfrak{K}$ represents the moment $K r$, for the proportion

$$o \mathfrak{K} : \mathfrak{r}_1 \mathfrak{r}_2 = O \mathfrak{R} : O \mathfrak{r}, \quad \text{that is,} \quad K : \mathfrak{r}_1 \mathfrak{r}_2 = 1 : r,$$

evidently gives $K r = \mathfrak{r}_1 \mathfrak{r}_2$. The rectangle $G C c_1 g_1$, constructed on the base $G C$ with $\mathfrak{r}_1 \mathfrak{r}_2$ as altitude, therefore represents the twisting moments, and by combining this with the bending moments we get for the diagram of the resulting moments the area $C H g g_2 h_2 c_2 C$.

The crank pin $D E$ is subjected to bending moments represented by the area $D E e_1 k_1 d_1 D$, when we make $E e_1 = e_4 e$ and $D d_1 = d_4 d$ and draw $e_1 k_1 \parallel A k$ and $d_1 k_1 \parallel H k$. In addition to this the crank pin is subjected to a twisting moment by the reaction R_1 acting with the lever arm r. We must here suppose R_1 to act at A perpendicular to the plane of the paper. If we lay off from A the distance $A a$ equal to crank arm r, we shall have $a a_1$ equal to the moment $R_1 r$, and with this ordinate as an altitude we may construct on $D E$ the rectangle $D E e_2 d_2$, which,

FIG. 594.



combined with the area $D E e_1 k_1 d_1$, will give the diagram $D E e_3 k_3 d_3 D$ for the combined twisting and bending moments.

The crank arm $E F$ is acted upon by the reaction R_1 at the bearing A. Let the perpendicular $A A_1$ be dropped on to the direction of the arm $E F$, and let the reaction R_1 be transferred to A_1 by the addition of a couple whose forces are equal to R_1 and its lever arm to $A A_1$; it is evident that the crank arm $E F$ will be subjected to a bending stress by a force R_1 acting at A_1, and to a twisting stress by the couple $R_1 . A A_1$. Therefore if the angle $E A_1 h_1$ is equal to $F A k$ or equal to the angle $\mathfrak{R}\, O o$ in the force polygon, the area $E F f_1 h_1$ will represent the bending moments of the crank arm. By making $A_1 a_2 = A_1 A$ we get at $a_2 a_3$ the twisting moment $R_1 . A A_1$, and by drawing the straight line $f_2 h_2$ through the intersection a_3 parallel to $E F$ we get the rectangle $E F f_2 h_2$ for the twisting moments. By combining these two areas of moments we get the diagram $E F f_3 h_3 E$ for the resultant moments of the crank arm $E F$.

The crank arm $D C$ is also subjected to bending and twisting by the reaction R_1 at A and by the piston force K at J, these two forces acting in opposite directions. Let us here draw two normals to the crank arm, one $J J_0$ from J and the other $A A_0$ from A; let us again suppose couples to be added whose moments are $K . J J_0$ and $R_1 . A A_0$, these couples causing the twisting, while the forces K and R_1 (supposed to be acting at J_0 and A_0) cause the bending. If we therefore make the angle $C J_0 M = \mathfrak{R} O \mathfrak{K}$ and $C J_0 L = C A_0 N = \mathfrak{R} O o$, the two triangles $J_0 L M$ and $A_0 N C$ will represent the bending moments of the crank arm caused by the forces K and R_1 respectively. The difference of these two triangles gives $C D p_1 q_1 l_1 C$ for the resulting bending moment. By making $A T = A A_0$ we get $T t$ for the twisting moment $R_1 . A A_0$, and by making $J_0 J_1 = J_0 J$ we get $i_1 i_2$ for the twisting moment $K . J J_0$. On account of the opposite directions of rotation of these moments, we must take the difference $T t - i_1 i_2$ for the resulting moment; and if we construct on $C D$ a rectangle $C D p_2 l_2$ with this difference as altitude, its area will represent the twisting moment of the crank arm $C D$. Finally, the two areas $C D p_1 q_1 l_1$ and $C D p_2 l_2$ can be combined in the well-known manner so as to form the diagram $C D p_3 q_3 l_3 C$ whose ordinates give the resultant moments.

All of the areas of moments are section-lined in the figure, in the directions of the ordinates representing the corresponding moments M. The dimensions at any point of the crank shaft can now be found by means of the general formula

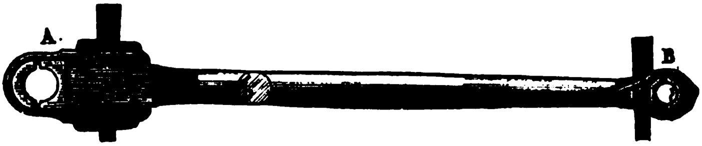
$$M = k \frac{W}{e},$$

where W is always the moment of inertia of the cross-section and *e* the distance of the outermost fibre from the neutral axis. We see from the figure that the diagram obtained gives a very clear view of the stresses to which the crank shaft is subjected at all its points. It may be remarked that in the examples here chosen, the piston force *K* and the pressure of the wheels *P* are assumed to act in the same direction. But as *K* acts alternately in opposite directions, safety requires that the diagram be drawn for the case where *K* and *P* have opposite directions, and that, from the two diagrams, the largest area of moments be chosen for each arm.

A fuller examination of crank shafts by graphical methods may be found in *Reuleaux's* Constructeur, the present discussion being based upon that work.

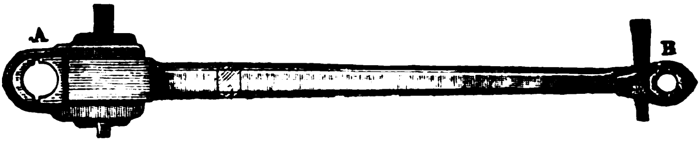
§ 158. The Connecting Rod.—The rod which transmits force between the cross-head and the crank pin is usually made of wrought iron. Formerly, when it was difficult to make heavy forgings, the largest steam engines were provided with cast-iron connecting rods having ribbed cross-sections, but the use of cast iron is almost entirely given up; this material is particularly unsuitable for high-speed engines because of the great stresses caused by the inertia of the masses. On the other hand, in rapidly running crank trains, wood as well as wrought iron has proved excellent material for connecting rods; and although wood is not used in steam engines, it is often used in gang saws. For this purpose, straight-grained fir is employed, because its specific gravity is comparatively small and its elasticity great, qualities which particularly fit it for resisting the variable action of the living forces (see Vol. I, § 375, Fifth Ger. Ed.). When the rod is subject only to tension, as in single-acting pumps, it is made cylindrical; but when the cranks are double-acting, the compressive or buckling stress requires that the diameter be greater at the middle than at the

FIG. 595.



ends, so that it has approximately the form of a body of uniform resistance to compression, Fig. 595. In slowly running cranks wrought-iron connecting rods are made circular and cast-iron rods cross- or star-shaped, for at low speeds they are principally subject to buckling stress and their resistance to bending outward or buckling must be equal in every plane.

FIG. 596.



On the other hand, connecting rods attached to rapidly running cranks, as in locomotives and gang saws, have greater dimensions in the plane of motion than in a direction perpendicular to this plane, for the action of the masses tends to bend the rod in the plane of motion. The cross-section chosen is usually a rectangle whose smaller sides are rounded as in Fig. 596, and at the same time the rod tapers from the crank pin *A* to the cross-head pin *B*.

FIG. 597.

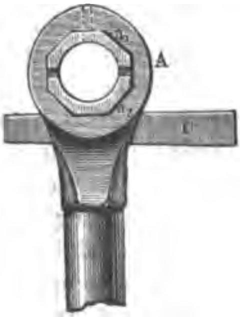
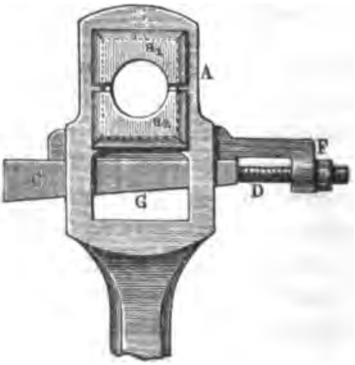


FIG. 598.



Each end of the connecting rod is provided with two ***brasses*** or *boxes* which can be adjusted to the pins by means of keys or screws, so as to take up all the play caused by the wear. This is of considerable importance where smooth and noiseless run-in general we may class them as *box-ends* or closed ends and *strap-ends* or open ends. Box-ends are shown in Fig. 597 and Fig. 598, the end of the connecting rod being forged into the form of a strong eye for receiving the accurately fitting brasses a_1 and a_2. The key *C* in Fig. 598 is moved by the screw *D* and nut *F* and the brasses may thus be kept in close contact with the pin. The object of the enlarged opening *G* in the box-end, Fig. 598, is to permit the introduction of the flanged brasses *a* into the eye. When the forces are small and when there is only a slight oscillating motion, as in the valve gear of steam engines, the adjusting arrangement is omitted and a closed eye is used, Fig. 599, the eye being provided with a hardened steel ring or bushing *a* to which a steel pin is carefully fitted. Strap-ends are shown in Fig. 600 and Fig. 601, where *C* represents the *key* or *cotter* for adjusting the brasses *a*, and *D* the *gibs* which guide the key and, by means of their hooks *E*, prevent the strap from spreading. It is evident that with the box-ends, Fig. 597 and Fig. 598, the driving in of the key eventually causes the centre of the brasses to move outwards. On the other hand, in strap-ends this centre is moved inwards. For this reason the connecting rods of steam engines are often provided with a strap-end at the crank pin and a box-end at the cross-head, so that the length of the connecting rod, measured from centre to centre of brasses, may be kept as constant as possible. Of course this object cannot be fully attained in this way, for the brasses wear less at the cross-head pin than at the crank pin, as there is much less motion at the

FIG. 599.

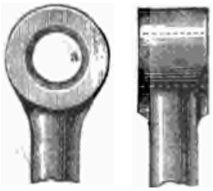


FIG. 600.

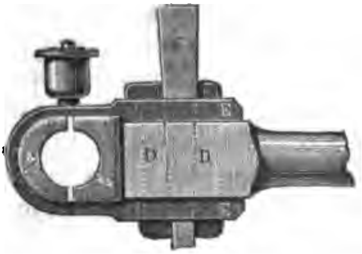
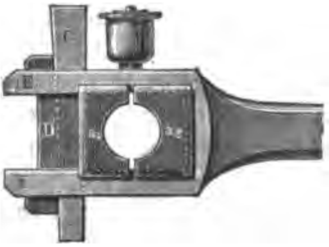


FIG. 601.



former than at the latter. For this reason the connecting-rod end is sometimes made with adjustments so that both brasses can be moved toward the centre of their pin. For the details of these constructions and of lubricating arrangements, reference must be made to the special works on machine design by *Wiebe*, *Redtenbacher*, *Reuleaux*, *von Reiche*, *Uhland*, *Unwin*, etc. The details of cross-heads and eccentrics with their straps and rods may be found in the same works.

When the rod is subjected to tensile stress only, its diameter D may be determined from the formula for tensile strength, thus:

$$D^2 \frac{\pi}{4} k = K,$$

where k is the permissible stress in the material and K the maximum force of the piston. If we also assume $k = 6$ kilograms [8500 lbs. per sq. in.], we get for the diameter of the rod

$$D = \sqrt{\frac{2}{3\pi} K} = 0.46 \sqrt{K}; \quad [D = 0.0122 \sqrt{K};]$$

and as the diameter of a wrought-iron crank pin is $d = 1.13 \sqrt{K}$ [$d = 0.03 \sqrt{K}$], we have the ratio

$$\frac{D}{d} = 0.41.$$

But in all double-acting cranks the connecting rod is alternately subjected to tensile and compressive stress, and the diameter must be determined according to the formulas for resistance to bending of long columns. We must here assume the case designated by II in Vol. I, § 266, the rounding of the rod ends permitting a certain inclination of the rod to its line of centres. Accordingly the dimensions of the cross-sections may be determined from the formula

$$K = \pi^2 \frac{WE}{l^2},$$

where l is the length of the rod, W the moment of inertia of the cross-section, and E the modulus of elasticity of the material.

With this formula we can employ a factor of safety of from 4 to 6, and may conveniently make use of Grasshof's formula, taking the coefficient in question from the table given in Vol. I, § 274, Fifth Ger. Ed., by first assuming a probable value for the ratio of the length l to the required dimension of the cross-section. According to *Reuleaux*, the diameter D of a wrought-iron connecting rod at the middle may be found from

$$D = 0.21\, d \sqrt{\frac{l}{d}},$$

where d again represents the diameter of the crank pin and l the length of the connecting rod. When the velocity of the crank is great, the connecting rod is subjected to a considerable bending stress by the forces of inertia of the elementary masses constituting the rod. These forces act only in the plane of oscillation of the connecting rod and explain why the connecting rods of rapidly running crank trains have greater dimensions in this plane than in a direction perpendicular to it. This is particularly true, as before mentioned, of the connecting rods of locomotives and gang saws.

In order that we may examine the influence of the accelerating forces on the connecting rod, let us assume that the crank pin has a constant velocity v, which is permissible in the present case, for the variations of crank velocity during a revolution, investigated in § 144, are very small in high-speed engines. In the present case the accelerations of two points of the connecting rod are known in amount and direction for every instant of the motion. The point of the connecting rod which coincides with the centre of the crank pin has only a normal acceleration, due to its uniform circular motion. This normal acceleration is directed toward the shaft, and may be determined from the expression for the centrifugal force

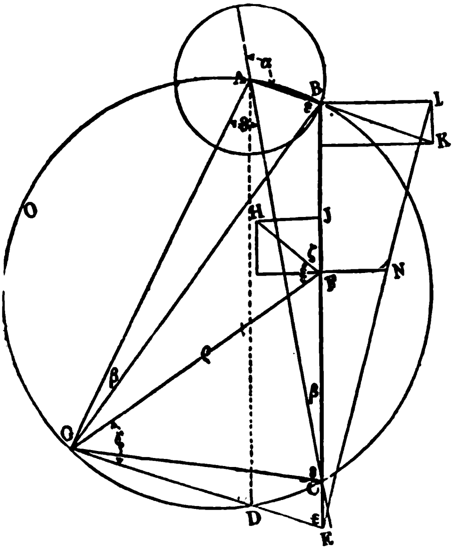
$$m\frac{v^2}{r} = p$$

(see Vol. I, § 44, Eng. Ed., and § 299, Fifth Ger. Ed.). This point has no tangential acceleration because v is constant. On the other hand, the point of the connecting rod which coincides with the centre of the cross-head pin has a tangential acceleration p_1 which is expressed by the general formula $p_1 = \frac{\delta c}{\delta t}$ where c again represents the velocity of the cross-head. This acceleration p_1 coincides in direction with the stroke; this point has no normal acceleration, for the expression $\frac{c^2}{\rho}$ becomes equal to 0 on account of the infinitely large radius of curvature ρ of the rectilinear path. The acceleration p_1 of this end of the connecting rod is the same as that determined for the reciprocating mass m_2 in § 151, namely,

$$p_1 = \left(\cos \alpha \mp \frac{r}{l} \cos 2\,\alpha\right)\frac{v^2}{r}.$$

In order to find the acceleration of the remaining points of the connecting rod, we must make use of the properties of the *centre of acceleration*, established in § 15 of the Introduction. It was there shown that in every moving system there is a point called the centre of acceleration which is itself without acceleration, and *whose rays to the various points of the system are not only proportional to the acceleration of these points, but make with these accelerations one and the same angle* ε. This centre of acceleration is constantly changing its position during the motion of the system and, while the crank is making a revolution, it traverses a certain looped line somewhat like the figure

FIG. 602.



8, for the centre of acceleration coincides twice with the cross-head, namely, at the instants when the acceleration of the cross-head is zero. We can easily determine the centre of acceleration for any position of the crank train, for instance $A\,B\,C$, Fig. 602. If we pass a circle O through the points A, B and C, this circle will contain the centre of acceleration G, for this is the only circle whose points have the property that rays drawn from them to B and C make equal angles ϵ with the directions of acceleration $B\,A$ and $C\,A$, these being angles at the circumference subtended by equal arcs. To find G itself on this circle it is desirable to find a relation for the angle ϑ, for generally ϵ is not known, its determination requiring a knowledge of the angular velocity ω about the corresponding instantaneous centre (see § 15, Introduction). To determine ϑ we write

$$G\,B = A\,B\,\frac{\sin\,[\pi - (\alpha - \vartheta)]}{\sin\,\beta} \quad \text{and} \quad G\,C = B\,C\,\frac{\sin\,\vartheta}{\sin\,\alpha},$$

from which follows, by division,

$$\frac{G\,B}{G\,C} = \frac{r\,\sin\,\alpha}{l\,\sin\,\beta}\,\frac{\sin\,(\alpha - \vartheta)}{\sin\,\vartheta}.$$

Now since

$$r\,\sin\,\alpha = l\,\sin\,\beta$$

and

$$\frac{G\,B}{G\,C} = \frac{p}{p_1}$$

are known, we have

$$\frac{p}{p_1} = \frac{\sin\,(\alpha - \vartheta)}{\sin\,\vartheta} = \frac{\sin\,\alpha}{\tan\,\vartheta} - \cos\,\alpha;$$

hence

$$\tan\,\vartheta = \frac{\sin\,\alpha}{\dfrac{p}{p_1} + \cos\,\alpha},$$

from which ϑ may be found for any angle α.

But for the present investigation it is unnecessary to determine the centre of acceleration, as will be seen from the following remarks. If we suppose that ray GE to be drawn from the center of acceleration G to the point E of the prolonged connecting rod, which makes with this rod the angle ϵ,* the acceleration p_0 of this point E will fall into the connecting rod CB, *i.e.*, this acceleration will have no component normal to CB and will therefore exert no bending action on the connecting rod. If now we take any point F, at a distance $EF = x$ from E, with an instantaneous radius $GF = \rho$, its acceleration p_x may be found from the aforesaid property of G:

$$p_x : p = \rho : GB \quad \text{or} \quad p_x = \frac{p}{GB} \cdot \rho.$$

This acceleration has the direction FH, and therefore a component normal to the connecting rod BC represented by

$$HJ = p_x \sin HFJ;$$

and since $HFJ = EGF = \zeta$, this component becomes

$$HJ = p_x \sin \zeta = \frac{p}{GB} \rho \sin \zeta.$$

Now as

$$\rho : x = \sin \epsilon : \sin \zeta,$$

we finally have the component

$$HJ = \frac{p \sin \epsilon}{GB} x = kx,$$

where the constant $\frac{p \sin \epsilon}{GB}$ is represented by k. Now if, at each point of the connecting rod, we erect the perpendicular component of the accelerating force, for instance, making $FN = HJ$, it follows, from the expression $HJ = kx$, that the ends of these ordinates all lie in a straight line passing through the point E, at which point the direction of the acceleration coincides with the connecting rod. This ordinate of the accelerating force at the end B may be obtained by resolving the centrifugal acceleration at this point

---

* This ray may be found by making $AD \parallel BC$ and then drawing GD through the intersection D of the parallel with the circle.

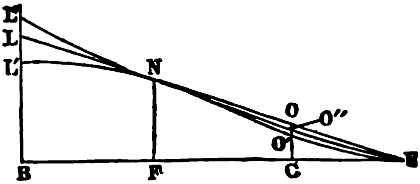
$$BK = \frac{m v^2}{r}$$

into two components, one in the direction of the connecting rod BC and the other, BL, perpendicular to the latter. The law here found, according to which the normal component of the acceleration of each element of mass m is proportional to its distance from a point E, is perfectly general and applies even when the rotation of the crank shaft is not uniform. Now if the mass of the connecting rod is uniformly distributed, in other words, if the rod is a prismatic body having everywhere the same cross-section f so that the mass of each element of length δl is

$$m = \frac{f \cdot \delta l \gamma}{g},$$

γ representing the specific weight, then will the straight line EL represent the law of variation of the stresses caused in the rod by the accelerating forces. In this case we may regard the rod as resting upon supports B and C and subjected to a load represented by the area $BLOC$, Fig. 603. On the other hand, if the rod is not prismatic but has different cross-sections f at different points, we can express the law according to which the load is distributed, by a curve each of whose ordinates is the product of the cross-section f, the corresponding ordinate of the straight line LE. For example, the load line of a connecting rod whose cross-section is greatest at the middle point F and becomes smaller toward the ends, is represented by a line like $L'NO'E$, while the curve $L''NO''E$ corresponds to a rod which is greatest at the crank pin B and gradually diminishes toward the cross-head C. These three load lines will have the point N in common, for this point corresponds to the point F at which the cross-

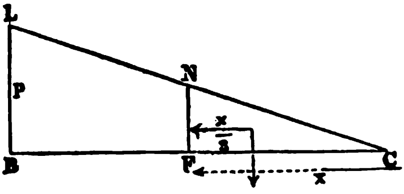
FIG. 603.



sections are equal. The form of these curves evidently depends on the longitudinal profile of the rod or on the law expressing the variation of the cross-section. In the following we will assume the simple case in which the mass is uniformly distributed, that is, we will assume the connecting rod to be prismatic, the straight line $L E$ representing the load line. It is evident that this straight line $L E$ is different for different positions of the connecting rod, and that the point of intersection E, in particular, is at very different distances from C. While the crank is at the dead points there are no normal accelerations in the connecting rod; the load line $L E$ will therefore coincide with the rod $B C$, but as soon as the acceleration of the cross-head is 0, *i.e.*, as soon as its velocity is a maximum, the point E will coincide with the cross-head C. In a previous article these positions, which principally depend on the ratio $\frac{r}{l}$, were more fully determined, and it was there stated that the maximum piston speed for the ordinary proportions of $\frac{r}{l}$ corresponds very nearly to that crank position at which the connecting rod is perpendicular to the crank. Now since in this position the acceleration of the element m at the crank pin B is perpendicular to the connecting rod, and therefore its whole amount acts transversely, this position agrees very closely with that at which the connecting rod is subjected to the greatest stress by the transverse action of the accelerating forces. If therefore we base the calculation on this crank position, we shall have the case of a beam resting on two supports B and C, Fig. 604, subjected to bending stress by a load distributed as shown by the triangle $B L C$. Here $B L = p$ represents the force accelerating an element of mass at B, whose length is unity, *i.e.*,

$$p = \frac{f \gamma}{g} \cdot \frac{v^2}{r}.$$

FIG. 604.



The total load acting on the rod and distributed as shown by the triangle BLC is equal to

$$P = p\frac{l}{2},$$

which causes at B and C the reactions

$$B = \frac{2}{3}P = p\frac{l}{3} \quad \text{and} \quad C = \frac{1}{3}P = p\frac{l}{6}.$$

For any point F at the distance x from C whose load ordinate is $y = \frac{p}{l}x$, the moment M of the exterior forces becomes

$$M = Cx - y\frac{x}{2}\cdot\frac{x}{3} = p\frac{lx}{6} - \frac{p}{6l}x^3 = \frac{p}{6}\left(lx - \frac{x^3}{l}\right).$$

This moment is a maximum for

$$\frac{\delta M}{\delta x} = 0 = l - 3\frac{x^2}{l},$$

i.e., for

$$x = l\sqrt{\tfrac{1}{3}} = 0.577\,l.$$

With this value of x we obtain the maximum breaking moment

$$M_{max} = 0.0642\,p\,l^2 = 0.064\frac{f\gamma l^2 v^2}{g\cdot r}.$$

If in this we substitute

$$v = \frac{2\pi r n}{60},$$

where n is the number of revolutions of the crank per minute and $g = 9810$ millimeters [386 ins.], the acceleration of gravity per second, we have

$$M_{max} = 0.0717\frac{f\gamma l^2 r n^2}{1000000} = k\frac{W}{e}$$

$$\left[M_{max} = 0.00000182\,f\gamma\, l^2 r n^2 = k\frac{W}{e}.\right]$$

If we assume a circular cross-section with diameter D, we have

$$\frac{W}{e} = \frac{\pi}{32} D^3 = 0.0982 D^3,$$

and $f = 0.785 D^2$; hence

$$D = 0.000000573 \frac{\gamma l^2 n^2 r}{k}. \qquad \left[D = 0.0000146 \frac{\gamma l^2 n^2 r}{k}.\right]$$

This formula will answer for determining, in the case of high-speed cranks, the diameter of rod needed to resist the forces of inertia; when, in consequence of the longitudinal stress, greater dimensions must be given to the rod, the formula will serve to ascertain the stress k caused in the rod by the accelerating forces.

If the rod is not of uniform cross-section, the maximum moment caused by the forces of inertia can best be determined graphically, by first finding the load line $L' O'$ or $L'' O''$, Fig. 603, from the law of variation of the cross section, and then drawing the equilibrium polygon in the manner formerly described (Vol. I, Appendix, § 45, Fifth Ger. Ed.). This investigation was undertaken by *Proell*, in an article* of which use has been made in the preceding discussion. In this article it was found that with a conical rod having the diameter D at the centre, the diameter $0.8 D$ at a distance $0.1 l$ from the crank pin, and the diameter $0.7 D$ at a distance $0.1 l$ from the cross-head pin, the relation obtained was

$$D = 0.000000559 \frac{\gamma l^2 n^2 r}{k}. \qquad \left[D = 0.0000142 \frac{\gamma l^2 n^2 r}{k}.\right]$$

For a rod of rectangular section and of constant breadth, whose depth at the crank pin is h and at the cross-head $0.6 h$, it was then found that

$$h = 0.000000567 \frac{\gamma l^2 n^2 r}{k}, \qquad \left[h = 0.0000144 \frac{\gamma l^2 n^2 r}{k},\right]$$

---

* *R. Proell*, Civil-Ingenieur, vol. xviii. Nos 2 and 3.

from which it may be concluded that this last-mentioned rod is more economical of material than the straight cylindrical rod.

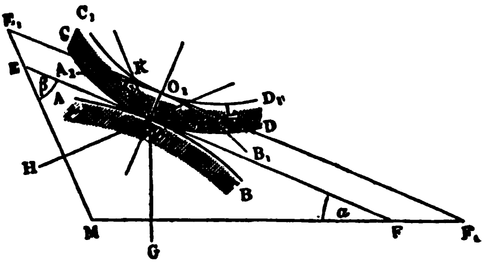
The literature of double cranks is quite extensive. There is an article on the subject by *J. Weisbach* in Polytechn. Centralblatte, 1843, vol. i. The first correct theory is due to *Eytelwein*. *Morin* discusses the theory of the double crank very fully in the third part of his Leçons de Mécanique Pratique. For the many mechanisms deducible from the crank train, reference must be made to the elegant presentation by *Reuleaux* in his Kinematics of Machinery. *Laboulaye* also discusses crank mechanisms in his Traité de Cinématique. The details of the separate pieces constituting the crank train are given in all the text books on Machine Design, *Reuleaux's* Constructeur giving special attention to graphical methods. Mention has already been made in the foregoing articles of *Radinger's* "Dampfmaschinen mit hoher Kolbengeschwindigkeit" and *Proell's* "Beschleunigungen in der Schubstange eines Kurbelmechanismus."

CHAPTER VII.

CAM TRAINS.

§ **159. Cam Surfaces.**—In designing machinery it is often necessary to transform the uniform sliding or turning of a machine part into a variable motion of another machine part, the law of variation being prescribed by the object to be attained. The non-circular wheels described in § 49 furnish one example of such mechanisms, for by their means the uniform motion of one shaft is transmitted to another shaft in such a way that the velocity ratio is subject to certain periodical variations. A peculiarity of these non-circular wheels is that the motion of the driven shaft is always in the *same direction*, so that no reversal of the direction of motion takes place. If such a reversal is to take place, other means must be employed, and from the preceding chapter it is evident that the crank train is such a means, for it can transform the continuous rotation of a shaft into the reciprocating motion of another piece, say a lever or rod. In this case, when the crank shaft has a uniform rotation the character of the motion of the driven part is fixed by the geometrical character of the crank train. Consequently this mechanism will not answer for cases where the motion must vary according to some other law, say one prescribed by the process employed in performing the work. *Cams* generally enable us to solve this problem: they are rigid machine parts with curved surfaces of such a *form* that the machine parts in *constant contact* with them describe perfectly definite motions. The cam and the machine element which it guides constitute, according to § 29 of the Introduction, a higher pair

FIG. 605.



of elements, which elements have the property of being reciprocally *envelopes* of each other, the two elements touching each other in *points* or *lines*, not in *surfaces* as in *closed pairs*. To explain the action of such mechanisms, let $A\,B$, Fig. 605, be the profile of the driving piece and $C\,D$ the outline of the driven element. Let the two surfaces touch at a point O or, more exactly, along a line projected at O. It may make the matter clearer to regard $A\,B$ as a wiper attached to a shaft whose rotation moves a lever partly bounded by the surface $C\,D$. Whatever the character of the motions of the two machine pieces may be, we may, for the instant in question, regard them as infinitely small rotations about their instantaneous axes (see Introduction, § 7). Therefore if we suppose the lines $O\,G$ and $O\,H$ to be drawn from the point of contact O of the two curves to the instantaneous centres of the two bodies, the elementary motion of the surface $A\,B$ can be regarded as an infinitely slight displacement in the direction $O\,L$, perpendicular to the instantaneous radius $O\,G$. In like manner the elementary motion of the body with a surface $C\,D$ can be regarded as an infinitely slight displacement in the direction $O\,K$, perpendicular to the instantaneous radius $O\,H$. After these elementary motions, suppose the curves to have reached the positions $A_1\,B_1$ and $C_1\,D_1$, contact taking place at O_1, and let $E\,F$ and $E_1\,F_1$ be the common tangents to the profiles at the points of contact O and O_1.

If the assumed elementary motion is regarded as infinitely small, the two tangents will be parallel and the two points J and L will coincide, these points representing the intersections of the path $O\,D$ of the point O on the curve $A\,B$ with the new position $A_1\,B_1$ and with the tangent $E_1\,F_1$. The same may be said of the curve $C\,D$ of the driven body, the points K and N coinciding; for these points represent the intersections of the path of the point O of the curve CD with the new position C_1D_1 and the tangent E_1F_1. Therefore if v and v_1 represent the velocities of the two points in contact at O, we have the general equation

$$v : v_1 = OL : OK.$$

If through any point M we draw two straight lines MF and ME parallel to the directions of the motions v and v_1 of the points in contact at O, till the lines intersect the tangent at O in F and E, we may determine from the triangle EMF formed by these three directions the ratio of the two velocities v and v_1, for we have

$$v : v_1 = MF : ME = sin\ \beta : sin\ \alpha.$$

Here α and β represent the angles EFM and FEM formed by the tangents to the curves, with the directions of motion of the point in contact at any instant. If these two directions are perpendicular to each other at any instant, the above equation becomes

$$v_1 = v\ tan\ \alpha.$$

In consequence of the infinitely small motion, the point O of the surface AB has moved to J or L, and the point O of the surface CD to N or K; a relative sliding w equal to NL must therefore have taken place between the two surfaces, and for this relative motion we obtain from the triangle EMF the equation

$$v : v_1 : w = MF : ME : EF = sin\ \beta : sin\ \alpha : sin\ (\alpha + \beta).$$

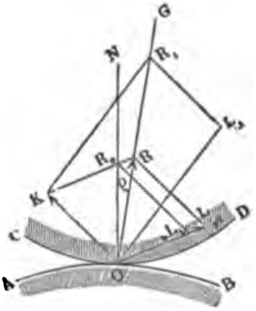
This relative sliding w of the surfaces on each other is of considerable importance in actual constructions, for it represents the distance through which the sliding friction between the two surfaces must be overcome while the supposed motion is taking place. The amount of this friction depends, in every case, on the normal pressure N between the surfaces at the point of contact and must be taken equal to ϕN, ϕ representing the coefficient of friction. We easily see from the figure and the above proportion that the ratio of the path w of this friction to the path v_1 of the driven point is greater the smaller the angle $\alpha = EOK$, and for the case when $\alpha = 0$, *i.e.*, when the tangent coincides with the direction of the driving point, we have

$$v_1 = 0; \quad w = v.$$

In this case, therefore, all the work will be expended in overcoming the frictional resistance, as when a load is slid along a horizontal plane. We already see that the mechanisms here discussed involve considerable friction, and that in general they consume so much power as to render them unsuitable for the performance of great quantities of work. They are usually employed only where the work to be transmitted is small. When the opposite is the case, as in hammer work, the angle α made by the tangent with the direction of motion of the driving point is taken as large as possible. It is also customary to reduce the sliding friction by means of rollers.

With reference to the angle α made by the common tangent EF with the direction OL of the motion of the driving point, the following additional remarks may be made: If ON, Fig. 606, is the common normal to the two surfaces at the point of contact O, and if we lay off from it the angle of friction $\rho = NOG$, we know from the theory of sliding friction that this straight line OG is the direction of the force with which the surface AB acts on the surface CD during the motion from O to L. Hence if OK represents the force which must be exerted upon the point O of the driven piece CD in order that motion may be possible, and if we draw on OK the parallelogram $OKRL$, whose other side coincides with the direction of motion of the point O on AB, and whose diagonal coincides with the direction OG of the reaction, then will OL represent the driving force which when applied at O is just sufficient to produce the desired motion. On the other hand, if friction did not exist, it would be necessary to take the nor-

FIG. 606.



mal ON as the direction of the reaction, and then the side OL_0 of the parallelogram OKR_0L_0 would give the theoretical driving force. Now if the direction of motion of the driving point O on the surface AB were not directed along OL, but along OL_1, the side OL_1 of the parallelogram OKR_1L_1 would give the driving force needed. This force will be greater the more the direction OL_1 approaches the direction OG of the reaction, and will be infinite when the two directions coincide. In this last case motion would be impossible, however great the driving force was taken, and we therefore conclude that the possibility of transmitting motion by two surfaces in sliding contact is governed by the condition that ***the angle made by the direction of motion of the driving point with the common normal of the surfaces shall be greater than the angle of friction.*** From the relation

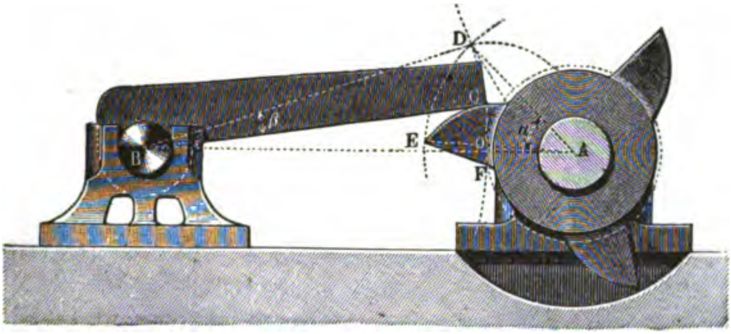
$$v : v_1 = \sin \beta : \sin \alpha$$

found above, it follows that the ratio of the velocities of the two shafts at any instant depends upon the inclination of the common tangent to the directions of motion of the points in contact, *i.e.*, on the *form* of the two surfaces. Therefore in every case the profiles of the surfaces acting on each other must be so determined that the transmission of motion will take place in the desired manner. How this may be done will be shown by a few examples.

§ **160. Wipers.**—In practice it is often necessary to convert the uniform rotation of a shaft into the reciprocating motion of a lever oscillating about an axis. This case occurs in the motion of tilt-hammers, the driving shaft lifting the arm of the hammer, and the return motion being effected by the weight of the hammer. The fall of the hammer is utilized in the forging of metals and in breaking hard bodies into small pieces. To accomplish this the driving shaft is provided with a wheel carrying one or more projections known as *wipers*, on whose form principally depends the manner in which the lifting of the hammer takes place.

We will first take the simplest case, that in which the uniform rotation of the shaft is to communicate a uniform motion to the axle of the hammer. This transmission of motion is the same as that between two toothed wheels with constant velocity ratio, and we can therefore at once apply the rules for the profiling of teeth established in Chapter II. For example, let A, Fig. 607, be the shaft carrying the wiper wheel, and B the axle of a trip-hammer whose lever is moved through the angle $CBD = \beta$ with uniform velocity by a wiper CE, and let $\alpha = EAD$ represent the angle through which the cam-shaft turns during this raising of BC. From the known relation

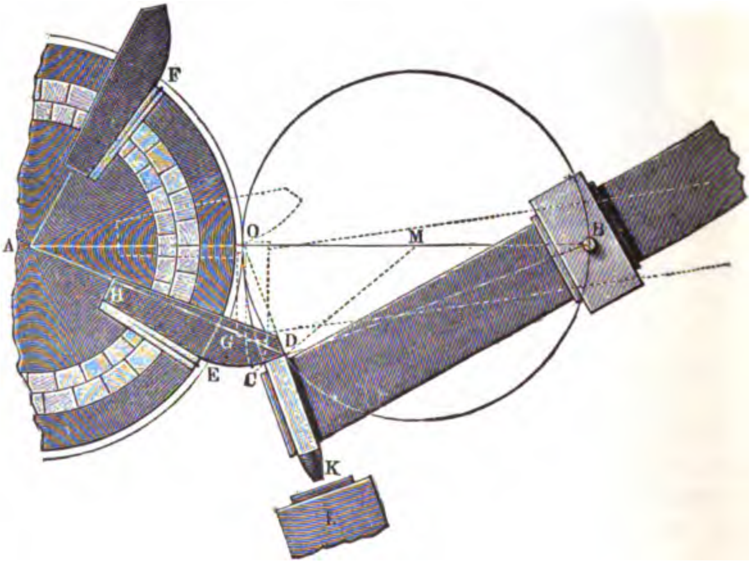
FIG. 607.



$$AO : BO = \beta : \alpha$$

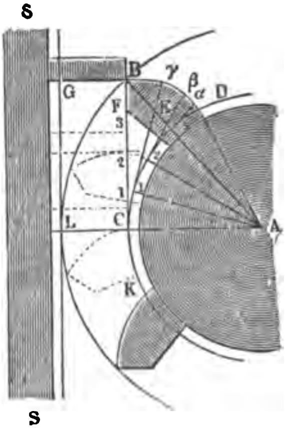
we find on the line of centres AB the point O at which the pitch circles of two spur wheels touch when the transmission of motion of the wheels is the same as that of the present system. After these pitch circles have been established there is no difficulty in determining the form to be given to the wiper CE. For instance, if we assume that the lever BC is always to be touched in the same point C, we must regard this point C as the tooth of the wheel B; and according to § 72, the corresponding profile of the wiper will be an epicycloid generated by rolling the pitch circle BO on the pitch circle AO, for we can regard the point C as generated by the rolling of the pitch circle BO within itself. The limit E of the wiper can be found from the extreme position BD of the lever by describing through D a circle about A. The back surface EF of the wiper never comes into action, and it may have any form provided it does not interfere with the fall of the hammer. The construction here described is not to be recommended, because the very considerable friction between wiper and lever can only take place at one point *C* of the latter, consequently there must be rapid wear. To avoid this evil we may assume a describing circle of smaller radius than *B O*, and then, according to § 70, we get a hypocycloid and an epicycloid for the profile of the lever and wiper respectively. For example, if in a hammer, Fig. 608, the surface *D C* of the lever acted upon is given the direction of the radius *B C*, *i.e.*, if a generating circle of diameter *B O* is chosen, we shall get for the profile of the wiper, according to § 71, the epicycloid generated by rolling the describing circle *B O* on the pitch circle *A O*. In like manner all the rules given for profiling spur wheels can here be applied to wipers; therefore by means of the general method given in § 69, for any surface assumed as the acting surface of the lever, we can find the corresponding form for the wiper.

FIG. 608.



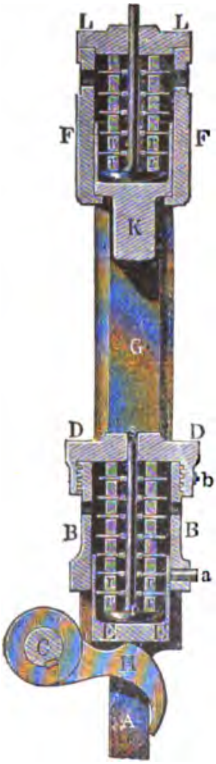
If we suppose the length *B C* of the lever to gradually increase, the circular motion of the end *C* will finally become a rectilinear sliding motion, illustrations of which may be found in stamp mills. In this case the form of the wiper is determined in the same way as the forms of the teeth of pinions gearing with racks. For example, let A, Fig. 609, be the cam-shaft which is to lift the stamp SS by acting on its horizontal plate or *tappet* BG. In order that the lifting may take place with uniform velocity, the wiper must be given a profile BD which is the involute of that circle described about A, which touches the direction of motion BC of the end B (see § 71). Here the tappet is always driven at its outermost point B, and owing to this one-sided action, the stamp SS is pressed against the upper and lower guides, causing considerable friction in them. To avoid this, the projecting tappet GB is omitted and the stamp is provided with a slot into which the wiper enters and acts against the upper horizontal surface of the slot. Fig. 610 shows this arrangement as it is applied in certain hammers. Here A represents the slide which is connected below with the hammer and above with a slot for receiving the wiper H of the driving shaft C, carrying at its upper end the buffer box DB. The surface acted upon by the wiper is the flat bottom EE of a hollow cylinder containing rubber disks $r\,r$. By means of these elastic disks the impulse received from the wiper is transmitted to the hammer slide with as little loss as possible. The second buffer box LF, fastened to the upper portion of the frame, contains other rubber springs rr for a second buffer K, which is pressed upward into its chamber by the living force of the rising hammer, thus increasing the energy of the subsequent fall by the elasticity of the rubber disks r.

FIG. 609.



In all cam trains where the driven piece has a velocity v_1 when the cam ceases to act, the driven piece will tend to move still further by virtue of the living force stored in it. Let M_1 be the mass of the hammer just considered, reduced to the driving point of the wiper (see Vol. I, § 283); this mass by virtue of its velocity v_1 possesses a living force $\frac{M_1 v_1^2}{2}$ which enables the hammer to rise through a distance $\frac{v_1^2}{2g} = h_1$. Now if the hammer is raised through the distance h by the wiper and then rises through the distance h_1 by reason of its velocity v_1, the effect of the subsequent fall will correspond to a total fall $h + h_1$. If we neglect the frictional resistances, this action will not be accompanied by any loss of work, for the work expended in throwing up the hammer is completely given back during the fall, but it is attended by a disadvantage, namely, the time needed for an up and down stroke of the hammer is considerably increased thereby. In rapidly running cam shafts it may readily happen that the wiper will again act upon the hammer before the latter has completed its fall. This explains why the buffer springs FL, Fig. 610, are applied to rapidly working hammers, the action of the springs resembling that of the *top steam* in quickly striking steam hammers. Also in rapidly running trip hammers *spring beams* are used, for without them, with a stroke of from 0.2 to 0.25 meter [8 to 10 ins.], it would be impossible to give 300 to 350 strokes per minute.

Fig. 610.



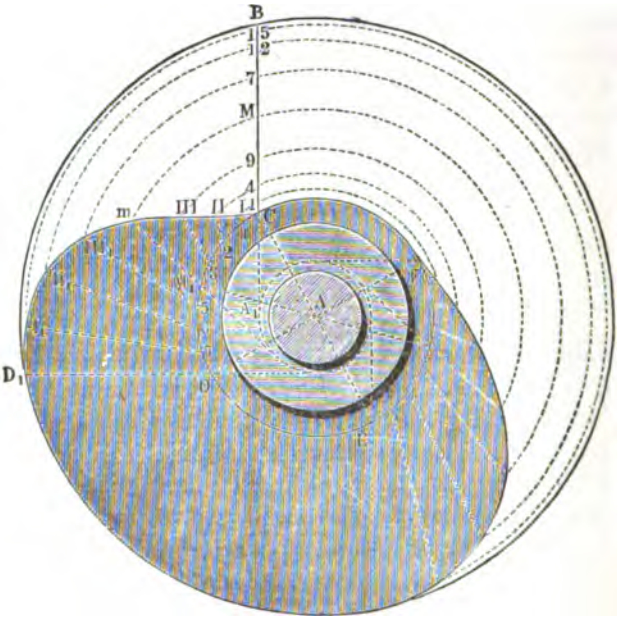
While the object of the buffer K is to reduce the time needed for an up and down stroke, the object of the buffer EE is to diminish the loss of living force accompanying the shock which takes place when the wiper acts on the hammer. Such shock is unavoidable with all wipers which, like those hitherto considered, are intended to impart a uniform motion to the driven piece, for in accordance with this condition the piece must suddenly pass from a state of rest to one with a velocity v_1. By using the buffer BD this blow is made as elastic as possible, for during the first instant of the action of the wiper on the plate EE the latter is pressed into the buffer box, thus compressing the rubber disks r. The upward motion of the hammer does not begin till this compressive force exceeds the weight of the hammer. As the rubber springs used for this purpose are never perfectly elastic, the loss of work in question cannot be wholly avoided, but may be considerably reduced. In trip hammers, in spite of their rapid motion, buffers for taking the blow of the wiper are not used and the loss occasioned by shocks must be diminished by making the mass M of the cam shaft as large as possible. For if M and M_1 represent the mass of the driving shaft, and the driven hammer reduced to the driving point, the loss of work due to the inelastic impact at the velocity v will be given by

$$V = \frac{MM_1}{M + M_1} \frac{v^2}{2},$$

which value becomes smaller the greater M is in comparison with M_1 (see Vol. I, § 346). In the slowly running hammers used in stamp mills this loss of work is usually insignificant.

Moreover the disadvantages attending the impact can be avoided by giving to the wiper such a form that it will gradually communicate motion to the lever or rod, for instance, according to the law of uniformly accelerated motion. When this is a condition, of course the rules developed for profiling the teeth of spur wheels cannot be employed for determining the profile of the wiper; this must be determined in each case from the law according to which the motion is to be gradually imparted. The construction may be best shown by an example. Let the problem be to so communicate motion to a rod sliding in the direction CB, by means of a cam disk attached to a uniformly rotating shaft A, Fig. 611, that the rise and fall of the rod between C and B will take place during a quarter revolution of the shaft A, the rod being uniformly accelerated during the first half of its path, and uniformly retarded during the second half, and having at the end of each up and down stroke a period of rest likewise equal to a quarter revolution. To make the problem more general we will assume that the direction of motion of the rod does not pass through the axis *A*. If we divide the circle passing through the initial point *C* of the rod into four equal parts, by the points *C*, *D*, *E* and *F*, these points of division will in turn occupy the position *C* when the transition takes place from one period to another. It is evident that for the periods of rest of the rod the cam disk must be concentric to *A*, the quadrant *CF* corresponding to the pause when the rod is at its lowest point *C*, and a quadrant of the circle described through *B* corresponding to the stationary period of the rod when it is in its highest position. To determine the ends D_1 and E_1 of this arc it is only necessary to lay off at *D* and *E* the straight lines $D D_1$ and $E E_1$ so as to make with the radii *A D* and *A E* an angle α equal to that in-

Fig. 611.



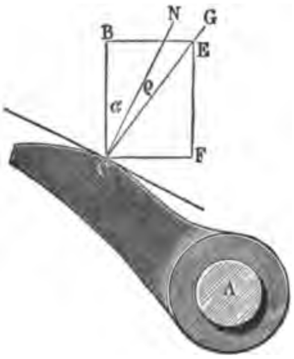
cluded by the direction BC of the rod and the initial radius AC. For this purpose it will be convenient to use the circle AA_1 which is tangent to the direction of the rod.

To find the curved profiles between the cylindrical portions of the cam, bisect the stroke CB at M and the quadrant CD at M_1. When M_1 has reached C the rod will have traversed the path CM with uniformly accelerated motion. As the spaces described during this motion are proportional to the squares of the times, we can use the following construction: divide the arc CM_1 into n equal parts, 4 in the figure, and the distance CM into n^2 equal parts, 16 in the figure; now draw circles concentric to A through the points of division 1, 4, 9 and M of the straight path, and tangents to the circle AA_1 through the points of division 1, 2, 3 and M_1 of the arc CM_1; then will the corresponding intersections I, II, III and m be points of the required curve, this curve flowing into and becoming tangential to the circle CF at C. The curve m III_1 II_1 I_1 D_1 for the uniformly retarded portion of the lifting motion is found in the same manner, MB being divided into 16 equal parts; but the circles are now passed through the points of division 7, 12, 15, for the paths traversed in the quarter intervals of the uniformly retarded motion are as the odd numbers 7, 5, 3, 1. The curve obtained likewise becomes tangent to the quadrant D_1E_1 at D_1. In the same way we may find the curve between E_1 and F for the motion of the down stroke. It is easy to see that the profiles will not be symmetrical in the present case on account of the eccentric position of the rod BC.

With the cam here found there is no shock at the beginning of the motion of the rod, and when motion ceases there is no loss of work by the destruction of stored up living force. On the other hand, there is a disadvantage attending this kind of construction, which consists in the pressing of the rod against its guides with a force which is greater the more acute the angle made by the profile of the cam with the direction of the rod. This latter pressure causes corresponding friction in the guides and, thus, loss of work. In Fig. 612, let CB be the direction of stroke of the rod and CN the normal to the profile of the wiper at C. By making the angle NCG equal to the angle of friction ρ and resolving the driving force CE of the wiper into two rectangular components CB and CF, the latter component will give the pressure S exerted at C perpendicular to the rod. If α represents the angle BCN of the rod with the normal, and $q = BC$ the resistance of the rod, the lateral pressure will be given by

$$S = CF = Q \tan(\alpha + \rho).$$

FIG. 612.



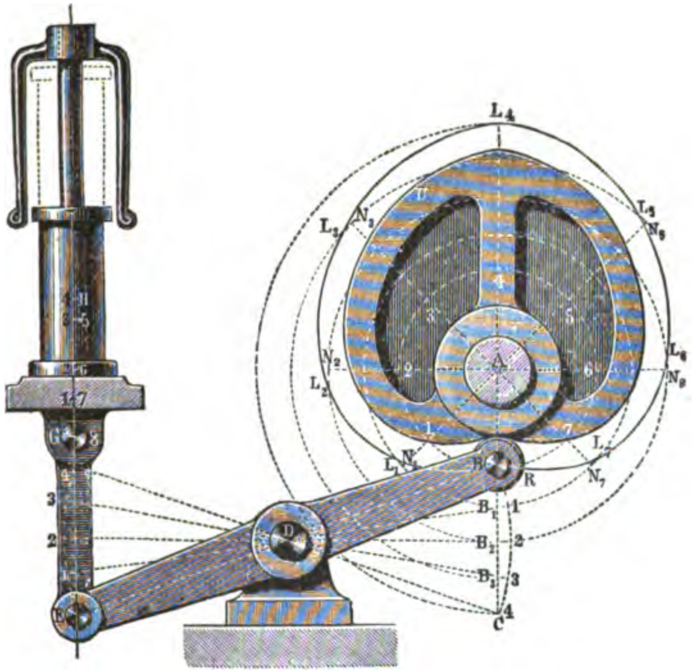
This lateral pressure, causing friction in the guides, increases with the angle α and does not wholly disappear even when $\alpha = 0$; for instance, in the example in Fig. 609, where the rod is pressed to one side by a force equal to the friction

$$Q \tan \rho = \mu Q.$$

If this lateral pressure is to be wholly avoided, the profile of the wiper must be given such a form that the stroke of the rod will deviate from the normal to the curve by the angle of friction ρ.

When the cam drives the rod or lever by means of a friction roller employed for the purpose of diminishing friction, the curve of the cam must be constructed as above and under the supposition that it acts directly on the lever at the axis of the roller. The curve thus found corresponds to the motion of the centre of the roller and is an equidistant to the profile of the cam. As an example of this construction, we will take the case so frequently occurring in spooling machines and throstles, in which the cam, by means of a double-armed lever, gives an up and down motion to the spool frame. Let GH, Fig. 613, be the rise and fall to be communicated to the spool frame by the lever BDE and connecting rod EG, the end B of the lever being provided with a friction roller R acted upon by the cam disk U of the shaft A. Moreover let the law for the up and down motion of the spools be given by the form of the wound thread (see § 141), and let us assume that this motion is a uniform one, as is required for winding the thread in *cylindrical* layers. Let us now divide the whole travel $G H + H G$ of the spool frame during the up and down stroke into any number of equal parts and then find the positions 11, 22, 33, etc., of the lever $E B$ corresponding to the points of division 1, 2, 3, 4, . . .

FIG. 613.



In like manner divide the circle, described about A and through B, into the same number of equal parts, and draw the radii A 1, A 2, A 3, etc. These radii intersect at $N_1 N_2 N_3 \dots$, the circles described about A through the divisions of the arc $B C$. It follows from the figure that we can obtain the *pitch line*

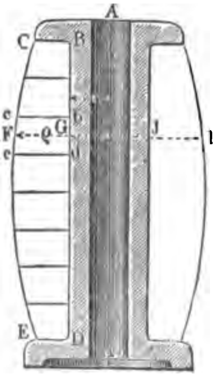
$$B L_1 L_2 L_3 \dots L_7 B$$

of the cam profile, for the centre B of the roller, by laying off from each point of intersection N_n the distance $N_n L_n$ equal to

$B_n\, n$, *i.e.*, by laying off that portion $D B_n$ included between the arc and the chord $B\,C$. If, about the points of this curve L, we describe arcs with the radius R of the roller, these arcs will envelop the desired profile U of the cam. We see from the figure that the curve L is not symmetrical, although the oscillations of the lever $E\,D$ are exactly the same in the two directions. The cam profile becomes symmetrical only when the motion of the point B takes place along a straight line passing through the axis A. The cam profile can be made approximately symmetrical by assuming, as in the figure, such an arrangement that the chord $B\,C$ of the arc described by B will pass through the axis A.

When the body of the thread is to have a different form, say one like the cross-section $B\,D\,E\,C$, Fig. 614, the stroke $B\,D$ must be divided into a great number of equal parts like $b\,d$. If a constant quantity of thread is delivered by the machine, the time during which the thread is being wound on the portion of the spool between the planes $d\,e$ and $b\,c$ must be proportional to the average cross-section $F\,H$ of the body of the thread, and consequently proportional to the area $\pi\,(\rho^2 - r^2)$ of the annular ring $F\,G\,J\,H$.* Therefore if we determine these cross-sections at the middle of the different parts of the spool, we shall get numbers proportional to the corresponding angles of rotation of the cam shaft, *i.e.*, the proportions according to which a half revolution must be divided in the construction for Fig. 613.

Fig. 614.



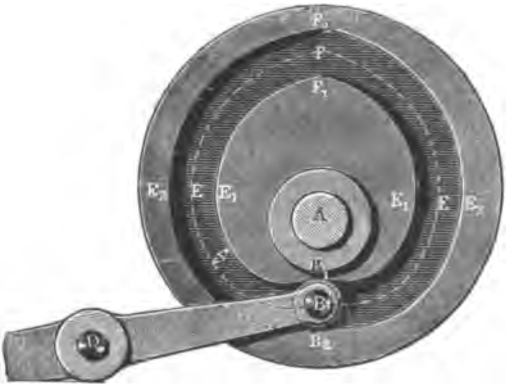
---

* This remark only holds when, as in *throstles*, the quantity of yarn supplied is always constant; the angular velocity of the bobbin dragged around by the thread of course varies with the changing diameter of the layers. In spooling machines, however, the number of revolutions of the spools is constant, and therefore the amount of thread wound on to the spools is variable. Hence the times needed by the thread for traversing parts of the spool like $b\,d$ are proportional to the number of coils contained in these parts, *i.e.*, to the areas of the axial sections $b\,d\,e\,c$; and when the altitudes $b\,d$ are equal and small, the times are proportional to the average radial thickness $F\,G = \rho - r$ of the layers.

§ **161. Flat Cams.**—Up to this point it has been tacitly assumed that the cam of the rotating shaft moved the lever or rod in only one direction, the return motion being effected by some exterior force, say the weight of the hammer. In hammers and stamps this return motion was a perfectly free one, the back of the cam being sufficiently cut away, but in the cases shown in Fig. 611 and Fig. 613 the back of the cam serves to give a certain determinate motion to the end of the lever or rod. During the forward motion the cam drives the lever, and on the return motion the lever tends to drive the cam, in these last-mentioned examples; but in hammers it is not permissible for these parts to act upon each other at all during the return motion.

But it often happens that the return motion of the swinging piece cannot be effected by an exterior force and therefore the cams must have such a form as will enable them to drive the swinging piece in both directions. This leads to the use of grooved disks. For example, let the curve $B\ E\ F\ E$, Fig. 615, be a profile of a cam on the shaft A which communicates the desired motion to the end B of a lever oscillating about D. The groove in the cam plate is bounded by two cylindrical surfaces whose bases are curves $B_1\ E_1\ F_1$ and $B_2\ E_2\ F_2$ parallel to $B\ E\ F$, and at the distance r from the latter. If the end B of the lever be provided with a fixed or loose pin of radius r and the groove be cut into the side of a plate, the arrangement will attain the desired object.

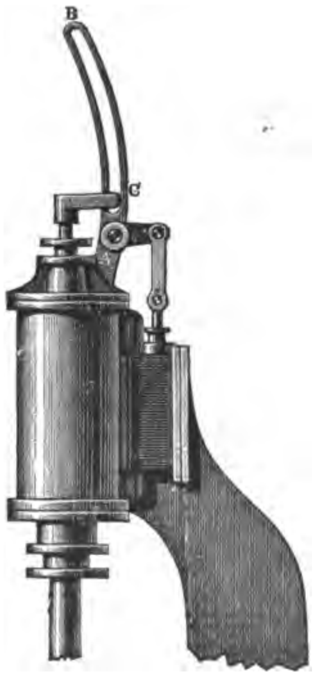
FIG. 615.



Such cam plates are very often used in machines in which it is necessary to communicate to a lever or rod from a uniformly rotating shaft a motion of a perfectly determinate kind such as is prescribed by the nature of the process or by the form to be given to the work piece. The form of the groove can be determined in the manner already described, from the kind of motion that is to be produced. The piece guided along the groove is usually cylindrical, being either a pin rigidly fixed to a rod or lever, or a loose friction roller. In the latter case the ordinarily considerable sliding friction is reduced, for it is transferred to the pin on which the roller turns.

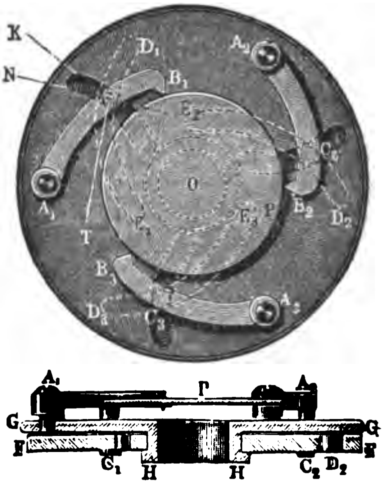
It occasionally happens that the reciprocating cylindrical pin drives, giving to the slotted plate a turning motion about its axis, this motion being limited to small *oscillations* which never become *complete revolutions*. An example of this may be found in an arrangement for moving the valve of a steam hammer, the principle of which is illustrated by Fig. 616. Here the cam plate has the form of a lever *A B* turning about the shaft *A*, the curved slot of the lever receiving the pin *C* that moves up and down with the piston rod. It is easy to see that this reciprocating motion of *C* will not cause an oscillation about the axis *A*, so long as the centre line of the groove coincides with the direction of motion of the pin. This permits an intermittent motion to be communicated to the shaft *A* by the continuous motion of the pin *C*.

FIG. 616.



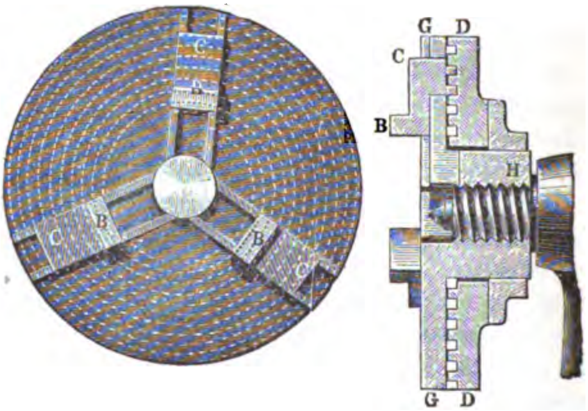
Cams are not only used as mechanisms but as clamping devices for fastening work pieces. An example of these is a little tool used by clock makers for holding a circular disk: here a disk-shaped frame *G H*, Fig. 617, carries at equal distances from the centre *O*, three pins *A* which are equally distant from each other. These pins are the axes of three similar levers *A C B*, whose ends *B* press against three points of the circumference of the plate *P*. In order that these levers may be firmly pressed against the plate, each is provided with a pin *C* which passes through a circular slit of the frame plate, the free end of the pin entering into a spirally shaped groove *D E* on the cam plate *F F*. This plate is arranged so that it can turn loosely around a projecting ring *H H* of the frame plate *G*, and it is easy to see that a turning of the cam plate *F* will cause the three jaws *B* to clamp the plate. In order that these jaws may not open of their own accord, it is necessary that in every position of the pin *C* its direction of motion *C K* (perpendicular to the radius *A C*) shall make with the normal *C N* to the curve *D E* at *C* an angle *K C N* which is smaller than the angle of friction between the pin *C* and the cam groove. Consequently the angle *A C T* included between the radius *A C* and the tangent *C T* to the curve of the cam must also be less than the angle of friction in every position of *C*. Accordingly the cam groove *D E* can be laid out so as to satisfy the condition that the aforesaid angle must be constant for all positions of *C*. If the pin *C* were guided in a radial slit of the frame plate *G* instead of a circular arc about *A*, a constant value for the aforesaid angle would cause the cam groove *D E* to become a logarithmic spiral (see § 50). Mention has been made in § 64, Fig. 226, of an expanding pulley in which a similar arrangement is employed.

Fig. 617.



A peculiarity of the arrangement shown in Fig. 617 is that the three points B of the jaws $A\ B$ always lie on a circle concentric with the axis O, making of it a self-centring device for holding work. Use is made of this property in certain lathe chucks whose object is to quickly and accurately centre pieces of work. Here the jaws are not jointed to pins, but are guided in radial slits, so that this mechanisn corresponds to the case in which a rod is moved in a straight line by a cam plate. Fig. 618 shows an arrangement of this kind known as the *scroll chuck.* The face of the cam plate D is provided with a spiral groove, which fits corresponding projections on the jaws $B\ C$. It is evident that when the cam is turned the jaws $B\ C$ will be moved along the radial guides in the face plate $G\ H$. In order that all the jaws B may move through equal distances for a given rotation of the cam plate D, it is necessary that the middle line of the cam groove be an *Archimedean spiral* in which the radius increases in the same proportion as the angle of rotation. This line is the only one which permits the jaws to have several projections fitting into different coils of the same spiral groove on the cam plate; it is plain that to do this the radial distance between two coils of the spiral must be everywhere the same, a property belonging only to the Archimedean spiral.

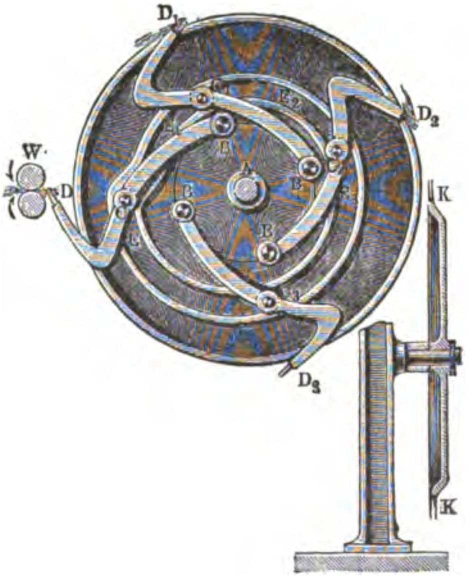
FIG. 618.



§ **162. Inversions.**—Hitherto we have assumed that the cam plate took part in the motion either as driver or as follower, but the cam can also be made stationary, and then we obtain mechanisms which, according to § 31 of the Introduction, may be regarded as inversions of the mechanisms hitherto described. These, like all other inversions, are obtained *by giving to the mechanism in question an additional motion which is, at every instant, equal and opposite to the previous motion of a link.* This causes the link to become stationary, the previously fixed link assuming simply the additional motion, while the other links have a compound motion made up of their previous motion and the additional one. Such cam trains with stationary cam are frequently applied in machinery, the machines for combing long wool furnishing interesting examples.

Fig. 619 gives the leading features of an arrangement for moving the combs in the combing machine of *Donisthorpe & Whitehead.** Here the cylindrical drum *B B* is provided with four levers *B D* turning on the pins *B*; the ends of the levers carry combs, and their projecting pins *C* are compelled to move in the stationary groove *E*. At E_1, E_2 and E_3, where this curve is concentric to the shaft *A*, the combs *D* keep at the same distance from *A*; but at *E*, where the groove curves outward, the lever is compelled to move outward so that its comb can take off wool while passing the feed rolls *W*. With further turning the form of the guiding groove is such as to

FIG. 619.



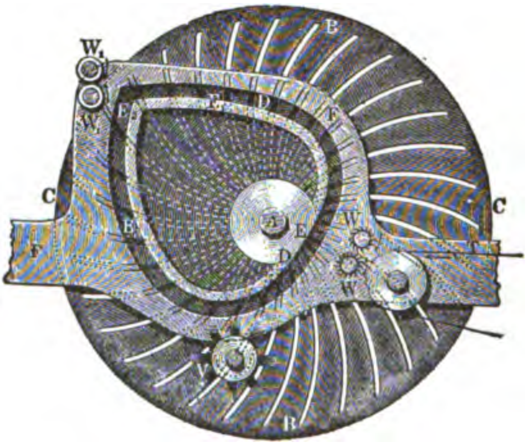
---

*For fuller details see *Prechtl's* Technolog. Encyklopaedie, Supplement, vol. 3, pl. 92.

compel the arm to return, so that the wool can be delivered at D_2 to a slowly rotating ring *K*, having first been combed by a suitable arrangement not here described while traversing the path $D\,D_1\,D_2$. In this case, therefore, we have the inversion of that mechanism which communicates an oscillatory intermittent motion to a lever by means of a rotating cam plate.

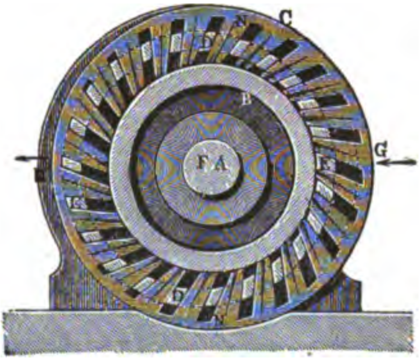
The same mechanism in a different form and for a different purpose is found in *Parpaite's* Démêloir, a machine employed for drawing apart and preparing the wool before it is given to the combing machines. Here also we have a uniformly rotating shaft

Fig. 620.



A, Fig. 620, which, as it turns, drags around a number of combs or gills *D D*. These combs are not connected with the shaft by pins and levers, as in the preceding example, but by two disks, *C*, each of which is fastened to *A* and provided with a system of circular slots *D B*. Through each pair of corresponding slots of the disks *C* pass the ends of a gill, their cylindrical terminations projecting into two heart-shaped grooves *E* cut in iron plates *F* screwed fast to the frame. It is clear that in this arrangement the gills *D* are compelled to follow the guide grooves *E* when the shaft *A* turns, and that each gill then describes, relatively to the shaft *A*, an arc represented by its slotted guide *B D*. The result is that the distance between two gills on the portion $E\ E_1\ E_2$ of the cam groove gradually increases, this distance diminishing again when the gills pass from E_2 to E by way of E_3. During their motion each gill is subject to certain slight rotations about its longitudinal axis. The endless feed belt T and the feed rolls W present the wool to the needles of the gills, and then, in consequence of the peculiar motion of the gills between E and E_2, the fleece is picked apart and delivered at E_2 to the rolls W_1, the emptied gills passing to V, where rotary brushes remove any hair still clinging to them.

FIG. 621.



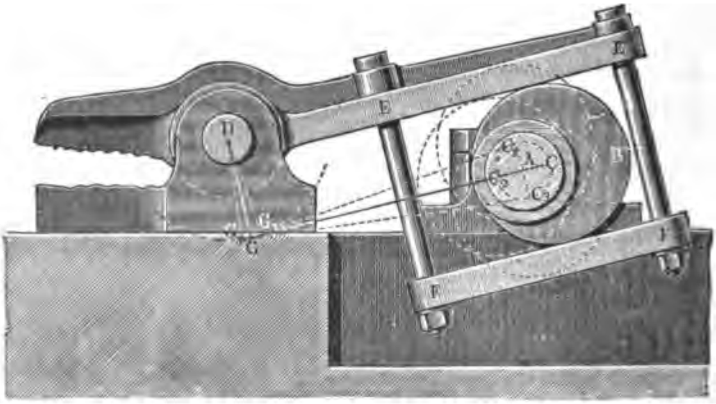
The inversion of the mechanism, in which the cam communicates a reciprocating rectilinear motion, is also occasionally found. An example of this is shown in the rotary gill or porcupine $A\ B$, Fig. 621, used in combing machines; for instance, in *Heilmann's* Démêloir.* This rotary gill consists of a drum B fastened to the shaft A and having steel needles N set in rows parallel to the axis. Between each pair of rows there is a rod D likewise parallel to the axis, and as in the previous example, passing through slots in the flanges C of the drum. These rods D are guided at each end by stationary circular grooves E whose centre F is eccentric to the shaft A. The wool is fed to the needles of the rotary gill at G and then the rods D move outward, gradually pushing the wool from the needles so that it may be delivered at H and there removed by suitable apparatus. A circular groove, eccentric to the axis, is employed to move the rods. The kind of motion thus obtained is discussed in the following paragraph.

§ **163. Eccentrics.**—If we give to the cam of the rotating shaft a circular form, the cam will likewise give an oscillating motion to the lever or rod on which it acts, provided the cam is placed eccentrically on the shaft. Such circular cams are often used in lever shearing machines and squeezers. In Fig. 622, A is a shaft driven by water or steam power and having an eccentric disk B, which touches the two flat bars $E\,E$ and $F\,F$. It is easy to see that with this arrangement the lever $D\,E$ is moved in both directions by the disk B. For a closer examination of this motion, let us suppose the disk B to gradually become smaller; this does not alter the nature of the motion, provided the centre C maintains its position, namely,

---

* See *Prechtl's* Technologische Encyklopaedie, Supplement, vol. 3, pl. 95.

FIG. 622.



its eccentricity $A\,C = r$. When the disk B finally reduces to its centre C, the two surfaces E and F, which have been gradually approaching each other with a parallel movement, will coincide with the straight line $G\,C$ passing through the centre C. Whatever the position of the centre C of the eccentric on the circle described about A with radius $A\,C = r$, the line *midway* between and parallel to $E\,E$ and $F\,F$ will always pass through the position of C. In order to find the extreme positions of this middle line, and from it the positions of the lever, let fall from D the perpendicular $D\,G$ upon this line, and describe a circle with D as a centre and $D\,G = d$ as a radius. It is then clear that the two tangents $G_1\,C_1$ and $G_3\,C_3$ to the two circles $D\,G$ and $A\,C$ will be the extreme positions of the middle line $G\,C$, and that these will enable us to find the extreme positions of the lever. Now if $G\,C$ is that position of the middle line which passes through the axis A, and if α_1 and α_2 respectively represent the angles $G D G_1$ and $G D G_3$ through which the lever swings from its middle position, then will the middle position $G C$ of the aforesaid middle line make the angle α_1 with $G_1 C_1$, and the angle α_2 with $G_3 C_3$.

As the eccentricity $A C$ is almost always small in comparison with the length $l = A G$, it is sufficiently accurate for practical purposes to assume

$$l \alpha_1 = l \alpha_2 = A C = r,$$

and then we get approximately for the angle of oscillation of the lever

$$\alpha = \alpha_1 + \alpha_2 = 2 \alpha_1 = \frac{2 r}{l}.$$

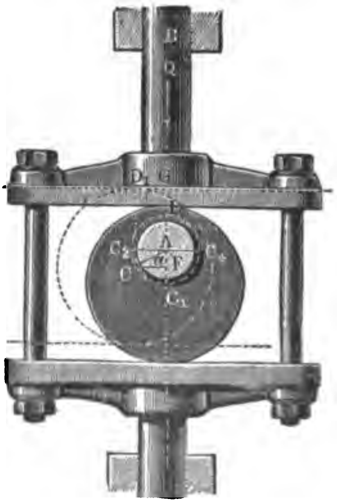
Strictly speaking, the angles α_1 and α_2, representing the swing of the lever from the middle position $D G B$, are not equal, but the difference is ordinarily so slight that we will omit the complex calculation necessary for a more exact determination. The velocity of the lever gradually increases from zero at the change points till it reaches its greatest value in some middle position and then gradually diminishes to zero, so that no shocks are caused by this mechanism. The motion itself is essentially like that of a corresponding crank train, and the resemblance is a perfect one when the swinging lever becomes a sliding rod $B D$, whose direction of motion passes through the axis A, Fig. 623. When the centre of the eccentric disks moves from its lowest position C through any angle $C_1 A C = \alpha$, the rod rises through the distance

$$s = D G = C_1 F = r (1 - \cos \alpha),$$

and at this instant the velocity v_1 of the rod becomes

$$v_1 = \frac{\delta s}{\delta t} = r \sin \alpha \frac{\delta \alpha}{\delta t}.$$

FIG. 623.



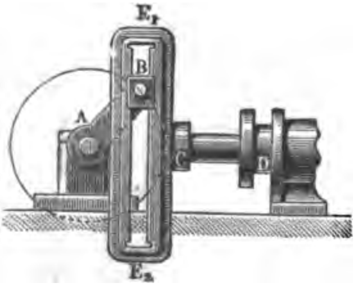
Therefore if we assume a uniform velocity for the centre of the eccentric, we have

$$v = r \frac{\delta \alpha}{\delta t},$$

and the ratio $\frac{v_1}{v} = sin\ \alpha$, exactly as in the crank train with slotted piston rod, *i.e.*, with infinite connecting rod, Fig. 624. We may here remark that the resistance Q of the rod BD is directly sustained by the shaft A only when the rod is in its highest and lowest position; in every other position the point of contact D_1 between the eccentric and the rod is at a distance $GD_1 = r\ sin\ \alpha$ from the axis of the rod. The vertical upward force P acting at D_1 then causes the rod to be pressed against its guides, generating friction which increases with the moment $Pr\ sin\ \alpha$. Consequently this friction is greatest for the middle positions C_2 and C_4 of the eccentric and disappears at the dead points.

The stroke of the rod DB in the mechanism, Fig. 623, is equal to twice the eccentricity r, and is therefore exactly like the stroke of the crank train shown in Fig. 624, the same law of motion holding for both mechanisms. Indeed the nature of the two trains is exactly the same, the whole difference consisting in the difference of diameter of the crank pin and the consequent difference in the width of the slot. The difference here is just like that existing between the ordinary crank train and the eccentric train, Fig. 588, deduced from it. Here, also, the frictional resistance in the mechanism in Fig. 623 is considerably greater than that in Fig. 624, the former construction being only used when it is desirable to have the shaft A extend in both directions without bend or double crank. Wherever the driving link can be attached to the free end of the shaft A, the arrangement of Fig. 624 is preferable.

Fig. 624.



When it is a question of moving a rod B according to some other law, the object may generally be attained by giving the slot in the rod the shape of a curved groove whose form may in each case be easily determined from the given law of motion of the rod. For instance let us suppose that the rod is to perform its oscillating motion $2r$ with *uniform* velocity when the crank r turns uniformly. To determine the form of the curved slot for this case, let us suppose the rod B, Fig. 625, to occupy its middle position, the crank occupying the position AC perpendicular to the direction of the rod BB. Now divide the quadrant CC_4 at $C_1 C_2 C_3 \ldots$ into any number of equal parts, and the travel AC_4 belonging to this quadrant into the same number of equal parts at $A_1 A_2 A_3 \ldots$; we then see that when the crank-pin stands at, say, C_2, the rod will have travelled the distance AA_2 from its middle position. Therefore if we draw through the points of division C lines parallel to the rod and lay off on these lines the distances $C_1 D_1 = AA_1$, $C_2 D_2 = AA_2$, $C_3 D_3 = AA_3$, etc., the points D thus obtained will give a line $C D_1 D_2 D_3 A$, which if constructed for all four quadrants, will pass through the points C and C' and twice through the centre A. The sides of the groove are obtained by drawing lines parallel to the curve D at a distance from the latter equal to the radius of the crank pin C. Moreover it is evident that the construction remains essentially the same when the stroke of the rod is not taken equal to $2r$. If the half stroke AOC_4,

FIG. 625.

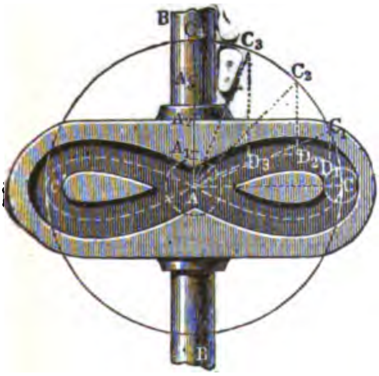


FIG. 626.

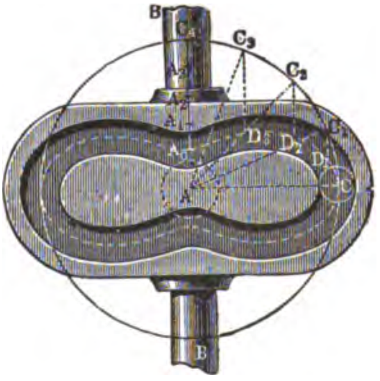


Fig. 626, is less than the length of the crank, the two halves of the curve will no longer meet at A, and if the stroke of the rod is greater than the diameter of the crank circle, the two branches of the curve will intersect at two points, Fig. 627. When the ratio of the revolutions of the crank to the travel of the rod is expressed by a different law from that here assumed, the construction can still be made in a similar manner, and we see that the rod will have a period of rest whenever the centre line of the groove is curved like the crank circle. The period of rest will then begin at the instant when this arc coincides with the crank circle, and the duration of this period will depend on the length of the circular portion of the groove. Such an arrangement exists in the mechanism for driving the needle bar in the *Singer* * sewing machine. Here the crank pin C is provided with a friction roller that moves along a heart-shaped groove $C D_1 D_2 D_3 C_4$ on the needle bar B, communicating to the latter an up and down motion which is broken by a period of rest, this rest occurring not at either end of the stroke, but after the needle has risen a short distance (say 3 millimeters [$\frac{1}{8}$ in.]) from its lowest position. This action of the cam plate can be explained as follows: If the crank moves from its middle position $A C$ to $A C_2$, the needle bar will move downward a distance $D_2 C_2$, for this distance represents the vertical deviation

FIG. 627.

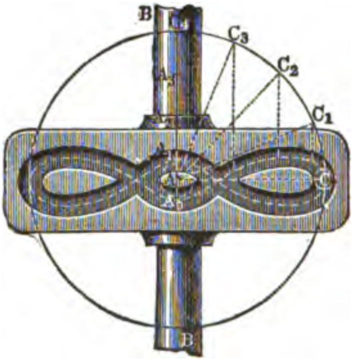
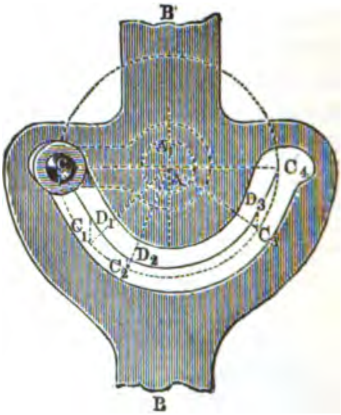


FIG. 628.

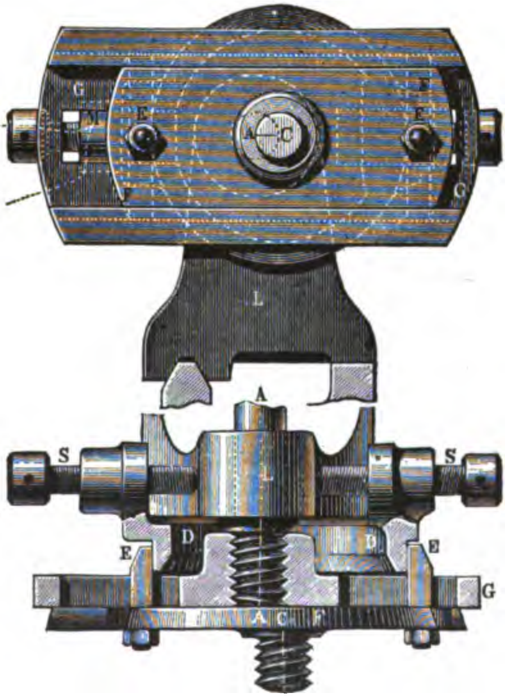


---

* See *Herzberg*, Die Naehmachine.

of the centre line of the groove from the crank circle $C C_1 C_2$. Now, since the centre line of the groove between D_2 and D_3 is a circular arc described with crank radius $A C$ from a centre A_1 which is located vertically above A a distance $A A_1 = C_2 D_2$, at the instant when the crank assumes the position $A C_2$ the needle bar will come to rest and remain in this condition while the crank turns from $A C_2$ to $A C_3$, after which the needle bar again rises. We can see from the figure that the beginning of a period of rest of the needle bar, represented by the crank position $A C_2$, does not correspond to the lowest position of the needle, for this lowest position corresponds to the crank position $A C_1$, for which the vertical distance $C_1 D_1$ included between the crank circle and the middle line of the groove is a maximum. The motion therefore takes place in such a way that the needle moves downward a distance $D_1 C_1$ while the crank turns through $C A C_1$, and then it moves from this lowest position a distance $C_1 D_1 - C_2 D_2$ while the crank is turning through an angle $C_1 A C_2$, whereupon it comes to rest, giving the shuttle an opportunity to pass through the loop of thread formed by the slight rise of the needle from its lowest position.

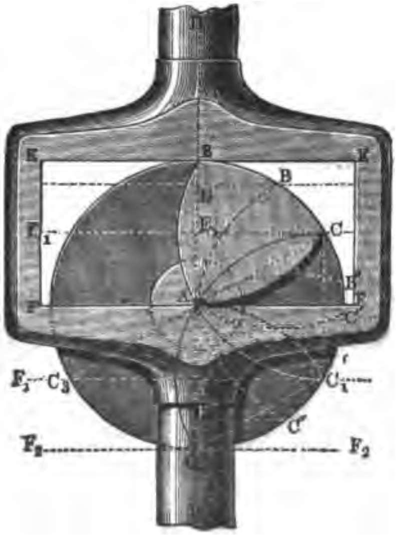
Fig. 629.



The inversion of the mechanism in Fig. 623 is occasionally employed, the eccentric disk being stationary and the slotted rod receiving, in addition to its reciprocating motion, a rotation about the axis. On this depends the well-known "elliptic chuck," Fig. 629. Here the circular ring *D* is eccentric to the axis *A* of the lathe, being fastened to the head-stock *L* by screws *S*. The rod here assumes the form of a plate *F* to which the work is fastened, this plate being guided by slide bars on the face plate *G*, which is screwed to the spindle *A* of the lathe. The cheeks *E* of the slide *F* project through openings in the face plate *G* and enclose the eccentric disk *D*, thus forming the slot. It is easy to see that any fixed point in space, for instance the point *M* of the tool, describes an ellipse relatively to the slide *F* carrying the work piece. The semi-conjugate axis of the ellipse is equal to the distance of the fixed point *M* from the axis *A* of the spindle, and its eccentricity is equal to the distance *A C* of the centre of the fixed guide disk *D* from the axis *A*, the centre of the disk *D* being fixed where desired, by the screws *S*.

§ **164. Triangular Cams.**—Eccentrics bounded by circular arcs are often employed for communicating intermittent motion to rods, *i.e.*, reciprocating motions broken by periods of rest. The triangular cam *A B C*, Fig. 630, is a common form; here the profile of the cam is composed of three arcs, *A B*, *B C* and *C A*, whose centres are the vertices of an equilateral triangle and whose radii are each equal to the side $A B = r$ of this triangle. If this cam is turned about an axis passing through the vertex *A* of the triangle, it is easy to see that the slot *E E F F* with width $E F = r$ will continually enclose the curve-triangle *A B C* in such a way that one side of the frame will touch an

FIG. 630.



arc of the triangle while the other side touches the vertex opposite this arc. Motion communicated to the rod by a uniform rotation of the eccentric can be divided into three distinct periods for a half revolution of the eccentric, each period corresponding to a rotation $\frac{\pi}{3}$ or 60°. If we choose ABC for the initial position in which the vertex B lies in the direction of the rod DD, this position will correspond to an extreme position of the rod, and the figure shows that at the end of the first period the triangle will be in the position ACC_1. The side EE of the frame then occupies the position E_1E_1, the rod having travelled downward a distance $BE_1 = \frac{1}{2}r$. During this motion the arc AC has acted as the driving surface, this arc rolling and sliding on the side FF of the frame. To prove this latter assertion it is only necessary to call attention to the fact that at the beginning of the motion the surface FF touches the eccentric at A and, at the end of the period, at C_1. Therefore while the point of contact traverses the arc of the triangle from A to C, *i.e.*, a distance equal to arc $AC = \frac{r\pi}{3}$, it also traverses on the surface FF the distance $F_1C_1 = r \cos 30°$. The sliding of the two parts on each other is therefore given by

$$\frac{r\pi}{3} - r \cos 30° = 0.181\, r.$$

In the following second period of motion there is a further turning through 60°, the triangle moving from the position ACC_1 to AC_1C_2 and the surface FF from its position FF_1 to F_2F_2, the amount of sliding being $F_1C_2 = \frac{1}{2}r$. This motion is caused by the driving vertex C, the sliding friction between this vertex and the surface FF of the slot being overcome through the distance $C_1F_1 = r \cos 30°$; there is no rolling of the surfaces in contact during this period.

Finally, when the shaft turns through the third sextant, the cam will move from the position AC_1C_2 to AC_2C_3 and the rod will remain at rest, for the arc BC, though in contact with the surface FF at C_2, cannot communicate a sliding motion to the latter because it is concentric to A. During the subsequent half revolution of the shaft, these three periods again occur in the same order, but the rod moves in an opposite direction. Therefore during a half revolution the rod is moved through the distance r, the first half of the sliding motion taking place with a gradually increasing and the second half with a gradually diminishing velocity, and the rod reaches and leaves each end of the stroke with a velocity 0, as may be shown in the following manner.

If, during the first period of the motion, the triangle moves to $B' A C'$ by turning through the angle $B A B' = \alpha'$, the rod will traverse the distance

$$s' = B E' = r(1 - \cos \alpha').$$

The velocity of the rod at this instant is therefore

$$v' = \frac{\delta s'}{\delta t} = r \sin \alpha \frac{\delta \alpha}{\delta t};$$

or, since $v = r \frac{\delta \alpha}{\delta t}$ is the constant circumferential velocity of the vertices B and C, we have

$$v' = v \sin \alpha.$$

This value is zero when $\alpha = 0$, and becomes a maximum when

$$v' = v \sin 60^\circ = 0.866\, v$$

at the end of the first period.

In like manner, during the second period, when the cam occupies any position $A B'' C''$ after turning through the angle $C_1 A C'' = \alpha''$, the travel of the rod from its middle position will be given by

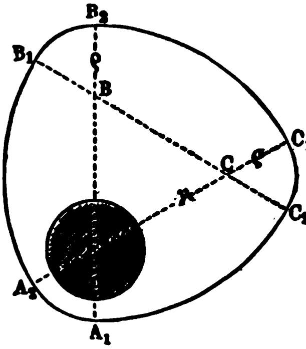
$$s'' = F_1 F'' = r \cos (60^\circ - \alpha'') - \frac{r}{2},$$

and the velocity of the rod by

$$v'' = \frac{\delta s''}{\delta t} = r \sin (60^\circ - \alpha'') \frac{\delta \alpha}{\delta t} = v \sin (60^\circ - \alpha'').$$

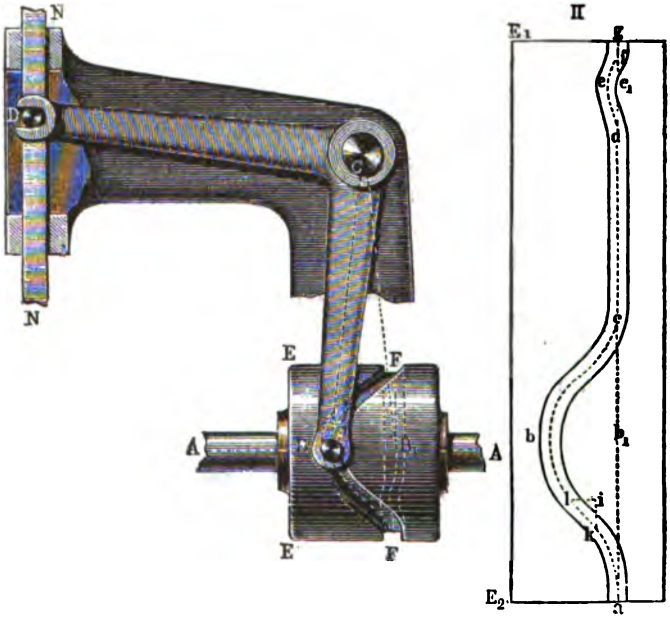
The rod therefore begins its motion in the second half of its travel with the same velocity $v \sin 60°$ with which it arrived at the middle position, and gradually diminishes this velocity till it is zero for $\alpha'' = 60°$. We see from this that shocks do not occur when the rod reaches and leaves its extreme positions, nor at the centre of the stroke where the acceleration passes into a retardation. The comparatively smooth motion resulting from this is to be regarded as a special advantage of this mechanism. It need only be mentioned that the velocities of the rod in every position are proportional to the perpendiculars $B' E'$ and $C'' F''$, while the corresponding travel is given by $B E'$ and $F' F''$ respectively. The triangular cam in the form shown in Fig. 630 must usually be placed at the free end of a shaft A like an ordinary crank, but a form can easily be given to it which will allow the shaft to extend to both sides without thereby essentially changing the law of the motion. For this it is only necessary to assume the radius of the arcs greater by an amount $\rho = A A_1 = B B_1 = C C_1$ than the length of the side r of the triangle $A B C$, Fig. 631, and to form the corners of the cam by arcs $A_1 A_2$, $B_1 B_2$, $C_1 C_2$, described with radius ρ from the centres A, B and C respectively. A little thought will show that the stroke r is not changed, but that the work of friction between the cam and the frame is increased because this resistance acts through a greater distance. On the other hand, its durability is also increased, for the pressure of the slot against the cam is no longer concentrated on the points B and C, Fig. 630. The mechanism here considered is one of a series of analogous mechanisms fully discussed by *Reulcaux* in his Kinematics of Machinery.

FIG. 631.



§ **165. Cylindrical Cams.**—In the cam trains hitherto considered it was always assumed that the axis of the cam was parallel to that of the reciprocating body, whether the axis of the latter was at an infinite distance, as in stamps, or at a finite distance, as in trip hammers. Now it often happens that these two axes cross without intersecting, being then most often at right angles to each other. When this is the case the cam becomes cylindrical, *i.e.*, the middle line of the cam groove lies on a cylindrical surface whose axis coincides with the rotating axis *A*. Fig. 632 shows such a mechanism applied to *Howe's* sewing machine, for the purpose of moving the needle bar *N N*.

FIG. 632.

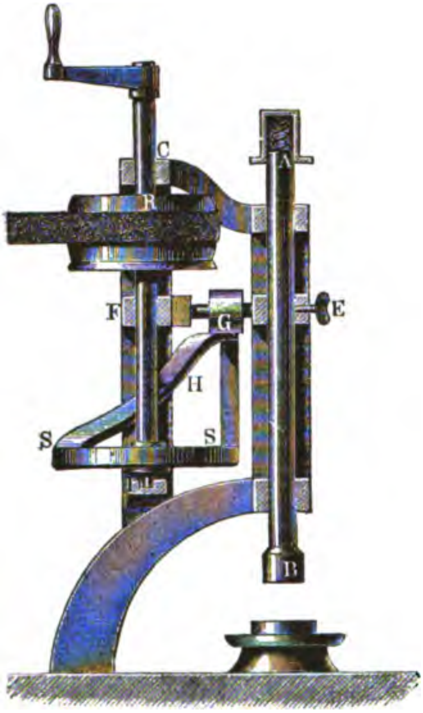


The cylinder *E E* is fastened to the driving shaft *A A*, the surface of the cylinder being provided with the guide groove *F F* for the friction roller *B* at the end of the bell-crank lever *B C D*. The breadth of this groove, measured in the direction of the shaft *A*, is everywhere equal to the diameter of the friction roller *B*. The course of this groove is best shown by Fig. 632, II, which represents the developed surface of the cylinder *E*. Here the curve *a b c d . . . g* is the development of the middle line of the groove. First of all it is clear that no portion of the groove lying in a plane perpendicular to the axis *A* can cause motion in the oscillating piece, for such a portion is a circular arc on the cylinder, and a straight line parallel to $E_1 E_2$ in the development. In the present example, therefore, the portions *g f* and *d c* correspond to positions of rest of the lever, the lever being then in its extreme and the needle in its lowest position. The slight deviation of the curve *f e d* causes a motion of the lever to the left and a return motion to the position of rest *d*, the slight rise of the needle accompanying this motion forming the loop through which the shuttle passes during the period of rest *d c* of the needle. The greater deviation *c b a* causes the needle to rise to its highest position, and to return again to its lowest, corresponding to the point *a* of the curve. The amount of axial motion communicated to the reciprocating point *B* becomes exactly equal to $e\,e_1$ and $b\,b_1$ (the distances between *a g* and the tangents to the middle line of the groove parallel to *a g*) when *B* has a perfectly rectilinear motion, and nearly equal to these distances when the arm *B C* is not very short. The velocity v_1 communicated to the guided piece, the roller *B*, while traversing the element *k l* of the groove, is represented by *l i*, the distance *k i* representing the uniform velocity *v* of the surface of the cylinder. Consequently $v_1 = v \cot \gamma$ when γ represents the angle *k l i*.

The position of the axis *C* of the oscillating lever had better be chosen in that plane perpendicular to the axis *A* which bisects the axial distance included between the extreme positions of the end of the lever, in order that the deviations on opposite sides of this normal plane may be equal. In like manner the normal distance of the axis *C* from the axis *A* must be so chosen that the rise of the arc $B\,B_1$ described by *B* will be bisected by the plane passed through the axis *A A* and parallel to the axis *C*.

In the stamper shown in Fig. 633 we have an example of a cylindrical cam communicating rectilinear motion to a rod. Here the rod *A B* carrying the stamp is lifted by the cam *S* fixed to the rotating shaft *C D*. The cam consists of a sort of helical surface *H* which passes under the friction roller *G* attached to the rod $A B$. It is evident from the figure that the hammer can fall freely after the roller G has been lifted into its highest position.

FIG. 633.



A mechanism of special interest is obtained by making the middle line of the groove on the cylindrical cam a plane curve. If we suppose the cylinder to be cut by any plane oblique to the axis, an ellipse is obtained which may be assumed as the middle line of the groove. Let us assume that the plane $B D$, Fig. 634, of this ellipse, makes the angle $B O G = \gamma$ with the plane normal to the axis $A A$, and let this curve serve to transmit motion to a rod $C S$ which can slide in fixed guides parallel to $A A$. It is evident that when the cylinder makes a complete revolution, the end of the rod will move from its initial point B to H, through a distance $2 B G = 2 r \tan \gamma$, and that it will return from H to B, the motion produced resembling that caused by a crank at O having the length

$$O F_1 = B G = r \tan \gamma,$$

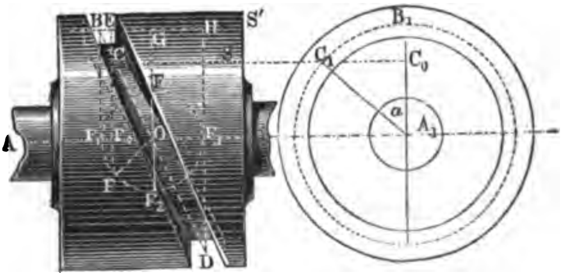
and a crank pin working in a corresponding slot of the rod $C S$. It is also evident that the nature of the motion imparted to the rod by the elliptical groove is exactly like that of the crank motion mentioned. For if we suppose the cylinder to be held fast, and the rod and its guide to turn around the cylinder through the angle $B_1 A_1 C_1 = \alpha$, thus moving from the position

BS' to CS, the sliding accompanying this motion of the rod will be represented by

$$BE = CE \tan \gamma = B_1 C_0 \tan \gamma = r\,(1 - \cos \alpha) \tan \gamma.$$

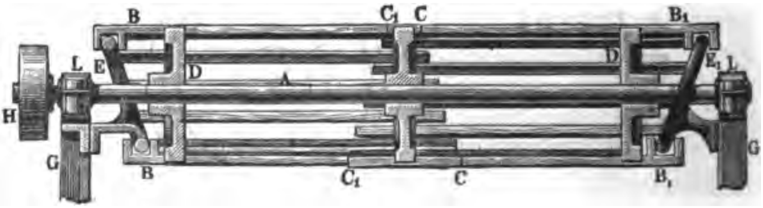
According to § 139, this expression agrees with that found for the sliding of a rod when its crank $r \tan \gamma$ has traversed the angular distance α from a dead point, provided the connecting rod is very long. This proves the statement made above that the present mechanism resembles a crank train with a slotted bar in which the crank has a length $OF_1 = r \tan \gamma$, and the dead points $F_1 F_3$ correspond to the vertices B and D of the ellipse. It is also evident that if through any position C of the rod a line CF is drawn perpendicular to the axis AA, the ordinate FF_0 will be a measure of the velocity of the rod, and OF_0 will represent its travel from the middle position (see § 139).

Fig. 634.



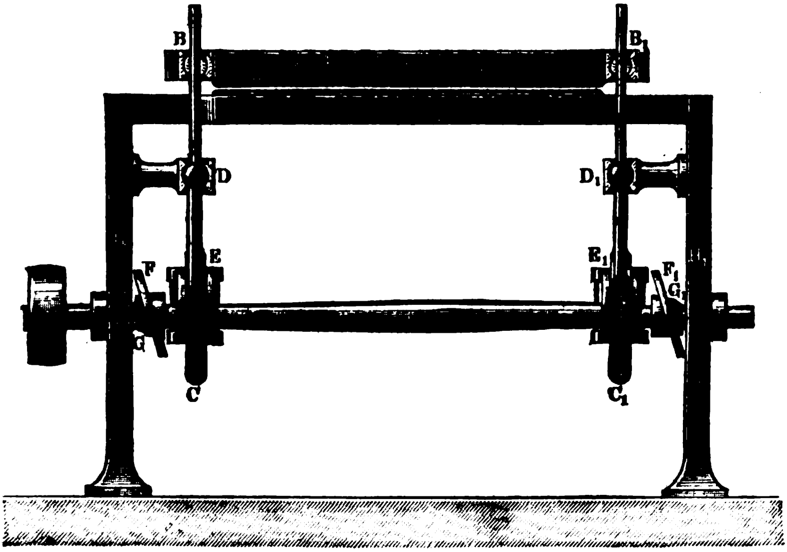
That the cylinder is supposed to be held fast and the rotation communicated to the rod, in the preceding discussion, does not affect the motion. Moreover, this case of a stationary guiding surface often occurs in practice, not one only but a whole system of rods being moved around it. An interesting example of this is found in the arrangement given to certain rolls in finishing machines for the purpose of maintaining the width of the cloth and smoothing it in the direction of its width. Fig. 635 represents such a roll used in spreading the cotton cloth passed over it. The uniformly turning shaft A lies in fixed bearings L, and has three disks D with rectangular notches in their rims for receiving rectangular bars BC and permitting them to slide axially, some suitable device, as rings, preventing them from falling out. These bars are arranged in two groups, *B C* and B_1 C_1, extending from each end to points beyond the middle, the bars of the two groups lapping at the centre. Of course the bars are so distributed that those on one side are placed in the spaces left by those on the other. The cam disks here employed are two elliptical rings *E* and E_1 fastened to the frame *G* of the machine. The rings are half surrounded by claws at the ends of the bars. In consequence of this arrangement a turning of the shaft *A* causes both sets of bars to move axially in opposite directions, the cam rings *E* E_1 having opposite inclinations. It is evident that the half of the surface of this roll, in which the rods move outward, will exert a smoothing action on the cloth passed over it, and it is only a question of so feeding the cloth, or of so placing the cams *E* and E_1, that the bars touched by the cloth shall have an outward motion.

FIG. 635.



The mechanism last considered with stationary cam can be regarded as an inversion of the rotating cylindrical cam, Fig. 634, which inversion is obtained by the introduction of an additional rotation, at every instant equal and opposite to the rotation of the shaft *A*. Another inversion can be obtained from the rotating cylindrical cam with reciprocating rod, by imparting to the whole system an additional motion which at every instant is equal and opposite to the *sliding* of the rod. This brings the rod to rest while the shaft *A* is subjected to a rotation about, and an oscillation in the direction of, its axis. An example of the application of this mechanism in the manufacture of woollen goods is shown in Fig. 636. The object of this machine is to arrange the nap of certain woollen goods in tufts, which is accomplished by passing the cloth over a fixed plate *T*, while a movable plate *P* with a peculiar vibrating motion is pressed against the upper, nap, side. For example, if the tufts are to be circular, the plate *P* must be so moved that all points will describe equal horizontal circles. To accomplish this the rubbing plate *P* is moved by two rods *B C* and $B_1 C_1$, which turn around stationary ball joints *D* and D_1 and receive an oscillatory motion in a plane perpendicular to the axis *A*,

Fig. 636.



from the eccentric *E*, at their lower fork-shaped ends *C*, Fig. 637. This mechanism, which is essentially the same as the eccentric, Fig. 622, causes oscillations of the plate *P* in a direction perpendicular to the length BB_1, *i.e.*, in the direction in which the cloth is moving. If these were the only oscillations, the nap would arrange itself in quite regular transverse ridges. But as there are two disks *F* and F_1 fastened obliquely to the shaft *A* and parallel to each other, which disks touch two projections $G G_1$ hinged to the frame $H H_1$ so that they lie in a line parallel to the shaft *A*, it is easy to see that with each revolution the shaft will slide to and fro in its bearings, the eccentrics $E\,E_1$ and their rods taking part in this sliding motion. That this will effect the desired operation is shown as follows: If we suppose a point to receive a reciprocating motion from A to B and back, Fig. 638, by means of an eccentric E, *i.e.*, according to the law of the crank train with slotted slide, this point will move from its extreme position a through a distance $a\,g = r\,(1 - \cos\alpha)$ when the driving disk has turned from the dead point through an angle α. If at the same time this point be reciprocated by an oblique disk, in a direction perpendicular to $a\,b$, through a distance $d\,e = 2\,r$, according to the same law and with the same duration as the other reciprocation, and if we assume that this point is at the middle C of its stroke by virtue of the reciprocation $D\,E$, and that it would occupy the extreme position a if it were only subject to the reciprocation $a\,b$, then it is evident that the oblique disks, for an angle of rotation α, will cause a travel $C\,F = r \sin\alpha$ from the middle. In consequence of the two motions the point in question will move from a to k when the shaft turns through the angle α, and, as this holds for any angle of rotation, it follows that the point will describe a circle having C as a centre, and $C\,a = C\,d = r$ as a radius when the strokes $a\,b$ and $d\,e$ are equal. When the two strokes are unequal, the path of the point is an ellipse which passes into the straight line $a\,b$ or $d\,e$ according as one or the other of the two motions becomes zero.

FIG. 637.

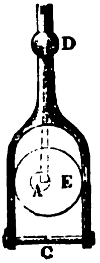
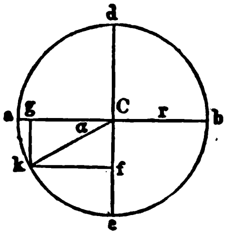
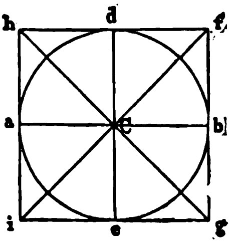


FIG. 638.



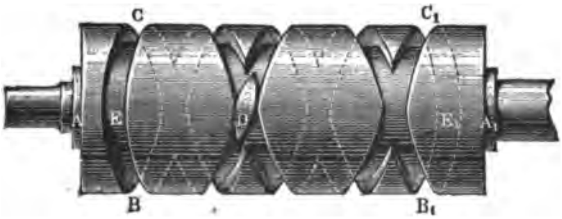
On the other hand, if we assume that each reciprocation acting alone would cause the point to assume an extreme position at the *same instant*, or that each reciprocation acting alone would cause the point to assume the middle position at the *same instant*, we shall find that the resulting motion of the point will be represented by the respective diagonals $h\,g$ or $i\,f$, Fig. 639. It is now evident that by suitably varying the two oscillations with respect to magnitude and relative position, a great variety of positions can be communicated to the rubbing plate *P*, thus adapting the machine to the different kinds of finish required by fashion. It is hardly necessary to mention that the stroke of the reciprocations can be varied by varying the eccentricity of the eccentric and the distance of the projections *G* from the axis *A A*.

FIG. 639.



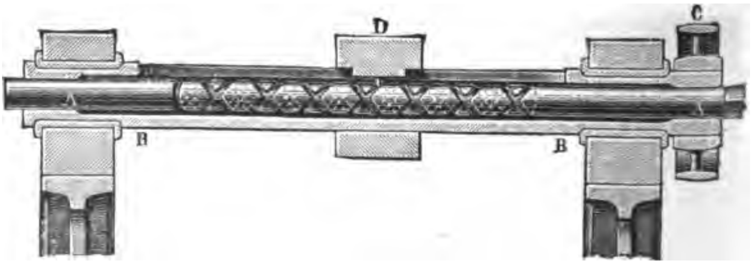
If the guide curve on the cylinder maintains its angle of inclination to the axis for a certain distance, this portion of the curve will take the form of a helix, and will communicate to the rod a uniform rectilinear motion when the cam turns uniformly. Here there is no objection to giving the helix as many coils as desired, but the guide curve must be a closed one, and must return if a continuous rotation of the cam in one and the same direction is to impart *continuous* motion to the reciprocating body. To accomplish this a cylinder $A A_1$, Fig. 640, is occasionally used, whose surface contains two helical grooves; one, *B*, right-handed and another, *C*, left-handed. If we unite the ends *B C* and $B_1 C_1$ of these grooves by suitable curves *B E C* and $B_1 E_1 C_1$, we see that a pin *D* projecting into the groove will receive a reciprocating motion $E E_1$ when the cylinder *A* turns. Along the portions $B B_1$ and $C C_1$ this motion will take place with uniform velocity, and will be the same for the forward and return stroke if the two helices have the same inclination to the axis. The velocity of the reciprocating piece when passing from one helical groove to the other depends, of course, upon the form of the curves *B C* and $B_1 C_1$.

FIG. 640.



For example, we may connect the two grooves of equal pitch by *plane* curves, and then the relations hold which were found for the oblique cam, Fig. 634. Such a plane connection is given by the ellipse which is the intersection of the cylinder by the plane passing through the parallel tangents to the two helices at the diametrically opposite points $C B$ or $C_1 B_1$, as the case may be. This mechanism is used in certain spinning machines to effect a regular winding of the bobbins, the pin D being connected with a carriage supporting the bobbin, which carriage is guided back and forth on fixed rails, in order that the thread, which is always delivered at the same point, may wind itself in regular cylindrical layers on the bobbin.

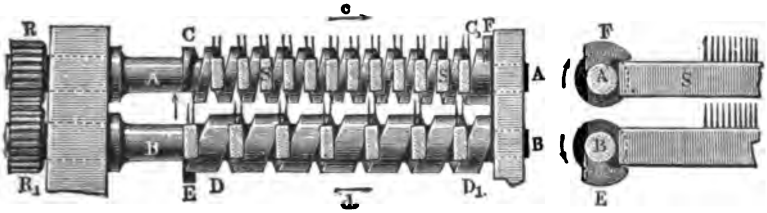
FIG. 641.



These mechanisms are also used with the grooved cylinder stationary, the rotation being communicated to the sliding piece. Fig. 641 shows this arrangement applied to a card grinder; here the grooved spindle $A A$ is stationary, and the cast-iron tube B is turned by the belt pulley C. An emery wheel D slides on the smoothly turned surface of the tube and carries a pin which passes through a longitudinal slot of the tube and projects into the helical groove of the spindle A.

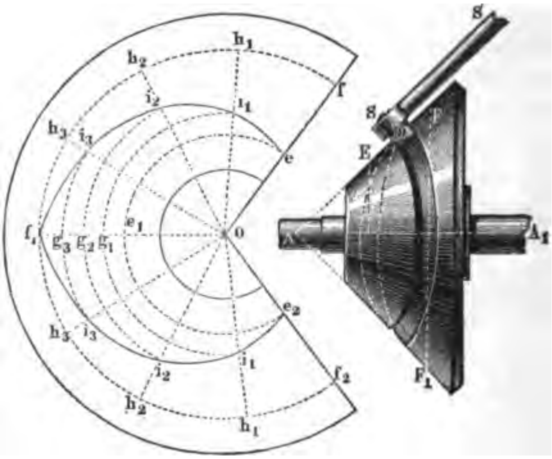
An interesting application of these helical grooves occurs in *drawing frames* for guiding the combs or gills which support the fibres and make them parallel. Here the ends of the gills S, Fig. 642, enter into the helical grooves of two screws like $A A$, whose uniform rotation causes the gills to move slowly in the direction of the arrow c. Under each screw A lies a parallel screw B which is turned in an opposite direction to the upper spindle by means of spur wheels $R R_1$. Now when a gill reaches the end of the upper screw A it falls vertically at C_1, its end entering the threads of the lower screw B and then the gill is guided from D_1 to D by the opposite rotation of B. Each of the screws B is provided with a cam E which passes under the gill on its arrival at B and lifts it till it again enters the threads of the upper screw A, where it is again moved in the direction $C\,C_1$. The upper screw is also provided with a cam F at C_1 to ensure the falling of the gills at this point. In this way the gills receive a perfect parallel movement from C to C_1 and the needles are introduced at C into the fibrous material at right angles to it and leave it in the same direction at C_1. To reduce the number of gills it is usual to give the lower screws twice the pitch of the upper ones.

FIG. 642.



In flat cams the motion of the rod is in a direction *perpendicular* to the axis of the cam, but in cylindrical cams this reciprocating motion is *parallel* to this axis. Now if the end of a rod is to be guided by a curve along a straight line $E\,F$, Fig. 643, which makes the acute angle $F\,A\,A_1$ with the axis A, this curve will be located on the conical surface generated by revolving the straight line $F A$ about the axis $A\,A_1$. To draw this curve, we must develop the conical surface $A\,F F_1$ and on the developed surface lay off the curve according to the principles given above, and then again wrap the developed surface on to the cone. An example will make this clear. Let $S\,S$, Fig. 643, be a rod which must be moved in the direction of its length so that its end D will traverse the distance $E\,F$ to and fro with uniform velocity while the shaft $A\,A_1$ makes one revolution. If we develop the conical surface we shall get the sector $O f f_1 f_2$ whose radius $O f = A\,F$ and whose arc $g f f_1 f_2$ is equal the circumference of the circle $F F_1$. Now if we lay off $O e$ equal to $A\,E$ and describe the arc $e e_1\, e_2$ about O, the desired curve will lie between the circles f and e. If we bisect the angle $f\,O f_2$ by $O f_1$, then will f_1, e and e_2 be points of the required curve. In order to determine the curve itself divide $e_1 f_1$ into n equal parts, and also each of the angles $f\,O f_1$ and $f_1\,O f_2$ into the same number of equal parts. The circles described about O through the points of division $g_1 g_2$ . . . intersect the corresponding radii $O\,h_1$ $O\,h_2$ . . . in the points i_1 i_2 of the required curve. Such conical cams are, however, seldom employed. When the direction EF of the end of the rod does not intersect the axis AA_1, but is askew to the latter, the principal form of the cam will be an hyperboloid of revolution to the axis AA_1, EF representing an element of this surface. But such a construction probably does not occur in practice.

FIG. 643.



§ 166. Cam Rails.—If the radius or distance of the curve from the axis is made infinite in a plane or cylindrical cam, the cam will become a sliding *rail*, the sliding of its curved profile communicating to the oscillating piece, a motion whose character depends upon the nature of this profile. All the principles developed in the foregoing articles without exception are applicable for determining the form of the profile from this motion. Of course, the motion of the rail which takes the place of the rotating cam is no longer a continuous one in the same direction, but now becomes a reciprocating motion.

Such cam rails are occasionally used in practice to produce a reciprocating motion in oscillating levers, and sometimes to produce a rectilinear motion in sliding pieces, for instance in certain rose-engine lathes.*

An interesting example of cam rails with rectilinear sliding motion occurs in the self-acting *mule*, where it serves to give a proper form to the *cop* or *bobbin*. The principal features of this arrangement are shown in Fig. 644. Here W is the carriage which supports the spindles S and has a horizontal reciprocating motion between O and Q. While moving out to Q each spindle draws and twists a portion of thread F which, when the carriage passes from Q to O, is wound on to the spindle. To properly place the coils of thread on the spindles, there is a *faller-wire* G which extends the whole length of the machine over the spindles and presses on the threads F while it passes from G_1 to G_2 and back again to G_1, thus giving them a direction which enables them to be properly wound on the spindles when the latter turn. The *counter-faller* is a wire moved by the lever VU turning on the axis V, and its principal use is to keep the threads taut; in correct winding the faller-wire must move rapidly from its highest position G_1 to its lowest position G_2, and then it must slowly rise from G_2 to a middle position G_3, from which point it must again move with greater velocity into the highest position G_1. To effect this motion use is made of the stationary *copping rail* A_1A_4, on whose upper surface runs the friction roller D attached to the lever NL. As the weight of the lever NL keeps the roller constantly in contact with the rail A, both the lever and the rod EL will receive a swinging motion when the carriage moves inward, the character of the motion depending on the form of the edge A_1A_4. The motion of EL is transmitted to the bell-crank lever EJG, and this gives the desired motion to the faller-wire G. We see from the connection of the pieces that the steep rise of the rail between A_1 and A_2 causes a rapid sinking of the faller-wire from G_1 to G_2, and that the following less rapidly descending portion A_2A_3 of the rail causes a comparatively slower rise of the faller-wire from G_2 to G_3. Finally, the sharp descent at A_3A_4 causes a quicker rise of the faller-wire in consequence of which the rest of the thread to be wound coils itself in the form of steep helices on the free end of the spindle between G_3 and G_1, which coils must be again unwound before the next inward motion of the carriage begins. Owing to the aforesaid motion of the faller-wire the thread to be wound will place itself in numerous close coils on the conical portion of the cop between G_2 and G_3, thus enlarging the cop by a conical

---

* See *Karmarsch*, Mechanische Technologie, vol. i.

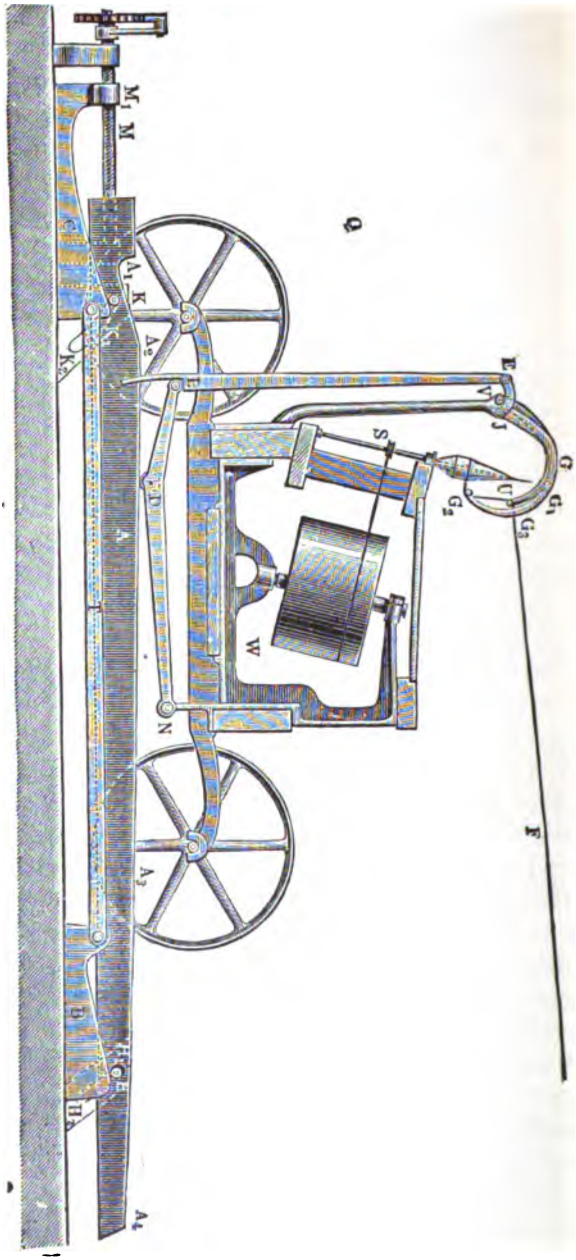


FIG. 644.

layer. For correct winding it is therefore necessary that the following layer be not wound on the same part of the spindle, but at another part higher than the former by the thickness of the layer. To accomplish this the rail A_1A_4 is lowered a little after each motion of the carriage inward, causing the arc described by E to sink and therefore the arc of G to rise as needed. To effect the lowering of the rail A_1A_4 two other cam rails B and C are employed, on whose upper guiding edges rests the rail A_1A_4 by means of the pins H and K. These pins rigidly attached to A_1A_4, are also guided in two stationary slots H_1H_2 and K_1K_2 of the frame. The two cam rails or *formers* B and C are united by the rod T. To lower the rail A_1A_4 we must, after each inward motion of the carriage, cause these formers to be moved in the direction Q to O by a slight rotation of the screw M turning in the frame and working in the nut M_1 attached to C. The kind and extent of the lowering motion of the rail depend principally on the shape of the formers B and C. It is clear that by giving different inclinations to the guiding edges of the formers the ends of the copping rail A_1A_4 can be lowered different amounts, which is necessary when the conical layers are to be gradually made more pointed, thus making the body of the thread more self-sustaining. It is also easy to see that a suitable choice of the form and direction of the guiding slots H_1H_2 and K_1K_2 gives a further means of regulating not only the *lowering* of the copping rail A_1A_4, but also its longitudinal sliding.*

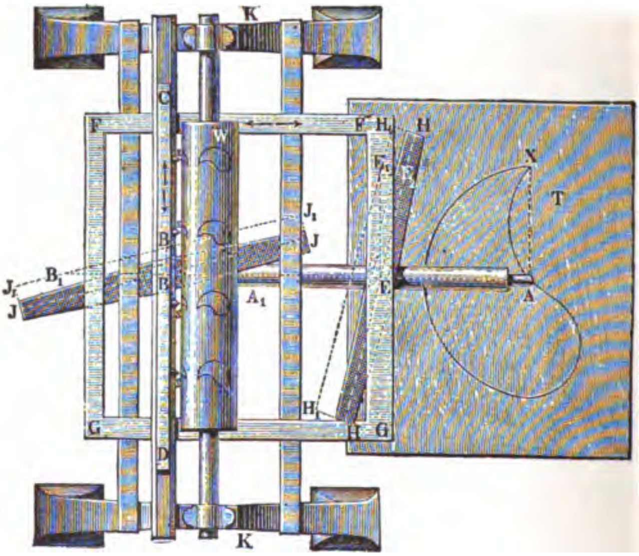
If the guiding rail is straight instead of curved there is at every instant a constant ratio between the motion of the rail and the piece guided by it. An example of this, as applied to a *copying machine*† used in cotton printing, is shown in Fig. 645. These machines serve to transfer a reduced copy of any pattern drawn on a plate T to different parts of the copper roll W. For this purpose the smoothly turned copper roll W is placed between the fixed bearings K that its rims rest on the sides FF and GG of the frame FG which can slide in the direction $F F$. A sliding of the frame in this direction therefore turns the roll in one direction or the other, so that the motion of the circumference of the roll is exactly equal to the sliding of the frame. This sliding is effected with the help of a prismatic guide $H H$ fastened to the frame $F G$ so as to make the angle α with its direction of motion. The lower part of this guide H is provided with a groove into which fits a slide block E through which passes a rod $A A_1$ parallel to $F F$, the rod $A A_1$ being capable of sliding in the direction of its length through the slide block E. Now if we suppose the rod $A A_1$ to be shifted at right angles to its length, say by moving the tracer at A to X so that the rod moves from the position $E A$ to $E_1 X$, it follows from the figure that the prismatic guide will move from its position $H H$ to $H_1 H_1$. If we place this shifting motion $E E_1$ of the rod $A A_1$ equal to a, the motion of the frame $F G$ and therefore the circumferential motion of the roll will be $E E_2 = a \cot \alpha = a_1$.

---

* For further details see *Huelsse*: "Die Baumwollspinnerei," and *Stamm*: "Der Selfactor."

† See Zeitschrift deutsch. Ingen., 1874, p. 608.

FIG. 645.



Let us also suppose a bar $C D$ to be so placed in the frame that it can slide parallel to the roll W, and let us suppose it to be rigidly attached to a prismatic bar JJ at B so that the angle included between the two directions JJ and CD will likewise equal α. Moreover, let the rod $A A_1$ be so connected with the slide block B of the guide J that $A A_1$ can move freely in B in the direction of the axis of the roll. It is evident from this arrangement that a sliding of the rod $A A_1$ in its own direction, say by the amount $A\ Y = b$, will cause the point B to move to B_1 and the guide JJ to $J_1 J_1$, and this sliding will be represented by

$$B B_2 = B B_1 \cot \alpha = b \cot \alpha = b_1 .$$

Therefore if we suppose the tracer at A to be passed over the outline of the pattern on T, every elementary motion of the pin A in the direction of the axis of the roll will transfer this motion to the circumference of the roll, but reduced in the ratio $1 : \cot \alpha$, while every elementary motion of the pin A in the direction $A A_1$ will cause a reduced sliding of the bar CD in the same ratio. Therefore if this bar CD is provided with a number of diamond points s which touch the surface of the roll, the motion of the pin A on the drawing $X Y$ will cause each diamond point to engrave on the roll a drawing which is a reduced copy of the pattern $X Y$ in the ratio $1 : \cot \alpha$. It follows from this that the desired reduction can be varied at pleasure by simply changing the angles α made by the guides with their directions of motion.

Cam trains are frequently applied in machinery, and numerous examples of them can be found in all technical periodicals. The theory of cams is treated in *Willis'* Principles of Mechanism, in *Lanz* and *Bétancourt's* Essai sur la composition des machines, in *Laboulaye's* Traité de Cinématique and in *Reuleaux's* Kinematics of Machinery.

CHAPTER VIII.

ENGAGING AND DISENGAGING GEAR.

§ 167. **Disengaging in General.**—Engaging and disengaging gear is used to set a machine or any part of it into or out of action, as may be necessary. The object is not only to start or stop a machine at will, but also to suddenly change the motion of a machine piece, the existing motion being removed by a disengagement, and the new motion transmitted to the machine piece by a subsequent engagement. To accomplish this the mechanisms in question are so arranged that the motion transmitted to a certain machine piece for the purpose of throwing it into gear will, when reversed, throw the same piece out of gear. It follows from this that every engaging gear is also suitable for disengaging.

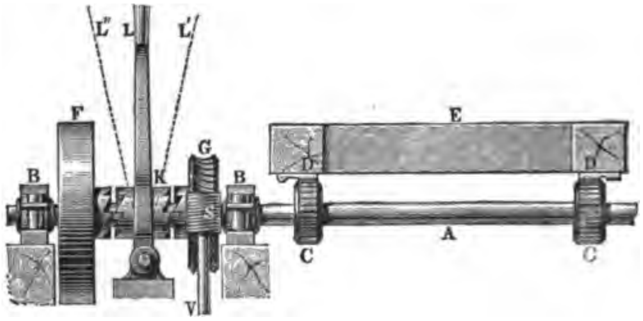
The engaging and disengaging can be effected by the attendant; this of course is always done when the machine begins and ends its work. But there is also a periodical or occasional engaging and disengaging which is effected by special automatic mechanisms, so arranged that they act either at regular intervals, as, for example, in pendulums, or in certain contingencies, as in stop-motions which act when a thread breaks in a loom.

The disengaging of a mechanism in motion can generally be effected in two different ways, either by causing the driving force to stop acting on the mechanism, or by opposing the motion by a resistance greater than the driving force can overcome. The first method is employed in *clutches* (see § 29), and the second in *clock escapements* and *hoisting apparatus*, the resistance of the pawl being too great to be overcome by the load. Brakes may also be classed with this latter kind of disengaging gear, provided they are employed to *stop* the motion, for instance the motion of a train. Usually, however, brakes are employed as in windlasses for *moderating* and *regulating* the motion, wherefore it is better to class them with *regulating mechanisms*.

We will first discuss disengaging gear, in which the throwing out of gear is effected by preventing the driving force from acting on the machine piece.

§ **168. Clutches.**—Disengaging couplings for shafts have been already discussed in § 29 to § 31, and the peculiar action of clutches and friction couplings was also considered there. In these articles it was assumed that the two parts of a coupling were fastened to different shafts in such a way that they could not turn on them, and that one shaft could receive rotation from the other or not as was desired. But couplings can be arranged so that a shaft can receive its rotation from a *loose* wheel, or other piece situated on it, which is in continuous rotation. For this it is only necessary that this driving piece be constructed like the half of a coupling, *i.e.*, provided with teeth, and that the other half of the coupling be formed like a muff, and so placed that it can slide but not turn on the shaft. Engaging and disengaging then take place by sliding the muff upon the shaft, the muff turning with the latter by means of a prismatic feather. Such an arrangement, often used in *gang saws* to effect the forward and return motion of the log carriage, is shown in Fig. 646. Here A is a sliding shaft supported by the bearings B, which moves the carriage E by the spur wheels C rigidly keyed to it, the frame E having racks on its lower

FIG. 646.



surface for this purpose. According as the shaft *A* has a right-handed or left-handed rotation, the log will move toward the saw or away from it. Here, to economize time, the return motion takes place with greater speed than the forward motion during the cutting. For this purpose the belt pulley *F* and the worm wheel *G* are placed so as to be *loose* on the shaft *A*, these two wheels receiving continuous motion in opposite directions. The pulley *F* is driven directly by a belt from the main shaft, while the worm wheel *G* receives a slow motion in the opposite direction from a worm *S* on the upright shaft *V*. The sliding muff *K* is placed between the pulley and the worm wheel, and provided at each end with clutch-teeth whose partners are attached to the pulley *F* and to the wheel *G*. When the muff is in the middle position the sliding shaft receives no motion, the carriage remaining at rest; but when the lever occupies the position *L'* or *L''* the muff meshes with the worm wheel or the belt pulley and communicates a slow forward or a quick return motion to the carriage. The shifting of the lever is here effected by the attendant.

FIG. 647.

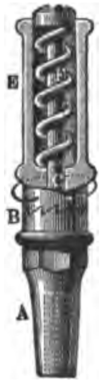
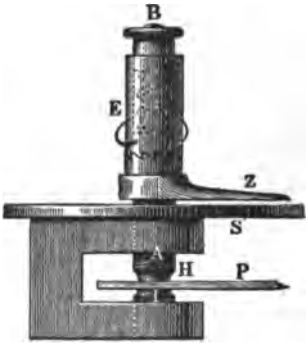


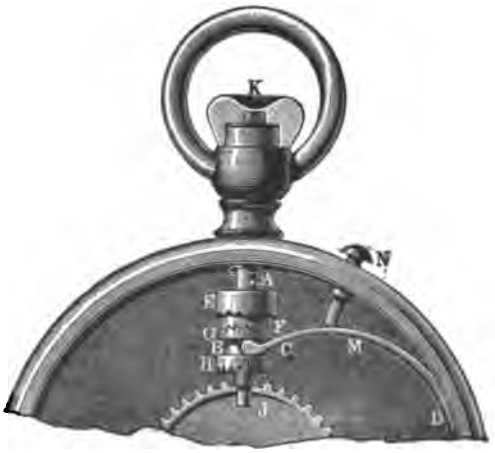
FIG. 648.



We may also mention here *Bréguet's* key for winding up clocks, Fig. 647. The clock key proper is at *A*, and is reduced at *B* to a small spindle *C* on which the loose sleeve *E* turns. The lower end of this sleeve and the shoulder of the key at *B* are provided with clutch-teeth, a little spring *F* maintaining contact between the teeth. Therefore when the sleeve is turned in the direction of the arrow, motion will be communicated to the teeth of the key *A* and the clock will be wound up. When the sleeve turns in the opposite direction it will be pushed upward by the oblique surfaces of the teeth and the spring compressed. During the return motion of the sleeve the key *A* is acted upon in a direction opposite to the arrow by a force corresponding to the *friction* between the teeth of the coupling. Use has been made of this in *micrometer screws* for measuring the thickness of plates, Fig. 648, in order that the clamping pressure between the plate *P* and the two cheeks *H* and *I* may be as constant as possible. Here the micrometer screw is also turned by the toothed sleeve *E*, but the oblique teeth of the coupling are so placed that the screw is only turned by the friction between the teeth. In this way the screw is prevented from pressing the plate to be measured with a greater force than corresponds to the friction, and thus renders it possible to make different measurements under the same circumstances. The manner in which the index *C* connected with the screw indicates the distance between *H* and *I* by means of the angle of rotation and the known pitch of the screw hardly needs explanation.

Another arrangement coming under this head is the mechanism known as a stem-winder for winding watches without a separate key. Here the button *K*, Fig. 649, is provided with a steel shaft *A* which is mostly cylindrical, but is square at the portion which carries the little coupling muff *B*, so that when the button *K* is turned in either direction the muff must turn with it. For winding up the spring a little wheel *E* turn-

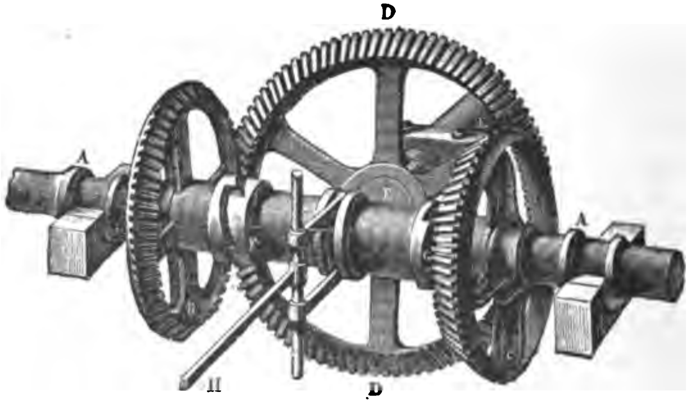
Fig. 649.



ing loosely on *A* is employed, the teeth of this wheel gearing with another wheel on the arbor of the spring. As the wheel *E* is connected with the muff *B* by the oblique teeth at *F* and *G*, the spring *D M C* ordinarily holding the two parts together, we see that the winding up can take place at *A* as with *Bréguet's* key, for the wheel is only turned when the arbor *A* and the muff *B* turn in the direction of the arrow. In order that the hands may also be set with this device, the muff must be thrown out of gear at *F* by pressing upon the pin *N*, whereupon the other toothed side *H* of the muff gears with the toothed wheel *J* connected with the hands.

The manner in which disengaging gear may communicate rotation from a shaft turning always in the same direction to another shaft alternately turning in opposite directions is shown by the two figures, 650 and 651. In the *reversing gear*, Fig. 650, the two wheels *B* and *C* turn loosely on the driving shaft *A*. By a suitable shifting of the muff *F G* along the shaft *A*, about which it cannot turn, either wheel, *B* or *C*, can

FIG. 650.



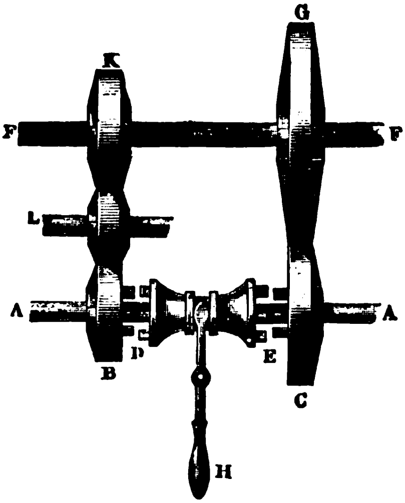
be coupled to the shaft *A*, thus turning the shaft *E* by means of the wheel *D* in one or the other direction. Here the two wheels *B* and *C* always turn with equal velocity in opposite directions.

On the other hand, Fig. 651 represents a reversing gear for parallel shafts; here also the wheels *B* and *C* turn loosely on the driving shaft A and can be coupled at will to the driving shaft by the muff $D\,E$, as in the preceding arrangement. It is now easy to see that the driven shaft will turn in one or the other direction, according as it receives its motion directly by the pair of wheels $C\,G$ or indirectly by the pair of wheels $B\,K$ and the idle wheel L. If the velocities are to be the same in both cases, the ratio of the radii of the wheels must be equal; thus

$$\frac{C}{G}=\frac{B}{K},$$

for the size of the idle wheel has here no influence. This mechanism has recently been used with advantage in reversing mills, *i.e.*, in rolling-mill trains in which the motion is alternately in opposite directions, in order that the iron or steel may receive the desired form with as little loss of time as possible. In this case we must suppose $A\,A$ to represent the engine shaft and $F\,F$ the shaft of the lower roll. The high speed and great mass of these rolls forbid the use of toothed clutches, consequently powerful friction couplings are used instead, generally in the form of plain or conical disks which are pressed together by hydraulic pressure to generate the requisite friction.

Fig. 651.



The engaging and disengaging may also be effected automatically, and such an arrangement should be chosen where it is necessary that the engaging and disengaging shall take place promptly in regular and rapid succession

The best examples of this are found in the self-acting spinning machine or *mule*, in which a series of different engagements and disengagements must be regularly repeated for the delivery rolls, the spindles, the carriage and the faller. As the intervals are small, 1 to 2 minutes, the engagements and disengagements must be effected by a self-acting mechanism which we may call the *directing-gear.* This gear, which is usually very complicated and ingenious, occurs in very different forms, but almost always has a shaft which at certain instants turns with sudden jerk-like motion through exactly a half or a quarter revolution, this partial rotation accomplishing the engagement or disengagement of the couplings. For this purpose each coupling of the shaft is provided with a cam (see § 161) whose groove acts on a pin in the end of the corresponding disengaging lever. The form to be given to the cam in order that the lever may have the motion necessary to effect the engagement or disengagement may be found from the methods employed in the preceding chapter. Moreover, it is clear that the shaft itself needs a certain engaging and disengaging gear to effect its sudden, intermittent motions; but this mechanism has a different arrangement from the kind now under discusion; hereafter there will be an opportunity of examining such a mechanism more fully.

It has already been stated (§ 29) that the engaging of a toothed clutch during the running of a machine is dangerous when the velocity and the moving mass are considerable, on account of the shocks accompanying such an engagement. During disengagement no shocks occur, but the greater the force transmitted, the more difficult it is to throw toothed clutches out of gear, for the frictional resistances encountered by the sliding muff and at the surfaces of the teeth depend upon the force which is transmitted. In the cases where it is a question of easily disengaging a heavy, toothed clutch, special arrangements must be provided for throwing it out of gear automatically. Fig. 652 gives a sketch of such a clutch, as given in *Wiebe's* "Lehre von den einfachen Maschinentheilen." Here the nave N of the half K of the clutch has a groove BCD whose middle line along BC is in a plane normal to the axis, but along CD gradually leaves this plane: therefore if a pawl turning about the stationary pin F is pressed against the nave of the clutch, the pawl will fall into the groove at B and will cause the nave to shift to the right on account of the oblique sides of the groove *C D*. Evidently we have here the cylindrical cam, Fig. 634, discussed in § 165, where the rod, represented here by the pawl *E F*, is stationary, and where the cam assumes a sliding motion in addition to its turning motion. In order that the pawl may rise out of the groove after disengagement is effected, the bottom of the groove is arranged to rise gradually till it loses itself in the surface of the nave *N*. A pump piston *A*, Fig. 653, is thrown out of action by withdrawing the key *D* which unites the driving rod *B* with the piston. The rod *B* then continues its reciprocating motion, but does not impart it to the piston *A*.

FIG. 652.

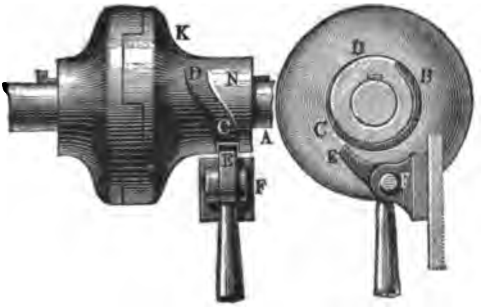
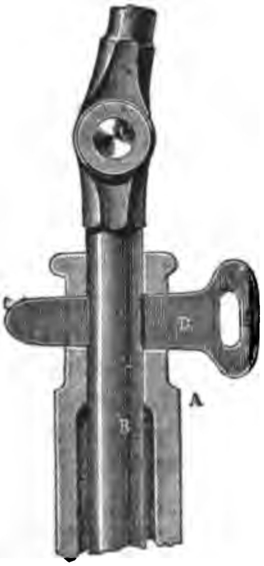


FIG. 653.



In large shearing machines the slide *A* carrying the knife *S*, Fig. 654, is thrown into gear by bringing the eccentric rods *D* and D_1 into such a position that their shoulders *N* and N_1 press against the projections *O* and O_1 of the slide. The rods cannot communicate any motion to the slide *A* in the position in which they are here drawn. The manner in which the engagement and disengagement are effected by the hand lever *H I K* and the coupling rod *E* is evident from the figure. In this arrangement the rods *D* can only move the slide downward, consequently means must be provided, say by a counterweight, for lifting the slide after the cut has been made.

§ 169. Toothed Wheels as Disengaging Gear.—If one shaft is driven by another by means of a pair of toothed wheels, the driven shaft may be brought to rest at any moment by destroying the contact between the wheels. This can always be accomplished by separating the wheels, either by moving one wheel axially or by separating the axes of the two wheels; both methods are common in practice.

Fig. 654.

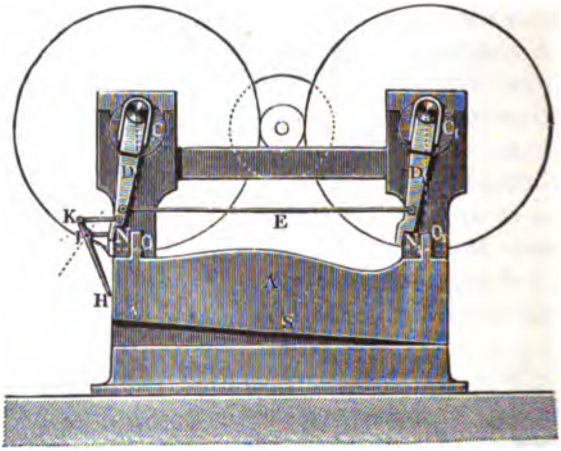
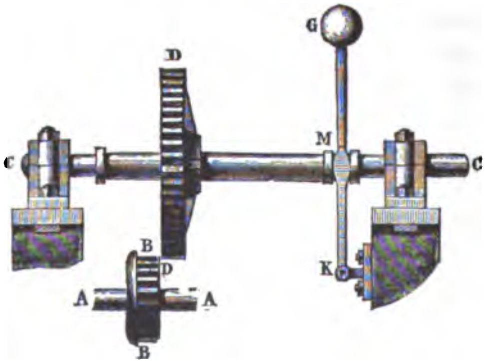


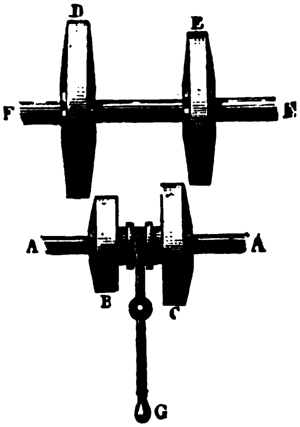
Fig. 655 shows how the axial sliding of the wheel can effect the disengagement. Here *A* represents the driving shaft which turns the shaft *C* as long as the two wheels *B* and *D* are in contact, the sliding of the shaft *C* and its wheel *D* interrupting the transmission of motion.

Fig. 655.



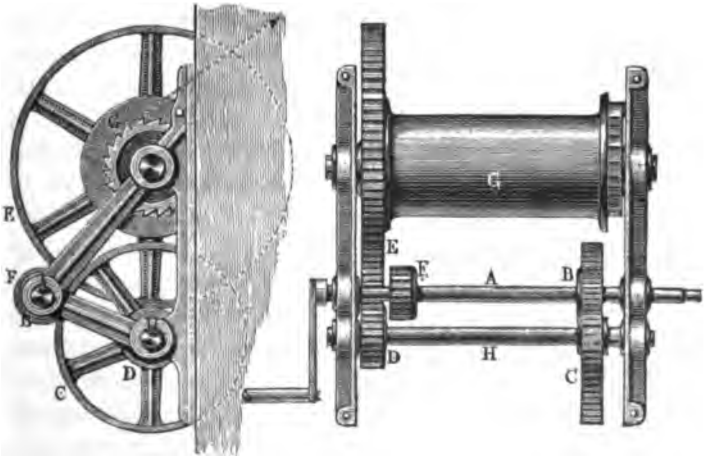
This arrangement is occasionally employed in hoisting machinery for varying the velocity ratio between the driving shaft and the drum, to suit the load to be lifted. For example, if *A*, Fig. 656, is a driving shaft, it can drive the shaft *F* of the drum either by the toothed wheels *B* and *D* or by the wheels *C* and *E* according as the lever *G* moves the rigidly united wheels *B* and *C* to the left or right of the middle position given in the figure. The shaft *F* turns in the same direction in both cases, but its velocity varies as $\frac{B}{D}$ or $\frac{C}{E}$ when *B*, *C*, *D* and *E* represent the radii or the numbers of teeth of the respective wheels.

Fig. 656.



This last arrangement effects only a slight variation in the velocity ratio, insufficient for most hoisting apparatus, where the driving pinion is always taken as small as convenient in order that the force may be increased as much as possible. Therefore to obtain the greater difference in velocity ratio needed for different loads, frequent use is made of the arrangement shown in Fig. 657.

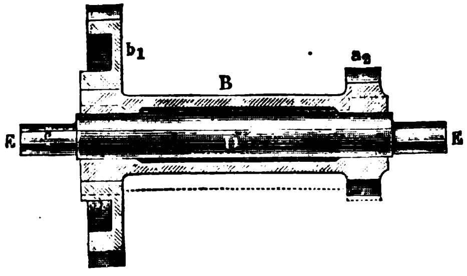
Fig. 657.



Here the crank shaft *A* can communicate a slow motion to the drum *G* either by allowing the pinion *F* on *A* to gear with the wheel E on the drum, or by permitting the pinion B on A to gear with the larger wheel C on the auxiliary shaft H, thus communicating to the wheel D on the same shaft a rotation which turns the wheel E on the drum. To use one or the other set of gears, it is only necessary to slide the shaft A in its bearings till the wheel F gears with E, or B with C. A hoisting apparatus with such an arrangement is said to work ***single-geared*** or ***double-geared***. With this arrangement one revolution of the crank shaft causes $\frac{F}{E}$ revolutions of the drum when F gears into E, while when working with double gear one revolution of the crank causes $\frac{B}{C}\frac{D}{E}$ revolutions of the drum, where the letters $B, C, D, \ldots$ again represent the number of teeth of the wheels. As the drum when working single-geared has a rotation opposite to that accompanying its double-geared condition, it of course follows that the workmen when raising the load must turn the crank in one or the other direction, according as the single- or double-geared arrangement is employed.

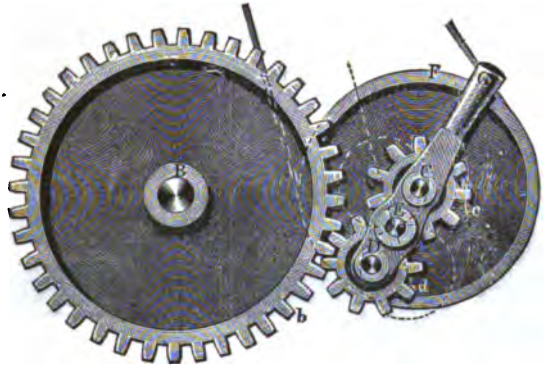
The other means mentioned above for throwing two wheels out of gear by separating their shafts is likewise often employed, particularly in cases where there is not sufficient space for imparting an axial motion to the wheels. One of the best known examples of this is the engaging and disengaging of the back gears of lathes. The double-geared arrangement of lathes has already been described in § 42. In order to throw the back gears in and out of gear, it is customary to move the back gear shaft B, Fig. 658, parallel to itself toward the lathe spindle or away from it. For this purpose the two back gears b_1 and a_2 are keyed fast to the hollow shaft B, which turns loosely on the solid shaft D. The ends of the shaft D are eccentric pins E, supported in fixed bearings on the frame. It is now evident that a half revolution of the

Fig. 658.



solid shaft *D* will bring the hollow shaft *B* into the dotted position, thus shifting parallel to itself a distance equal to double the eccentricity *e* of the journals *E*.

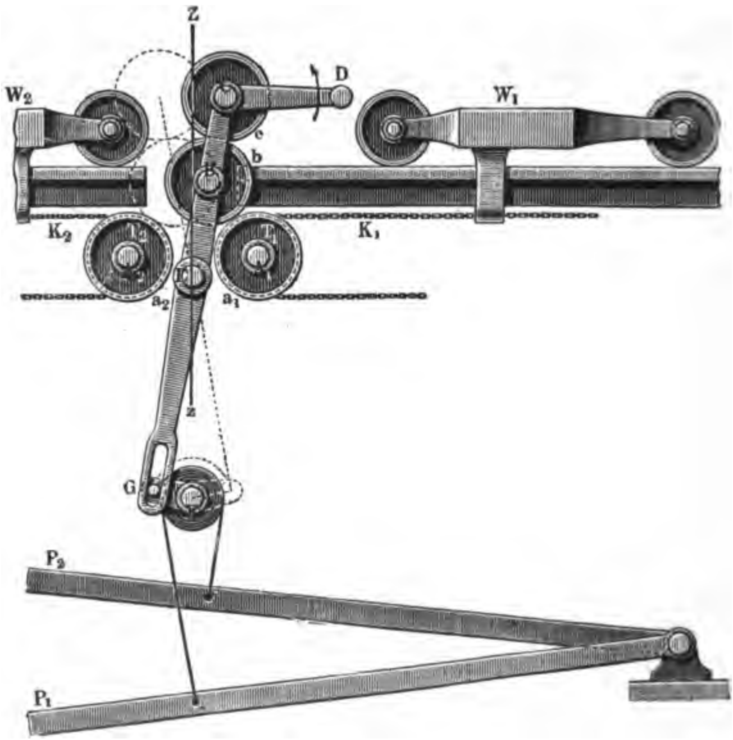
FIG. 659.



This method of disengaging is also employed in reversing gears, for example in the change gears of lathes for reversing the direction of the tool by reversing the leading screw. In Fig. 659, *B* represents the screw which gives longitudinal motion to the slide rest. This screw is driven by a cord pulley *F* which communicates its rotation to the pinion *c* rigidly attached to it, both turning about the fixed stud *C*. The wheel *c* gears into a second and equal wheel *d*, which turns loosely on the pin *D*. In consequence of this arrangement the pinions *c* and *d* turn constantly with equal velocity in opposite directions. As the pins *C* and *D* are placed on the lever *C E D* at equal distances from its pivot *E*, we have the means of bringing the pinion *c* or *d* into gear with the toothed wheel *b* fastened to the leading spindle. Hence the spindle *B* and the rest to which it communicates motion can be alternately moved in opposite directions.

An interesting mechanism coming under this head is employed in *Heilmann's* embroidering machine, for moving the needle carriages. Here *z z*, Fig. 660, represents the vertical web to be embroidered, on each side of which is a carriage carrying the pincers for grasping the needles, these carriages W_1 and W_2 rolling back and forth on horizontal rails. This motion is effected by means of drums T_1 and T_2 on opposite sides of the web, and by endless chains K. The reciprocating motion of the drums originates with the crank shaft C, which is turned by hand. The crank shaft turns the toothed wheel c, which turns an equal wheel b on a stud B that is fastened to the lever CBE swinging about E. Now if this lever is so arranged that in its extreme positions the wheel b gears with the wheels a_1 and a_2 of the drums A, we see how a given rotation of the crank D will cause the carriage in action to move in a certain direction, which direction is reversed when the crank D is turned in the opposite direction. In order that the position of the lever BE may be easily changed at any instant the shaft F is employed, which can alternately turn in either direction through a half revolution, according as the attendant

FIG. 660.



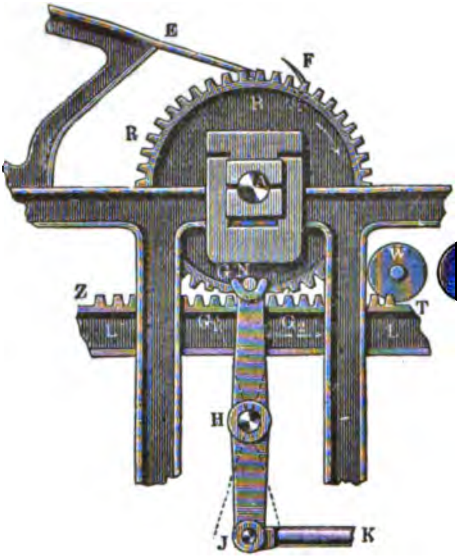
steps on one or the other treadle P_1 or P_2. How the turning of this shaft F and its crank G can effect the oscillation of the prolonged and slotted lever $B E$ is evident enough from the figure. The use of this mechanism can be explained in a few words. In the position drawn, the workman presses down the treadle P_1 and moves the crank D in the direction of the arrow. As a consequence the carriage W_1 with the needles approaches the web from one side, while the other carriage, W_2, is at rest on the other side close to the web. When the threaded needles on the left have pierced the web, the workman, without stopping the crank, steps on the other treadle, which causes the carriage W_2 to retreat from the web till the thread is drawn up or taut. The workman must now turn the crank in the opposite direction without altering the treadles, which will bring back the carriage W_2 to the web, when a treading down of P_1 will cause the carriage W_1 to retreat in turn, whereupon the operations described are repeated. We may remark that one effect of working the treadles is to cause the needles on one side of the web to be grasped by the pincers on the other, while the opposite pincers open.*

The disengagement of a toothed wheel may also be effected by omitting the teeth at a certain part of the wheel. An example of this is found in the mechanism employed in *power printing-presses* for connecting the cylinder with the reciprocating type-bed during the forward stroke, and rendering it stationary during the return stroke of the bed. Fig. 661 is a sketch of the arrangement, where B is the cylinder fastened to the shaft A, which receives the sheet of paper from the inclined table E, the movable fingers F taking hold of the paper and carrying it along with the cylinder as the latter turns. The cylinder B turns only in the direction of the arrow; turning each time through exactly one revolution, followed by an interval of rest. This motion is communicated to it by the sliding bed $L L$, which receives a continuous reciprocating motion from a rotating crank. The form T is alternately presented to the ink-rollers W and the paper on the cylinder. In order that the

---

* For further details of *Heilmann's* embroidering machine see *Prechtl*, Techn. Encyklopaedie, Supplem., vol. v.

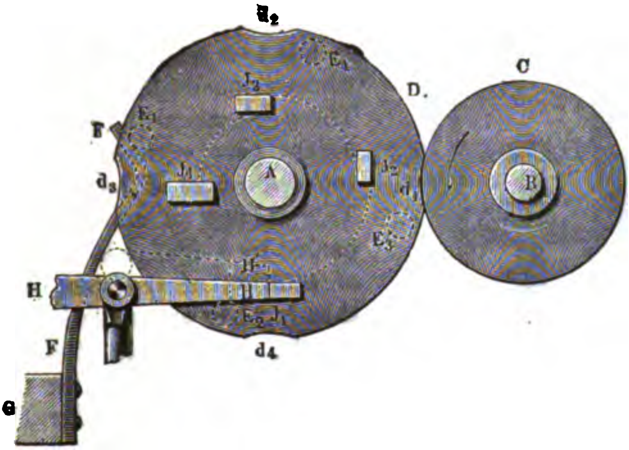
Fig. 661.



cylinder may move with the bed when the latter moves to the right, the bed has a rack *Z* which gears with a toothed wheel *R* attached to the cylinder, the diameter of the pitch circle of the wheel being exactly equal to that of *B*. Now a part of the teeth of the wheel *R* is planed away so that the rack during a portion of the stroke may pass under it without moving it. The cylinder therefore remains at rest when it occupies the position shown in the figure, *i.e.*, while the bed is moving to the right for the purpose of inking the type. When this is done, the bed moves to the left and the cylinder *B* must by some means be turned sufficiently to gear with the rack, which will then turn the cylinder till the planed portion of the toothed rim again stands over the rack. To effect this engagement the two-armed lever *J H G* turning about *H* is provided with a fork *G* which acts on a pin *N* projecting from the end of the cylinder *B*. Now if the lever *G J* is oscillated by the cam rod *K J*, it is evident that the fork *G*, moving from its middle position *G* to G_1, will cause the cylinder to begin its rotation in connection with the rack, and when it has completed its return stroke at G_2 it again takes hold of the pin *N* and leads it into the middle position *G*. It need only be mentioned that the rod *K J* does not receive its motion from an eccentric, but from a cam whose form is such that the point *J* is at rest during the return stroke of the bed, and completes its double stroke from the middle and back again while the bed is making its forward stroke. How the cam should be laid out to fulfil this condition is plain from the discussion on cams, in the preceding chapter.

There is a certain relation between the preceding mechanism and that shown in Fig. 662, the latter serving to impart the intermittent motion needed by the *directing shaft* of a *self-acting* spinning machine (see § 168). Here *A* represents the shaft which is to make a quarter revolution at certain times. It receives this motion from a continuously rotating shaft *B* by means of two friction wheels *C* and *D*, of which *D* is provided with circular notches d_1, d_2, d_3 and d_4, 90° from each other. When a notch is opposite the wheel *C*, as d_1 in the figure, the wheel *D* will not be turned for want of friction. Motion is also prevented at this instant by the end of a rod *H H* pressing against a projecting lug J_1 of the wheel *D*, thus preventing right-handed turning of the latter. When this obstacle is removed by lifting the rod *H* into the position *H'*, the wheel *D* will obey the impulse received from *C* as soon as it (*D*) is turned sufficiently to allow its unnotched periphery to come in contact with *C*. The slight turning needed to bring the wheels into contact is effected by a strong spring *F* attached to the frame *G*, and pressing against a pin E_1 so as to produce a right-handed rotation of *D*. As a consequence the directing shaft *A* is turned through 90° till the next notch d_2 is opposite to the

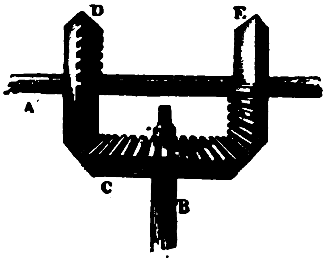
FIG. 662.



wheel C, when the second lug, J_1, strikes against the rod H'. At the same time the next pin, E_2, takes the place of E_1, having pressed back the spring. This is repeated three times during the period of the spinning process, the rod H receiving two more lifts and then returning to the initial position H. The motion of this rod does not take place at equal intervals, but is governed by the spinning process.*

Fig. 663 shows how sector wheels, *i.e.*, wheels whose circumferences are only partially provided with teeth, can be employed as *reversing gear*. Here the continuously rotating shaft A has two bevel wheels D and E whose opposite halves are toothed. These wheels will therefore alternately gear with the completely toothed wheel C on the shaft B. This change of contact is accompanied by a change in the direction of rotation of the shaft B, which thus receives an oscillating motion. If d represents the radius of the pitch circle of one of the wheels D or E, and c the corresponding radius of the wheel C, the angular extent of the oscillation of the shaft B will be

FIG. 663.



$$\frac{d}{c}\pi.$$

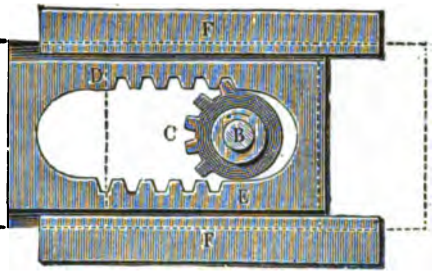
Within certain limits, therefore, the angular swing can be assumed at pleasure; for example, with equal wheels c and d the shaft B will oscillate through 180°. This mechanism cannot be used with B as driving shaft, for at best the shaft A could only make a half revolution before it came to rest. On the other hand, if half the wheel C and the whole of the wheels D and E are provided with teeth, it is necessary that C be the driver, in which case it will communicate to the shaft A an oscillation measured by

$$\frac{c}{d}\pi.$$

---

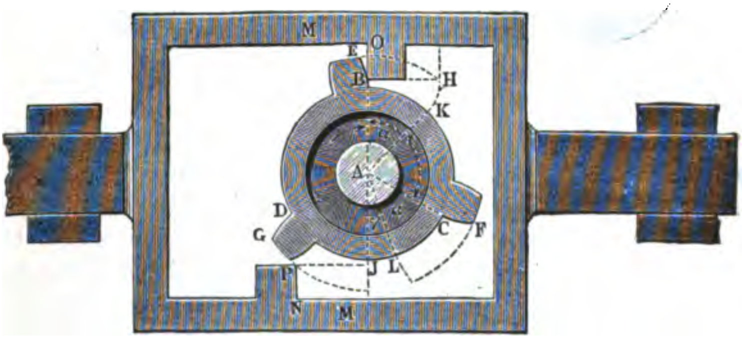
* For further details see *Huelsse*, Die Baumwollspinnerei.

Fig. 664.



If in this case we suppose the half-toothed wheel C to become a spur wheel, and the toothed wheels to become two parallel racks, we shall have the mechanism shown in Fig. 664. Here the two racks D and E form a frame which slides back and forth in the guides F, with stroke $c \pi$ for every revolution of the wheel C, c again representing the radius of the pitch circle of C. If the toothed portion of C were confined to less than a semi-circumference, say to the *arc* $n 2 c \pi$, where n is of course less than $\frac{1}{2}$, the sliding would only amount to $n 2 c \pi$, and at every reversal of motion the frame would be at rest for an interval $(\frac{1}{2} - n) t$, where t represents the time required for one revolution of the shaft. A similar mechanism is shown in Fig. 665. Here the rotating shaft A has three teeth, whose involute flanks $B E$, $C F$ and $D G$ are the correct profiles for working with the straight lugs $B O$ and $P N$ of the frame M. When the shaft A turns, the frame will be turned from the extreme position shown in the figure, through a distance $B H = P J$ to the right, this requiring a rotation of the shaft through the angle $\alpha = B A K = C A L$. If, as in the figure, this angle is taken less than the half pitch angle $B A C = \tau$ (in

Fig. 665.



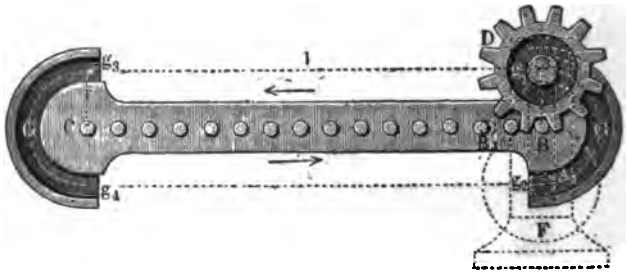
this case less than 60°), the frame will remain in its extreme position at the right, while the shaft travels through the angle

$$\gamma = \tfrac{1}{2}\tau - \alpha = L A J.$$

Therefore we must make $\alpha = \frac{1}{2}\tau$ when the frame is to reciprocate without periods of rest.

From the preceding mechanism we may deduce one which is employed in mangles for pressing clothes. In the arrangement shown in Fig. 664 the frame had but a slight stroke, at most equal to the semi-circumference of the driving wheel, but in *mangle gear* the stroke can be made as large as desired, for several revolutions of the driving wheel can be employed to slide the frame in the same direction. This mechanism occurs principally in two different forms. In the form shown in Fig. 666 the frame connected with the mangle box is provided with a pin rack ***B C*** which gears with the pinion D on the driving shaft A. The shaft A of this pinion has bearings of such a kind that it can rise and fall in the vertical guide F. If, therefore, we suppose the shaft A and its pinion to be above the rack, as in the figure, a right-handed rotation of the pinion will cause the rack to move to the left, and with a velocity equal to the pitch circle of the pinion. This continues till the last pin B is exactly under the shaft A. A further rotation of the driving pinion D will cause A to gradually slide down the slotted guide F, and occupy the position B when it has made a quarter revolution. This will cause the last pin to move from B to B_1, a still further distance to the left equal to the radius $B B_1 = r$ of the pitch circle of the driving pinion. While the

FIG. 666.



pinion is making another quarter revolution, its shaft passes to A_1, and at the same time the pin rack moves to the right a distance r. The uniform return motion of the pin rack now takes place till the extreme pin C arrives at the other end, where the same movements are repeated in the inverse order.

The driving shaft A is either guided along a vertical slot F, as above, or it is placed at the end of a swinging lever whose other end is counterweighted. The motion communicated by A in a circular arc differs but little from that described above. To insure continual contact with the wheel, each end of the rack is furnished with a grooved piece G concentric with the extreme pins B and C. Moreover, the shaft A can have fixed bearings, provided the pin rack is pivoted at its centre on a pin fixed to the mangle box. The pin rack will then place itself in the inclined position required; the groove guides G are indispensable in this case.

The motion of this mechanism is very simple. Let r again represent the radius of the pitch circle of the wheel, and l the distance between the extreme pins C and B measured from centre to centre; then each stroke of the rack will be equal to

$$r + l + r = l + 2r.$$

The number of revolutions corresponding to each stroke is

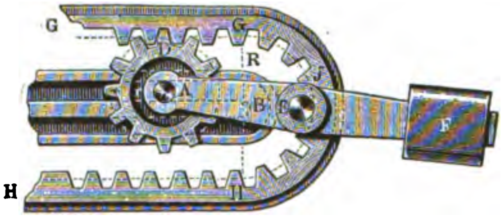
$$\frac{1}{4} + \frac{l}{2\pi r} + \frac{1}{4} = \frac{1}{2}\left(\frac{l}{\pi r} + 1\right);$$

consequently there will be $\frac{l}{\pi r} + 1$ revolutions for a double stroke.

It should also be noticed that a uniform rotation of the driving pinion communicates a uniform sliding motion to the pin rack only when the latter is at the middle portion l of its stroke. At the beginning and end of the stroke when the shaft is ascending or descending, the velocity gradually decreases to or increases from zero according to the law of the crank train with a slotted sliding bar, *i.e.*, the velocity of sliding is proportional to the cosine of the angle described by the wheel D while ascending or descending. If we draw the tangents $g_1 g_3$ and $g_4 g_2$ to the middle arcs of the curved guides G, the area $g_1 g_3 G g_4 g_2 G$ can be regarded as the diagram of the motion, the ordinate at any point of the stroke $G G$ giving the velocity of the sliding. Therefore in this mechanism the change of motion is unaccompanied by shock.

Instead of the pin rack, two toothed racks, G and H, Fig. 667, can be employed in this mechanism, their ends being connected by semi-circular internal gears $G J H$. Here, too, the shaft A of the pinion must receive an up and down motion equal to $2 (R - r)$, where R represents the radius $B J$ of the pitch circle of the semi-circular toothed arc, and r the radius of the pinion. To accomplish this the shaft A is carried by a lever $A E F$ counterweighted at F. The motion does not essentially differ from that of the mechanism in Fig. 666. If l again represents the length of the straight portion of the rack, the whole sliding in either direction will be $l + 2 (R - r)$.

FIG. 667.



The rotation of the driving pinion, while change of motion is taking place, can be determined as follows: The pinion begins to descend when contact takes place at G, and the descent ceases with contact at H. The toothed distance between these points is equal to πR. Consequently the distance between the points of the pinion that come in contact at G and H is also πR, if the distance is measured along the circumference of the pinion. As this is a case of a circle D rolling within another circle $G J H$ (see § 47), we must subtract from πR the angle subtended by $G J H$ expressed in terms of the circumference of D, and then

$$\pi R - \pi r$$

represents the actual rotation of D in terms of its circumference. From this we obtain the number of revolutions of D equal to

$$\frac{\pi R - \pi r}{2 \pi r} = \frac{1}{2} \frac{R - r}{r}.$$

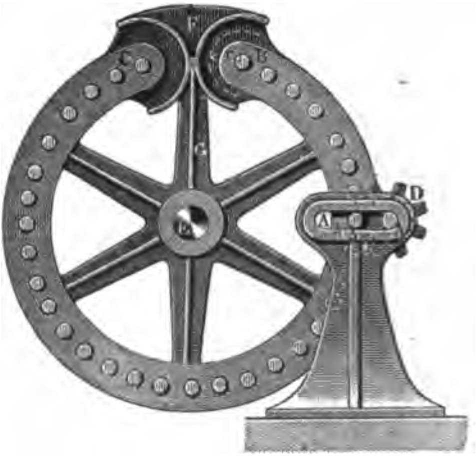
Consequently the number of revolutions of the pinion for a whole period of the motion, *i.e.*, for a forward and return stroke of the frame, is equal to

$$\frac{2l}{2\pi r} + \frac{R-r}{r} = \frac{l}{\pi r} + \frac{R-r}{r}.$$

The remarks made concerning the velocity of pin racks apply here also. While the frame moves over the middle portion l its velocity is uniform, but when it traverses the portions $R-r$ at the ends its velocity diminishes to and increases from zero.

If we suppose the pin rack, Fig. 666, to be bent into a circular wheel whose continuity is broken at one place and whose axis is E, Fig. 668, we see that continuous rotation of the driving shaft A will cause the shaft E to have a regular reciprocating rotation which never reaches a complete revolution. Here, also, the pinion D must be capable of shifting its position from A_1 to A_2 or from A_2 to A_1 when the rotation changes at B and C. The semi-circular guides F are also needed to secure internal or external contact. The motions are analogous to those of the pin rack, Fig. 666. If R represents the radius $EB = EC$ of the *mangle wheel* and l the length of the toothed periphery, so that $l = R\alpha$, α representing the angle of the toothed arc, we shall have, for each forward and return rotation of the mangle wheel, the angular motion

FIG. 668.



$$\frac{l + 2r}{R} = \alpha + \frac{2r}{R},$$

which corresponds to

$$\frac{1}{4} + \frac{l}{2\pi r} + \frac{1}{4} = \frac{1}{2}\left(\frac{l}{\pi r} + 1\right)$$

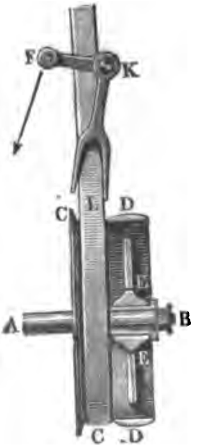
rotations of the driving pinion *D*. Here, also, the reversal of motion takes place without shock, and the variations of velocity resemble those of the pin rack, Fig. 666. The mangle wheel is seldom used; perhaps it occurs most frequently in spinning and spooling machines for generating the up and down motion of the spool frames, but usually better means are employed.

§ 170. Belts as Disengaging Gear.—Mention was made in the second chapter of the means that may be employed for putting belt pulleys into and out of action. These are principally either a lateral shifting of the belt, or else a producing or removing of the belt tension needed for running.

The first means, a shifting of the belt on the pulleys in the direction of their axes, is very frequently applied in practice. Most machines, especially the lighter ones which run rapidly, are thrown into and out of gear in this way.

This method of disengagement is very simple. Let *A B*, Fig. 669, be a driven shaft carrying two pulleys, *C* and *D*, of which *C* is keyed fast to the shaft *A B*, while *D* turns loosely on it. It is evident from this arrangement that when a belt runs on the pulley *C* it will turn the shaft *A B*, but when the belt runs on the loose pulley *D* the pulley will turn but the shaft will not, for the friction between the nave *E E* of the loose pulley and the surface of the shaft is too small to overcome resistance due to the work on the latter. To diminish this frictional resistance, which causes loss of work even when the machine is at rest, it is necessary that the nave of the loose pulley be well lubricated.

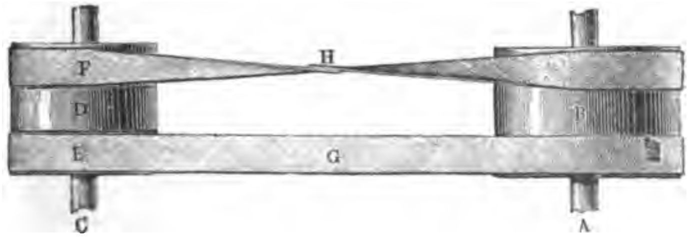
FIG. 669.



The *belt shifter* or *shipper K L* is provided with a forked end through which the belt passes, a lateral movement of the fork shifting the belt from one pulley to the other. It hardly needs to be mentioned that the driving pulley must have a width double that of the belt. According to § 57, the shifting of the belt requires a very slight force when the fork acts on it near the point at which it runs on to a pulley. The point where the belt runs on to the *driving* pulley could be taken instead of the point at which it runs on to the *driven* pulley, but the latter arrangement is the better and more usual one, for the engaging and disengaging of the shaft take place more promptly. The belt is easily shifted when it is in *motion*, but when at rest the shifting is difficult. It follows from this that the fast and loose pulleys should be placed on the *driven* shaft, while the wide pulley must be placed on the *driving* shaft; with an opposite arrangement it would be very difficult to effect the engagement. That the shifting of the belt may not be unnecessarily difficult, it is customary to make the swell on the faces of the fast and loose pulleys quite small, the driving pulley being made cylindrical, for if it were convex the belt would tend to run off when on either side.

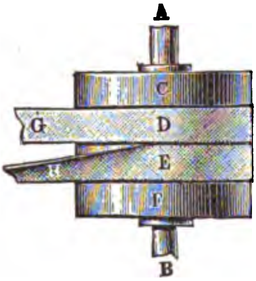
As a crossed belt imparts to the driven pulley an opposite rotation to that imparted by an open belt connecting the same pulleys, we have a convenient means of reversing the motion. If on the shaft *C*, Fig. 670, we place three equal pulleys *D*, *E* and *F*, the middle one, *D*, keyed fast and the others, *E* and *F*, turning loosely on the shaft, and employ an open and a crossed belt, the shaft *C* can be turned in one or the other direction according as one or the other belt is shifted on to the fast pulley. Of course when the shaft *C* is at rest, both belts must run on the loose pulleys. In this arrangement each belt has a

FIG. 670.



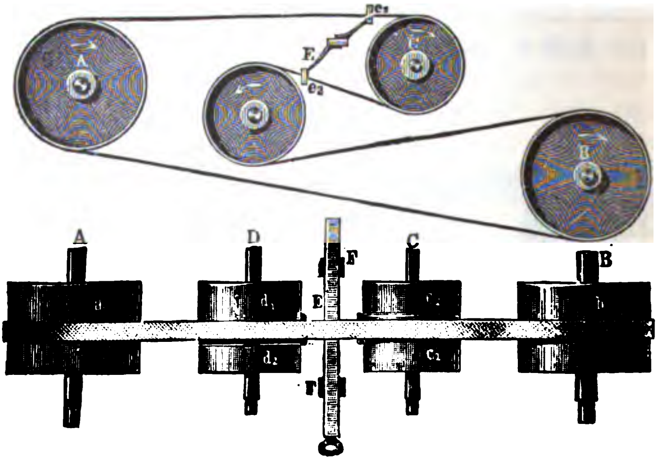
shipper which will move it independently. But to avoid as much as possible mistakes in handling the shippers the belts may be arranged so that only *one* shipper will be needed. For this purpose the driven shaft *A B*, Fig. 671, has two inner loose pulleys, *D* and *E*, and two outer fast pulleys, *C* and *F*. The belts *G* and *H* now always run close to each other, and it is clear that in the position shown the shaft *A B* is at rest and may be moved in opposite directions according as both belts are shifted through the width of a pulley to one side or the other.

FIG. 671.



An interesting belt-gear by *Sellers*, Fig. 672, shows how a single belt can effect reversals. Here the belt runs from the driving pulley *a* on to a tightening pulley *b* on the shaft *B*, after it has passed around two pulleys on the shafts *C* and *D*. It is evident from the arrows that the two pulleys of the shafts *C* and *D* turn in opposite directions, and this would be true of the shafts themselves if the pulleys were keyed fast to them.

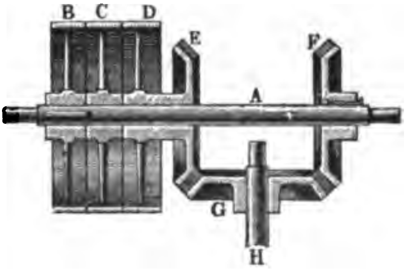
FIG. 672.



Now, to set the shafts in motion at pleasure, each of them is provided with two pulleys c_1, c_2 and d_1, d_2, of which c_1 and d_1 are loose and c_2 and d_2 fast. All these pulleys have equal diameters but the loose pulleys are twice as broad as the fast ones. When the belt is in the position shown in the figure it will move neither of the shafts, *C* or *D*, but when it is shifted to one side or the other it will either give to the shaft *C* its right-handed rotation or to *D* the opposite rotation. The shipping bar *E* sliding in the guides *F* carries two forks which surround the belts at the running-on points e_1 and e_2.

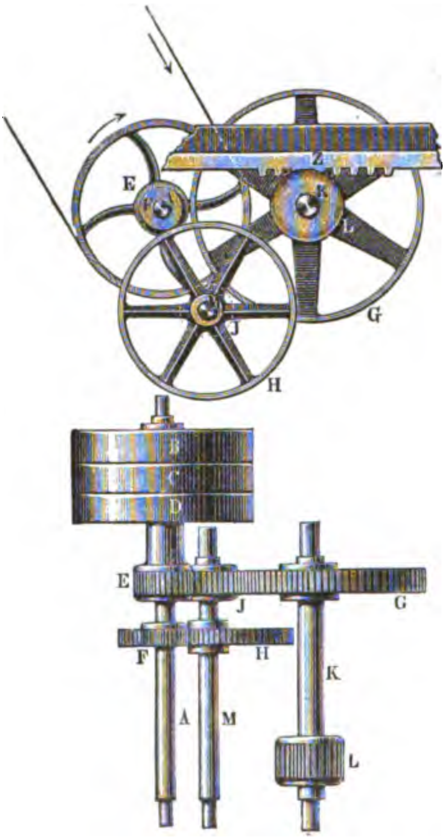
There are other reversing gears containing a single belt which at the same time use toothed gears. Two common mechanisms of this kind will now be described. In Fig. 673 there is a reversing mechanism with bevel gears in which the driving shaft *A* has three equal pulleys *B*, *C* and *D*; of these *B* is keyed fast, while *C* and *D* are loose. Here *D* is in one piece with the bevel wheel *E*, and the other bevel wheel, *F*, is rigidly attached to the shaft *A*. Both the driving wheels *E* and *F* gear into the wheel *G* on the driven shaft *H*. From this we see that *A* as well as *H* is at rest when the belt runs on the loose pulley *C*, but when the belt is shifted on to *B* the shaft *A* and its wheel *F* turn, the latter communicating motion in one direction to the wheel *G*. On the other hand, when the belt runs on the loose pulley *D* the wheel *E* will turn with *D* and rotate *G* in a direction opposite to the one just mentioned. In the latter case the shaft *A* does not remain at rest, for though it does not directly turn with its loose pulley *D*, the turning of *H* and the gearing of *G* and *F* cause the latter wheel and with it the shaft *A* to turn, and indeed in a direction opposite to the direction of *E*. The same holds for the wheel *E* when the belt runs on the pulley *B*, and in general we may say that with every motion of this mechanism the wheels *E* and *F* turn with equal velocity in opposite directions.

FIG. 673.



If the driving wheels *E* and *F* are of different size, the shaft *H* must be provided with two, different, corresponding, gears, and then the number of revolutions of *H* can be different in the opposite directions, provided the velocity ratios of the two pairs of wheels are suitably chosen. Such an arrangement has been occasionally used in ***planers*** to effect a quick-return motion of the table; for this purpose, however, the arrangement shown in Fig. 674 is in much more common use. Here only one belt is needed, and it is shifted to any one of the pulleys *B*, *C* or *D*, as may be desired. Of these pulleys, *B* is fast to the shaft, and *C* and *D* turn loosely on it; moreover, the prolonged nave of *D* is united with the spur wheel *E* which gears with the larger spur wheel *G* on the shaft *K*. This shaft turns the pinion *L* which transmits motion to the rack *Z* of the planer table, thus sliding the latter along the guides. Therefore when the belt runs on *D*, the shaft *K* will receive a left-handed rotation directly through *E* and *G*. This transmission is employed for the quick return of the table from right to left. To move the table more slowly in the opposite direction, the belt is shifted on to the pulley *B*, which communicates rotation to the shaft *A* and its toothed wheel *F*. The wheel *F* now gears into a larger wheel *H* on the intermediate shaft *M*, causing this shaft to rotate more slowly than *A* and in an opposite direction. A driving-wheel *J* on the shaft *M* gears with the wheel *G* and thus communicates to

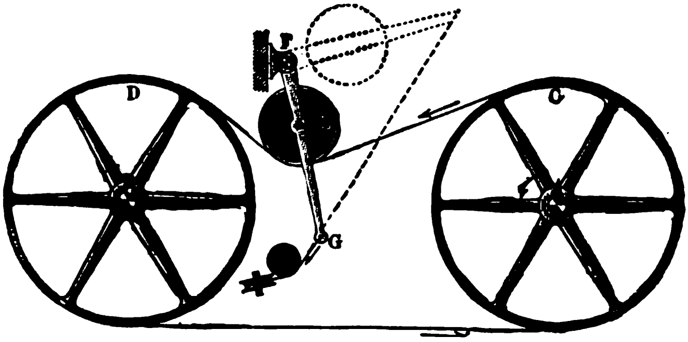
FIG. 674.



the shaft *K* a slow right-handed rotation. The ratio of velocities for the forward and return stroke of the table is evidently given by $\frac{F}{H}\frac{J}{G} : \frac{E}{G}$ when these letters represent the radii of the wheels as well as the wheels themselves. Here also the pulleys *B* and *D* move simultaneously in opposite directions.

Belts cannot be shifted with upright shafts, for instance with the vertical spindles in grist mills, for the belt's own weight tends to make it slide down the pulleys, and this tendency must be opposed by flanged pulleys. Moreover, when several belts are led off from the main upright shaft to separate spindles, there is not generally sufficient space to arrange two pulleys for each belt; it is customary therefore to effect the engagement and disengagement by means of tightening pulleys. When these act on the ordinarily slack belt they produce sufficient friction between the pulleys and the belt to transmit the motion. Fig. 675 shows this arrangement. We here see that

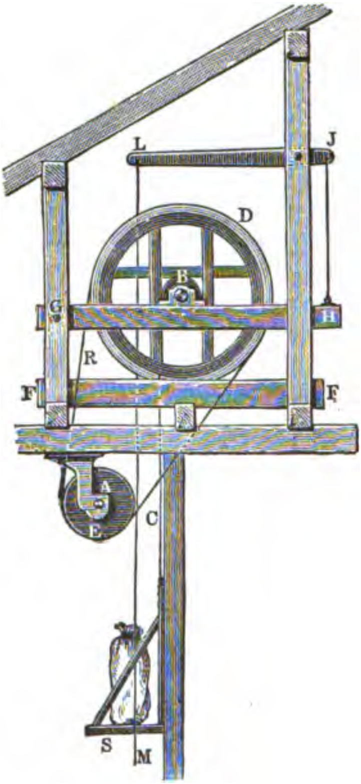
FIG. 675.



the tightening pulley is pressed against the ***driven*** or ***slack*** side of the belt. With this arrangement the force with which the tightening pulley is pressed against the belt is less and therefore the journal friction of the pulley is less than when the pulley is pressed against the ***tight*** side of the belt. The force needed for this tightening has been already discussed in § 56.

In a few cases the belt is tightened by increasing the distance between the shafts of the pulleys connected by the belt, the driven shaft being the one usually shifted. The best known example of this kind is found in the hoists of grist mills, Fig. 676. Here *B* is a drum located in the upper story of the building, and *C* a belt or rope from which the platform *S* is suspended, and which winds on to the drum when the latter has a right-handed rotation. This drum *B* can be driven by a belt connecting the pulley *E* on the driving shaft *A* with a larger pulley, *D*, on the hoisting shaft, provided the belt receives the necessary tension. In order that this tension may be communicated to the drum at any time, the single bearing of the hoisting shaft is fastened to a beam *G H* pivoted at *G*, and by means of the two-armed lever *L J* the bearing of *B* can be lifted by simply pulling on the cord *L M* which extends to the lower story of the mill. Thus the usually slack belt *R* is tightened and the hoisting drum *B* is turned as long as there is a pull on the rope *M*. When the pull ceases, all motion ends and the platform *S* does not sink because the friction between two lateral brakes *F F* prevents a reversing of the pulley *D*. Not until a moderate pull on the rope *M* has lifted the hoisting drum sufficiently to remove the friction between pulley and brake, without, however, tightening the driving belt *R*, does the load sink by its own weight with a velocity regulated by the friction brake *F F*.

Fig. 676.



§ **171. Click Gear.**—In all the arrangements hitherto discussed, disengagement was effected by removing the driving force. But, as stated at the beginning of the chapter, there is another way which consists in opposing the motion of the piece that is to be brought to rest by resistance which is too great to be overcome by the driving force. Such an arrangement we will call *click gear* or *catch gear*.* An example of a very common form of this gear is shown in Fig. 677. Here the rotating shaft *A* carries a click wheel *B* provided with wedged-shaped teeth between which the *pawl* or *click C D* can enter. When the pawl is lifted the wheel *B* is free to rotate in both directions, but when the pawl falls, either by its own weight or by the action of a spring, so that its end comes within range of the teeth of the click wheel, the motion of the wheel in the direction of the arrow will *suddenly* cease. This sudden stoppage of *A* is of course always accompanied by a shock which strains the pawl and the tooth against which it strikes, and from this alone we may conclude that this means of stoppage is less applicable the greater the living force of the shaft. For example, if the heavy fly wheel of a powerful steam engine were to be thus suddenly stopped it would certainly break the stop-gear. Click gear is therefore mostly used with small velocities, for practically living forces cannot be absorbed *instantaneously*, but must be expended by performing work on brakes or other stopping gear for more or less time.

FIG. 677.

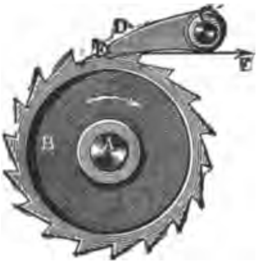
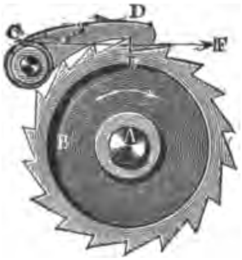


FIG. 678.



---

* This is better known as ratchet gear, but as we wish to distinguish between the two cases, one in which the pawl or click acts to prevent motion, and another in which it is used to drive the wheel or rack, we will follow Professor Kenedy in calling the first kind click gear, and reserving the common name of ratchet gear for the second and more important kind.

From the shape of the pawl and of the teeth we see that the former only prevents the rotation of the wheel in the direction of the arrow, and that the wheel is always free to rotate in the other direction, even when the pawl is not lifted out of contact. For instance, if the click wheel has a left-handed rotation, the inclined surfaces of the teeth will act as cams on the pawl, lifting it at each tooth.

The same remarks apply to the pawl shown in Fig. 678, which differs from that in Fig. 677 principally in that it is subject to tension instead of pressure, and that it must be turned in an opposite direction to be lifted out of contact.

In order that either pawl may lock the wheel a simple condition must be fulfilled by the position and form of the surfaces in contact. Neglecting for the present the sliding friction between the teeth and the pawl, we see that the direction in which these parts act upon each other must be normal to the common surface of contact. Let E be the point of contact between the pawl and a tooth, and $E\ F$ the normal to the surface of contact. Then will the wheel act on the pawl with a certain force by virtue of its tendency to rotate in the direction $E\ F$, and we see from the figures that locking will only take place when the two shafts A and C are upon *opposite* sides of the line of pressure in Fig. 677 and on the *same* side of this line in Fig. 678. If we connect the centre C of the pawl, Fig. 679, with the point of contact E between tooth and pawl, the line $E\ F$ perpendicular to $E\ C$ will be the direction of the tangent to the surfaces in contact for which locking is still possible whether the pawl is subject to tension or compression. But as sliding friction occurs between tooth and pawl when the latter is lifted, we may suppose the surfaces in contact to act on each other in a direction deviating from the normal by the angle of friction ρ. Consequently an automatic lifting of the pawls will not occur when the tangent to the surfaces of contact has the direction $E\ G$ in the pressure click and the direction $E\ H$ in the tension click, provided the deviations $G\ E\ F$ and $H\ E\ F$ do not exceed the angle of friction ρ. Therefore if the wheel has a notch $J\ K\ L\ M$ whose sides are parallel to the limiting directions $G\ E$ and $H\ E$, the pawl fitting this click will lock the wheel B in both directions. This may be called a fast click-gear, while the single-acting train shown in Fig. 677 and Fig. 678 may be called a free click-gear. It is evident that the fast click-gear, Fig. 679, maintains its character as such even when the angle formed by the sides JK and LM is taken less than 2ρ; it is usual in practice to make these flanks parallel, as at N, but as the angle of inclination is taken greater than 2ρ, as for example at O, the mechanism would not lock perfectly in either direction. Such imperfect locking gear is nevertheless sometimes used, for instance to secure nuts, the nut B of a bolt A, Fig. 680, being provided with a scalloped-edge flange D into which the spring pawl C enters. This arrangement always permits the nut to be turned by a wrench, and at the same time sufficiently resists the tendency of the nut to loosen.

Fig. 679.

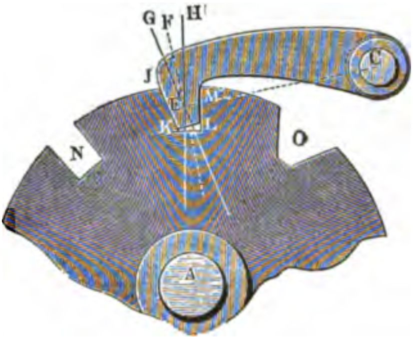
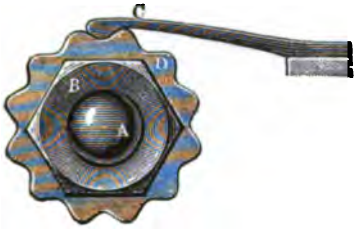


Fig. 680.



It may be required of a stop-gear that the pressure of a wheel shall press the pawl well into the notch. In this case, Fig. 681, the form of the teeth at DH must be so chosen that this straight line will deviate from the line FE perpendicular to CE by at least the angle of friction ρ. But such devices do not often occur in ordinary machinery, because the lifting out of the pawl is thereby rendered more difficult, for as the point D describes an arc DD' concentric to C, the lifting out of the pawl can only be accomplished by turning back the wheel through the arc HD'. This device for making the lifting out of the pawl more difficult is used in ***gun locks***, the ***cocking*** being effected by means of an ***undercut*** notch in the ***tumbler*** out of which the pawl cannot be lifted by the ordinary force of the trigger.

The ordinary single-acting click-gear is principally used in hoisting apparatus, the click wheel being attached to the drum and the pawl to the frame, thus permitting the drum to turn and wind up the rope, but preventing the opposite rotation which would be caused by the sinking of the load when the driving force ceased. This device is also employed for winding clocks without stopping them. If we suppose the shaft A,

FIG. 681.

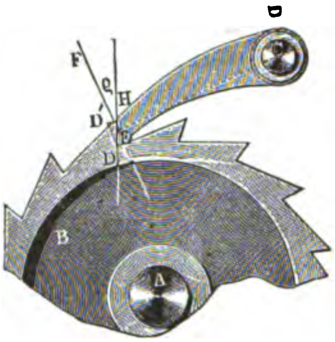


FIG. 682.

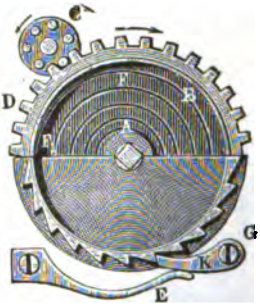
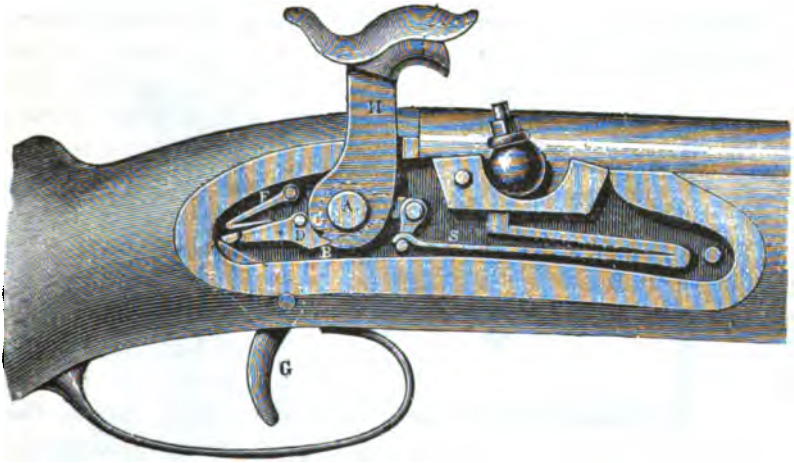


Fig. 682, to be attached to one end of a spiral-shaped spring F whose other end, F_1, is attached to the spring casing B turning loosely about A, the wound-up spring, when the shaft A is held fast, turns the spring-casing to the right, communicating motion through the spur wheel D to the pinion C and the rest of the clockwork. Now the shaft A is rendered stationary by a click wheel attached to it into which a pawl K enters, pivoted at G to the frame of the clock, the spring E keeping the pawl in contact with the wheel. This permits the winding up of the spring F at all times by simply turning the shaft A to the right, the action of the spring on the driving pinion continuing during the winding process.

The application of single-acting click-gear is shown in Fig. 683. Here H is the ***hammer*** which turns with its shaft or ***tumbler*** A, and two notches B and C on the tumbler permit the nose of a pawl or *sear* D to enter, the little sear-spring F being provided for this purpose. The figure shows how the trigger G can free the hammer, and how the main spring S acts to turn the freed hammer. As regards the two *cock-notches*, it has been remarked that B is formed for half cock, so that pressure on the trigger will not free the hammer.

If the click wheel gradually increases, it finally becomes a toothed rod into whose teeth the pawl enters just as it did into the wheel. These click-racks are occasionally used, but principally in such a way that they are stationary, their teeth preventing the machine pieces carrying the pawls from moving in a certain direction. For example, on inclines a click-rack is placed between the rails, and the car is provided with a pawl which prevents any unintentional rolling back of the car. In *gunpowder-hammers* the guides have a click rack whose pawl is attached to the hammer. When the latter is thrown upward by the explosion of the cartridge upon which it has fallen, the teeth of the rack are arranged to permit this upward motion, but catch the pawl as soon as the hammer begins to fall, and hold it till the pawl is released by a cord.

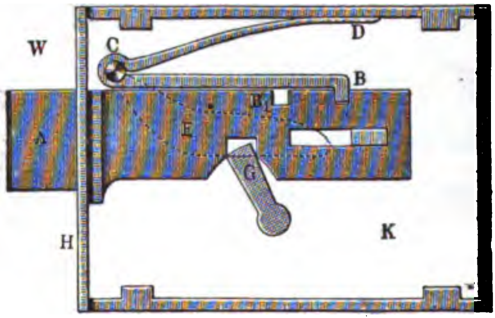
FIG. 683.



The double-acting or fast click-gear is also much used in practice; for example, to hold fast the levers in reversing gear, for dividing engines, ringing apparatus, etc. Most of the locks used in doors, drawers, etc., may be regarded as double-acting click-gear. This gear plays a particularly important part in safety locks, which in general may be characterized as lock-gear where the pawl is represented by the bolt, and where the freeing of the pawl, *i.e.*, the shooting back of the bolt, requires that the tumblers or additional locking arrangements opposing the motion of the bolt be first released. The releasing of the pawl is accomplished by the key, the safety of the lock depending on the difficulty of freeing the tumblers without the proper key.

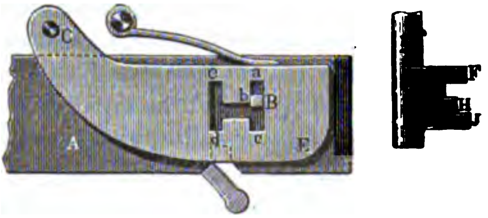
The arrangement of a common lock is shown in Fig. 684. Attached to the door is a metallic casing *K* through whose side *H* the bolt *A* passes and enters a corresponding recess in the wall *W*. The bolt is kept in this position by the tumbler *B C*, whose tooth or stump is forced into the notch *B* of the bolt by the spring *C D*. To shove back the bolt it is first necessary to lift the tumbler; to do this a part of the key-bit *G* presses against the piece *E* connected with the tumbler, and after the tumbler is lifted another part of *G* enters the notch of the bolt, shooting back the latter. The second notch, B_1, of the bolt receives the stump of the tumbler when the bolt is shoved back, thus preventing any unintentional sliding.

FIG. 684.



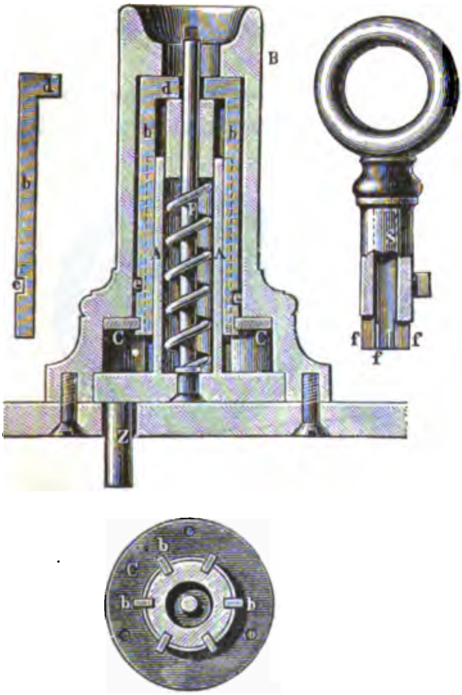
The many forms of key-bit and key-hole are simply a means for rendering the admission of lock-picking tools as difficult as possible. For this purpose many different arrangements have been applied, but the best safety locks employ *several* tumblers all of which must be lifted, and lifted certain definite amounts so that *too small* or *too great* a lift of any one of the tumblers prevents the sliding of the bolt. In the *Chubb lock*, for example, the tumblers are formed like *CE*, Fig. 685, each tumbler having an *H*-shaped slot *a c d e*, into which enters a stud *B* attached to the bolt *A*. It is clear that, in order that the bolt *A* may move, the tumbler must be lifted to exactly the height at which the stud *B* is opposite the horizontal slot in the tumbler, and that this tumbler must have a particular length of key-bit. Now if three, four or more such tumblers are thus applied, all turning about the common pin *C* and receiving the same stud *B*, then if the lift of the separate tumblers is different, a key-bit is needed of different lengths at different places. Such a key with stepped bit is shown at *S*, and is applicable to a lock with three tumblers which are lifted by the steps *G*, *H* and *J*, the spur *F* serving to slide the bolt.

Fig. 685.



In *Bramah's* lock, Fig. 686, a key turns the cylindrical sleeve *A* as soon as its locking arrangements are withdrawn. These locking arrangements are five or six radial plates *b* which slide in grooves of the sleeve *A*, and are kept upright by the

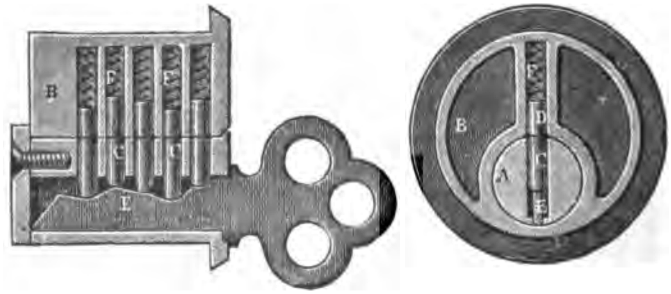
Fig. 686.



spiral spring *F*. As these plates project from the sleeve *A* and each enters into a notch of the fixed annular plate *C*, a turning of the sleeve *A* is ordinarily impossible. But if we suppose each plate *b* to have a notch *e*, and suppose all the plates to be simultaneously moved downward so that the notches *e* come opposite to the plate *C*, the latter will no longer prevent the turning of the sleeve, and by this turning sleeve-tooth *Z* shoves back the bolt. Now if the notches *e* are so placed in the tumblers that the sliding of the latter must be unequal, the key *S* must evidently have as many acting surfaces or steps as there are tumblers, these surfaces being located at different heights, according to the amount of sliding needed by each tumbler. For this purpose the key has the form of a tube fitting the sleeve *B*, and its wall is provided with notches *f* for the tumblers. These notches enclose the upper ends *d* of the tumblers *b*, and the latter are moved unequally when the key is pressed down, if the notches *f* are of unequal lengths.

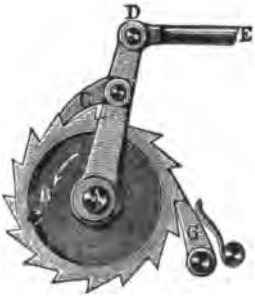
Yale's lock, Fig. 687, is distinguished for its simplicity and safety. Here the cylindrical box *A*, situated in the lock-box *B*, is prevented from turning by a number of spring pins *D* and tumbler pins *C* which enter *A* to different depths. By introducing a flat key *E* with suitable steps which pass under the pins *C*, these and the pins *D* are lifted so that their ends coincide with the periphery of the cylinder *A* and allow its rotation in the box *B*, during which rotation the bolt is shoved back.

FIG. 687.



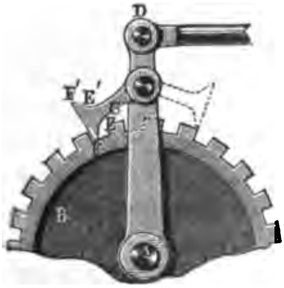
§ **172. Ratchet Gear.**—The various kinds of lock-gear discussed above serve to prevent motion of the click-wheel or rack; but these mechanisms can also be employed to impart motion. Suppose a pawl *C*, Fig. 688, to be pivoted to a lever *A D* turning loosely on the shaft *A* of the click wheel, and that the lever receives regular oscillations, say from an eccentric rod *D E*; it is evident that the wheel will thus receive an intermittent rotation in the direction of the arrow, for the form of the click teeth is such that there is an automatic lifting of the pawl *C* when the lever *A D* turns in a direction opposite to the arrow. Such an arrangement is called a ***ratchet gear***, and is employed wherever an intermittent motion is desired. An application of this gear to the feed motion of a saw has been already mentioned in § 156, Fig. 589, the feed motion in this case being communicated to the work. In a similar manner ratchet gear is extensively employed in planers and other machines to communicate a feed motion to the tool. In order that the friction of the pawl on the teeth may not cause the wheel to turn back with the lever *A D*, it is customary to add another pawl *G* to the ratchet gear.

FIG. 688.



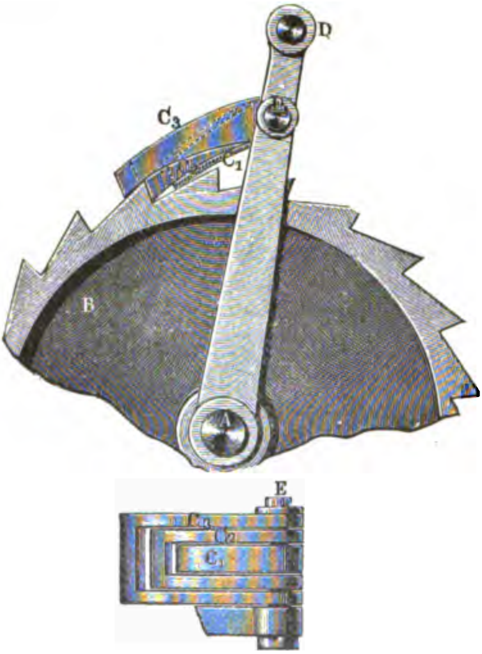
It is clear that the arrangement in Fig. 688 will allow the wheel to move in but one direction, and to move the wheel in the opposite direction it will be necessary to give the teeth an opposite inclination. But it can be arranged so that a wheel and a pawl will suffice to communicate intermittent motion in either direction as desired. Such an arrangement is shown in Fig. 689. Here the notches of the wheel are radial as in the fast click-gear, and the pawl is so formed that, instead of using the point *F*, we may employ the point *F′* by swinging the pawl over into the dotted position. That there may be an automatic lifting of the pawl on the return motion, the backs *E* and *E′* of the two pallets *F* and *F′* are inclined or rounded.

FIG. 689.



The motion communicated to a wheel by a ratchet gear at each stroke can be varied by varying the angle of oscillation of the lever, but the motion will always depend on the pitch of the teeth, for the wheel can only be turned through a whole number of pitches during each stroke of the pawl. But by arranging several pawls on the same lever it becomes possible to impart to the wheel at each stroke an angular motion equal to a fraction of the pitch angle. In Fig. 690 the lever *A D* has three pawls, C_1, C_2 and C_3, which turn on the same pin *E* and are so formed that C_1 is enclosed by C_2, and C_2 by C_3. These three pawls move independently of each other on the pin *E* and are of different lengths, C_2 being one third and C_3 two thirds of a pitch longer than C_1. It is evident from this that oscillations of the lever corresponding to a third of a pitch bring each pawl successively into action, each causing the wheel to turn through a third of the pitch angle. This will still be the case if the motion of the lever is somewhat greater than a third of the pitch angle. However, if the lever swings through two thirds of the pitch angle, all three of the pawls will come successively into action, but the shifting will then be two thirds of the pitch during the action of each pawl. If the lever were always to swing through a whole pitch, only one pawl would come into action. Of course the retaining gear which prevents an unintentional

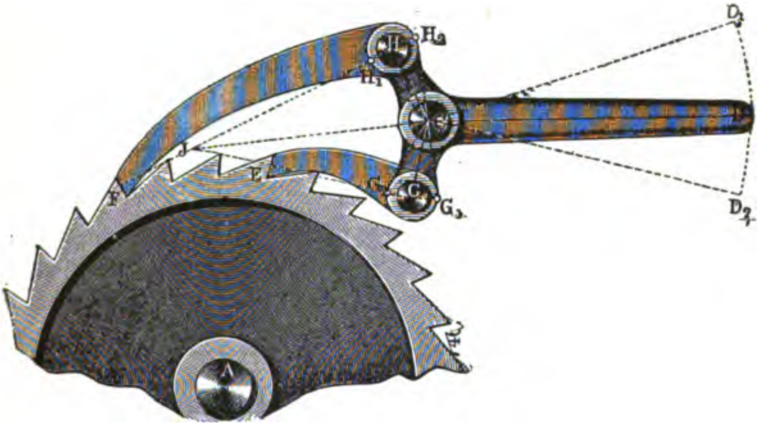
FIG. 690.



return of the ratchet wheel must be provided with three different pawls. This mechanism is used where the wheel has small rotations without employing a correspondingly fine pitch.

The retaining gear of a ratchet train can be made a driving pawl by giving it an independent motion. In this way we obtain two sets of ratchet gear which act alternately and render a special retaining gear unnecessary. The arrangement of such a double-acting ratchet-train is shown in Fig. 691. The two

Fig. 691.



pawls *G E* and *H F* no longer receive their motion from a lever swinging about the shaft *A* of the wheel, but from a bell-crank lever *H G D* turning about *C* and moved by, say, the handle *D*. If we suppose this lever to be moved from the position $D_1 G_1 H_1$ to $D_2 G_2 H_2$, the pawl *G E* will push the tooth *E* of the wheel through an arc $G_1 G_2$ corresponding to a pitch *t* of a tooth, while the pawl *H F* will be drawn back an equal distance. Here a portion of the circumference equal to twice its motion (2 *t*) slips under the pawl *H F*, on account of the opposite motion of pawl and wheel. On the return swing of the lever from D_2 to D_1, the pawl *H F* pushes the wheel forward a distance *t* while the returning pawl *G E* slips over two teeth. It follows that a double stroke of the lever *D C* advances the wheel by two pitches. If it were desired to turn the wheel through but one pitch during a complete oscillation, as in a single-acting ratchet-train, it would be necessary to proportion the mechanism so that each pawl would move the wheel through $\frac{1}{2}t$.

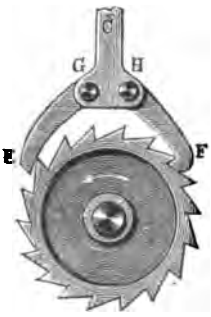
Such a ratchet train is easily arranged. If *E* and *F* are the positions of two teeth at which the pawls act when the lever is in its middle position, it is only necessary to draw tangents to the pitch circles at these points, to obtain the directions *E G* and *F H* for the stroke of the pawls. Therefore if we bisect the angle *G J H* formed by these directions, we shall find, somewhere on the bisector, the axis *C* of the lever, the point being so chosen that the perpendiculars $C G = C H$ have lengths each equal to the arm *l* of the lever. This length *l* must be determined with reference to the angle of oscillation λ allowed to the lever. For this we have approximately

$$l \lambda = r \tau,$$

where *r* is the radius of the ratchet wheel and τ the angle through which it is to be turned by each pawl. It is evident that this mechanism can be constructed with tension- instead of pressure-clicks or pawls.

Fig. 692 shows a double-acting ratchet-train with a push-click *G E* and a pull-click *H F* which act on opposite sides of the wheel. These clicks or pawls are not moved by an oscillating lever, but by a rod *C* having a reciprocating rectilinear motion. Here too the pawls act alternately on the forward and return stroke and turn the wheel in the direction of the arrow.

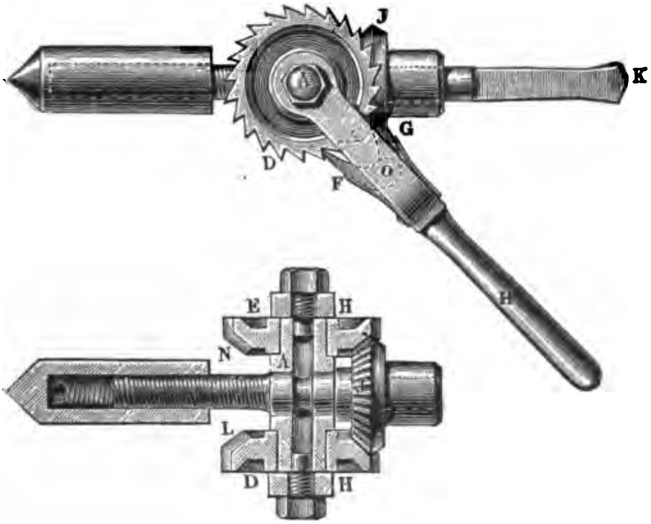
FIG. 692.



The mechanism shown in Fig. 693 is a *ratchet drill* used to effect the turning motion of a drill by the oscillating motion of a hand lever. The handle *H* swings about a pin *A* and, by means of the pawls *F* and *G*, communicates opposite motion alternately to two ratchet wheels *D* and *E* which turn loosely on *A*, and whose teeth have opposite inclinations. As each of these ratchet wheels has also the form of a bevel wheel, *L* or *N*, which gears with a third wheel *J*, we see that the repeated opposite rotations of the ratchet wheels will cause an intermittent rotation of the wheel *J*, but one having always the same direction, and therefore suitable for driving the wheel *K*.

All the click- and ratchet-gear hitherto discussed have the defect that the action of the pawl is accompanied by small shocks. For if we suppose the pawl of a ratchet gear on its return stroke to pass beyond its next point of action, this pawl, when its motion is reversed, must traverse a portion of its path before it reaches the tooth on which it is to act, and by the velocity thus obtained a shock occurs when it comes in contact with the tooth. This lost motion and the shock become greater as the pitch of the ratchet teeth becomes greater. To avoid this defect, mechanisms which work by jamming friction have been constructed which prevent lost motion by the friction between the pieces that tend to slip on each other. Such arrangements act as click-gear to prevent motion and as ratchet-gear to impart motion. All clamping gear is so arranged that the force acting to produce motion in the pieces to be locked calls forth a frictional resistance whose moment is greater than that of the moving force.

FIG. 693.

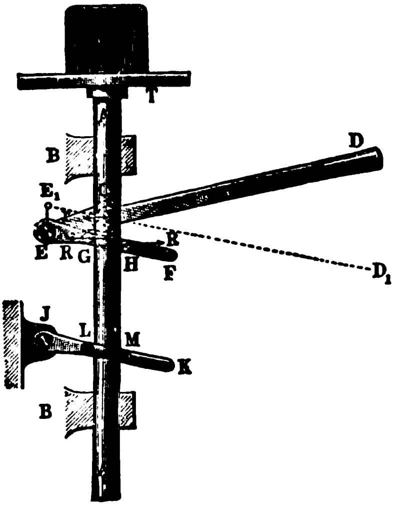


The action of this clamping gear is best explained by the ratchet motion invented by *Saladin*, Fig. 694; it has been applied in simple hoisting gear. Here a cylindrical rod A carries a plate T to support the load Q, and is guided vertically at $B\,B$. The lever $D\,E$ turning about the fixed fulcrum C lifts the rod through the distance $E\,E_1$, when the lever is pressed down into the position $E_1\,D_1$. The connection of the lever with the rod A is effected by a bar $E\,F$ which is jointed at E to $D\,E$ and provided with an oval eye $G\;H$ through which the rod A passes. With this arrangement the rod A and its load Q will follow the motion of the lever end E, for the eye $G\,H$ cannot slip on A during this motion if the proportions have been correctly chosen. Thus the eye, on account of its oval form and its weight, presses against the rod at two points G and H whose vertical distance from each other is equal to a. At these points the rod and the eye act upon each other with certain normal pressures R. If we neglect the friction of the rod A in the guides B and suppose the lever to exert a vertical force Q at the eye E of the bar $E\,F$, this force Q will call forth the reactions R at G and H, which may be found from

$$Q\,r = R\,a, \quad \text{or} \quad R = Q\frac{r}{a},$$

where r represents the distance of the force Q from the axis of A.

Fig. 694.



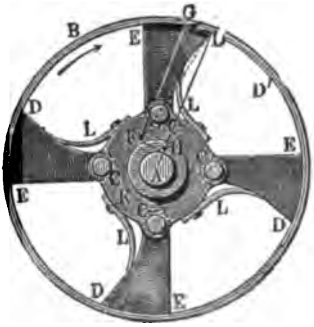
Now if we suppose the rod A to slide down under the influence of the load, it must overcome not only the frictional resistance of the guides, here neglected, but also the resistances ϕR at G and H, and consequently Q must then at least equal

$$Q = 2\,\phi R = 2\,\phi\,Q\frac{r}{a}.$$

So long as Q is smaller than $2\,\phi\, Q\frac{r}{a}$, or so long as $\frac{a}{r} < 2\,\phi$, no sliding downward of the rod can occur, hence the load must follow the upward motion of the lever end E. It is also clear that the second bar JK hinged at the fixed point J, by means of its eye LM opposes the sinking of the load Q in the same manner, so that it exerts a locking action on the rod when the lever moves from the position $E_1\,D_1$ to $E\,D$ to begin a new stroke. It follows that in order that the mechanism shall act, the condition $\frac{a}{r} < 2\,\phi$ must be fulfilled. For example, when the coefficient of friction $\phi = \frac{1}{12}$, the lever arm r must be at least six times as great as the distance a, a condition always satisfied by the form of the eyes $G\,H$ and $L\,M$. The resemblance between this clamping gear and that described in § 95, Fig. 342, is at once evident.

In a similar manner frictional ratchet-gear may be applied to rotary motions, for if we suppose the straight rod A bent in the form of a circular ring or rim, and give to the eye a corresponding form, we see that the oscillating motion communicated to the eye by a lever will communicate an intermittent rotation to the rim of the wheel. We will here examine only two of the many forms in which frictional ratchet-gear has been applied to wheels.

FIG. 695.



The clamping gear invented by *Dobo*, and shown in Fig. 695, has a pulley B smoothly turned on the inside and running loosely on the shaft A. The box K is keyed to A and provided with four pins C on which are placed the sector-shaped levers $C\,E\,D$. These levers are pressed by springs L against the inner circumference of the pulley so that they press with moderate force against the pulley. Now if the pulley B is turned, say, in the direction of the arrow, the slight friction between the sectors and the pulley will bend the springs back a little, but the force will not be sufficient to take along the shaft A. But if the pulley B is turned in the opposite direction, the friction $D\,G$ acting at D in the direction of the motion will act on the sector $C\,D$ as if it were a toggle joint $A\,C\,D$, and thus develop so considerable a pressure at D that the friction there will suffice to take along the shaft A. The conditions under which this occurs are determined as follows. If the pulley, when it tends to turn, communicates a radial pressure $N = D\,H$ to the sector, the friction caused at D will be $D\,G = \phi \,.\, D\,H$, where ϕ is the coefficient of friction. This tangential force combined with the normal pressure $D\,H$ gives a resultant force $D\,F$ which forms with the normal or radius $D\,A$ an angle $A\,D\,F$ equal to the angle of friction; in other words,

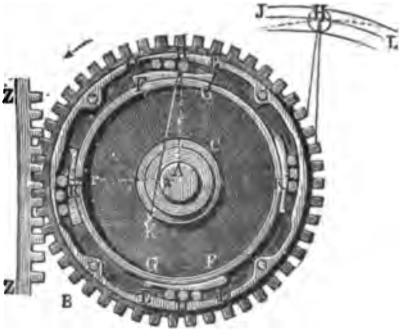
$$tan\ A\,D\,F = \frac{D\,G}{D\,H} = \phi.$$

Now if this direction of the force passes between the shafts A and C it will cause a separation of the surfaces at D; but if, as in the figure, it passes through or beyond C so that A and C lie on the same side of the resultant, this force must tend to turn the sector so that the pressure N and the friction at D become continually greater. This proves that there is clamping action, for here the condition is satisfied that the ray drawn from the hinge C to the extreme point D of the sector makes an angle with the radius $D\,A$ which is smaller than the angle of friction. It is not difficult to see that by widening the sectors, for example, by transferring the extreme point from D to D', the clamping property can be destroyed.

A more effective frictional ratchet-gear much used in *Langen & Otto's* gas engine is shown in Fig. 696. The toothed rim B turns loosely about the shaft A, the rim gearing with the reciprocating toothed piston rod Z of the gas engine. The coupling between the shaft A and the piston rod is so arranged that when the rod descends and the wheel turns in the direction of the arrow, the shaft turns too; but when the piston rises, the toothed rim has the opposite rotation, without disturbing the left-handed rotation of the fly-wheel shaft A.

To accomplish this the smoothly turned pulley *C* is keyed to *A* and the inner surface of the toothed rim *B* is given, at various places, a spiral-shaped profile *D H E*. Corresponding to each of these spirals there is a bent wedge *F G*, which on the inside fits the pulley *C* and on the outside is parallel to the spiral-shaped profile *D H E* of the rim. Between each pair of surfaces *D E* and *F G*, little steel rollers *K* are placed. The action of this arrangement is as follows: while the shaft *A* and pulley *C* turn continuously in the direction of the arrow, the up-stroke of the piston rod *Z* turns the toothed rim to the right, but exerts no special pressure on the rollers *K*, for during this motion the two oblique surfaces *D E* and *F G* are shifted so as to increase their distance from each other. On the other hand, when the piston is in its highest position it is pressed downward by the atmospheric pressure and the toothed rim *C* is turned in the same direction as the pulley *C* and, at first, with a greater velocity. This causes the surfaces *D E* to shift over *F G*, thus diminishing the distance between them and communicating to the rollers *K* a pressure which is transmitted through the wedge *F G* to the pulley *C*. This causes friction at the circumference of the pulley, and the moment of this friction must be greater than the moment of the driving force *P* at the toothed rim, in order that the piston force *P* may overcome the useful resistance on the shaft *A* instead of the friction at the pulley *C*.

FIG. 696.



To ascertain the condition that must be satisfied to accomplish this, let r be the radius of the pitch circle of the toothed wheel *B* and hence Pr the moment of the turning force exerted by the piston rod. Moreover, let r_1 be the radius of the smooth pulley *C*, $\mathfrak{r}$ the radius of a middle point *H* on the spiral-shaped centre line of the groove formed by the surfaces

$D\,E$ and $F\,G$, and γ the angle made by this middle spiral $J\,L$ at H with the circle described through H around A. The pressure K acting on the roller is then at a distance $a = \mathfrak{r} \sin \gamma$ from the axis A. Therefore to determine K we have the equation

$$P\,r = K\,a = K\,\mathfrak{r} \sin \gamma,$$

from which follows

$$K = P . \frac{r}{\mathfrak{r} \sin \gamma}.$$

This pressure K causes the wedge $F\,G$ to press the pulley C with a normal pressure

$$N = K \cos \gamma = \frac{P\,r}{\mathfrak{r} \tan \gamma}.$$

Consequently the moment of the friction at the pulley C is

$$\phi . N r_1 = \phi \frac{P\,r}{\mathfrak{r} \tan \gamma}\, r_1;$$

and in order that slipping may not occur, this moment must be greater than the moment $P\,r$ of the force. The condition to be satisfied is therefore

$$\phi \frac{r_1}{\mathfrak{r} \tan \gamma} > 1,$$

or

$$\frac{\mathfrak{r}}{r_1} \tan \gamma < \phi.$$

If we were to regard the slightly different radii $\mathfrak{r}$ and r_1, as equal, we should have the condition, that the deviation at any point of the spiral-shaped middle line of the roller groove from the circle through the point be less than the angle of friction.

We will now examine a few of the automatic disengaging gear or stop-motions, *i.e.*, arrangements which bring the moving piece to rest when an accident occurs. They are often employed in the manufacture of thread to stop the machine when a thread breaks. As the thread or filament can sustain but a slight tension, the machine pieces must, of course, be as light and sensitive as possible.

One of the simplest arrangements of this kind is the stop-motion, Fig. 697, occurring in ***silk-doubling machines.*** The object of the doubling frame is to wind three, four, or more filaments regularly on a spool *C*, the filaments being drawn from the spools *A* for this purpose. The spool *C* receives motion from the shaft *E* by means of the friction wheel *F*, the smooth pulley *G* resting on the circumference of *F*. The silk filaments are led over glass rods *O*, and when one of them breaks, the spool *C* must be brought to rest to avoid inequalities in the product. For this purpose each filament *B* is drawn through an eye *H* of a wire *J H* turning about *J*, which wire is carried by the filament. When the filament breaks, this wire falls and is caught by another wire *M L* which can turn easily about the pin *K* and is brought into the dotted position by the falling wire *H*. In this position the end *M* butts against a tooth on the ratchet wheel connected with the spool *C* and thus stops the spool. The driving shaft *E* which turns a great number of spools is not stopped by this, for the pulley *F* turns past *G*, though in contact with it. Stop-motions form an important part of ***looms***, two kinds being generally employed with each loom, one of which stops the loom when the shuttle sticks in its path, while the other acts when the shuttle-thread breaks or runs out.

Fig. 697.

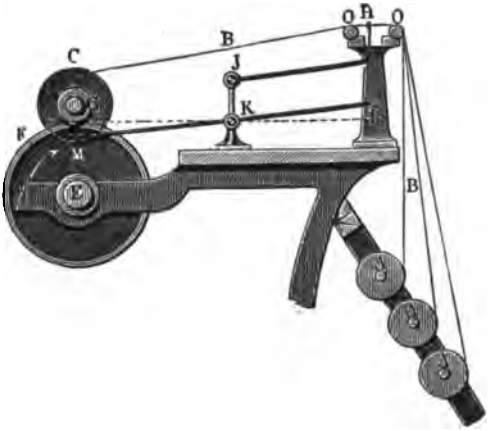
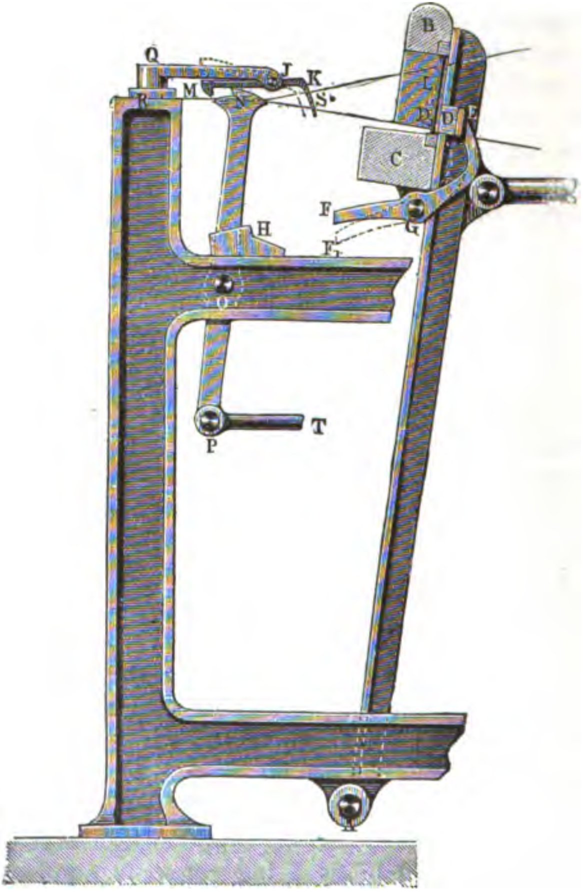


Fig. 698 represents the *lay* swinging about the fixed shaft *A*, the upper part of the lay between the *lay-cap B* and the *lay-*

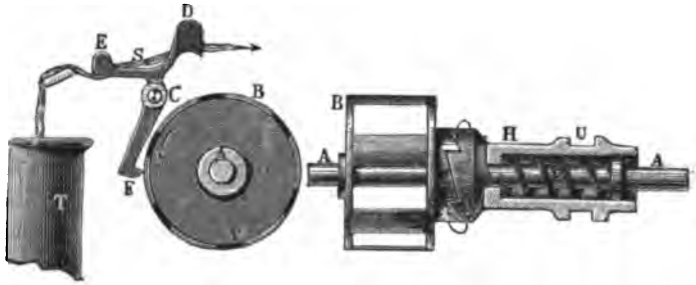
beam C containing, on each side of the web, a shuttle box for receiving the shuttle at the end of its fly. The back *D* of this box is hinged vertically like a door so that it can be pressed outward as shown in the figure at *D*. This is pressed open from D_1 to *D* by the advancing shuttle, consequently the bell-crank lever *E G F* which turns about the fixed pin *G* on the lay is so placed that the end of the lever *F* can pass freely over the projection *H* when the lay swings forward. But if the shuttle has not completely entered the box the clack *D* will remain in the position D_1, and the lever *G F* will remain in its

FIG. 698.



lower position F_1 and will strike against the projection *H*, pushing it so that it will shift the driving belt from the fixed to the loose pulley. The second disengaging gear comes into action when the shuttle thread gives out, and is constituted as follows: A sliding bar *R* is employed to shift the belt; on this bar is an arm *Q J* whose pin *J* carries an easily moved lever *MK* which has a hook at *M* and a fork at *K*, the fork being capable of passing through a light grating *L* in the lay, when the latter swings to the left. Now if the shuttle thread *S* has been correctly laid by the shuttle, the advancing grating *L* will press the thread against the tines of the fork *K* and cause the light lever *MK* to turn about its fulcrum *J*, thus lifting the little hook *M*. Now *N O P* is a lever oscillated about the fulcrum *O* by the rod *P T* which is driven by a cam. The upper end of this lever is also provided with a hook *N*, and we see that the oscillations of *N* will not act on the lever *M J K* when the little hook *M* is lifted; but when *M* is not lifted, *i.e.*, when the shuttle thread is not properly laid, the hook *N* will take hold of *M* and move it to the left, which motion is communicated to the bar *R* through the arm *Q J*, thus moving the belt shifter.

FIG. 699.

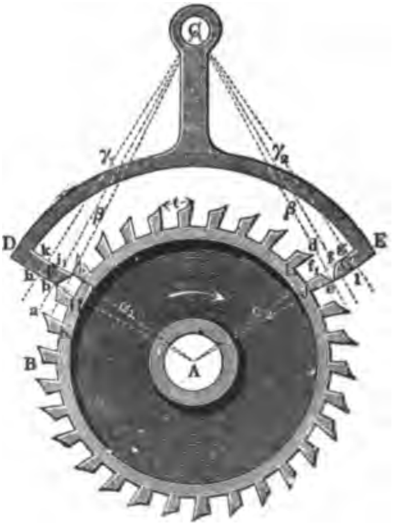


Another stop-motion shown in Fig. 699 is employed in *drawing frames* and effects a disengagement whenever the cotton sliver *S* breaks. To accomplish this the sliver is led from a vessel *T* to the drawing rollers over a light lever *C D F* which is held in such a position by the slight tension of the sliver that the hook *F* is held away from the rotating drum *B*. But when the sliver breaks, the hook *F* by its weight passes below one of the bars *V* of the drum and prevents it from turning. This drum turns loosely on the continually rotating shaft *A* and receives its motion from the friction between the oblique teeth of the hub *J* and the sliding sleeve *H* which is ordinarily pressed against *J* by the spiral spring *K*. It is therefore clear that when the drum *B* is held fast and the sleeve *H* continues to turn, the oblique teeth will cause the sleeve to slide along *A* and will act on the disengaging fork which surrounds the groove *U*, and thus shift the driving belt. All such stop-motions are so arranged that the stopping is effected, not by the weak sliver itself, but by a special moving piece brought into action by the breaking of the sliver.

§ **173. Escapements.**—The escapements of clocks and other time-pieces belong to that class of disengaging gear which acts by the insertion of a resistance. The hand of a clock would not receive uniform motion if the force of the weight or spring were allowed to act without interruption on the mechanism moving the hand, for the force would then become an accelerated one and therefore unsuitable for measuring time exactly. Instead of this the motion of the whole mechanism is interrupted at regular intervals and again resumed as regularly, so that the motion becomes an intermittent one, the periods of rest and motion being very small. Now although the motion during such a short interval of time cannot, strictly speaking, be uniform, yet the total action of the mechanism approaches uniform motion more closely the greater the regularity with which the engagement and disengagement are effected. The *regulator* which interrupts the motion at regular intervals in stationary clocks is the *pendulum*, while in portable clocks it is the balance spring. The *escapement* of a clock is a device intervening between the driving train and the regulator and converts a rotary motion into an oscillating isochronous movement. It is acted upon by both driving train and regulator. The train imparts through the escapement an impulse to the pendulum or balance spring sufficient to overcome the friction of the latter and the resistance of the atmosphere, and thus keeps up the vibrations. On the other hand, the pendulum or balance spring acts through the escapement to render the motion of the train intermittent. Each escapement is therefore to be regarded as a click-gear in which the pendulum or balance wheel acts as a pawl. Many different escapements have been constructed, but we will only discuss the principal ones in a general manner, referring the reader to special works on clock-making for details.

The escapement most frequently used in clocks with pendulums is that invented by *Graham*, known as the *dead-beat* (*anchor*) *escapement*. Here the wheel *B*, Fig. 700, is the click-wheel, better known as the escape wheel. The weight of the clock tends to turn this wheel continuously in the direction of the arrow, but by means of the escapement this rotation is regularly interrupted and renewed. The anchor *CDE* swinging about the fixed arbor *C* acts as pawl; it is so connected with the pendulum that it takes part in the oscillations of the latter. The pendulum, to be sure, does not swing about the same shaft *C*, being attached to a spring above *C*, but for the purpose of explaining the escapement we may suppose *C* to be the common axis of oscillation of both pendulum and anchor. The figure shows the form of the two hooks or pallets *DF* and *EG* attached to the arms *D* and *E* of the anchor. The ends of these pallets are bounded by oblique surfaces *bi* and *ef*, while their sides *bh*, *ik*, *el*, and *fg* are concentric to the axis of oscillation *C*. In the position of the pendulum and anchor, shown in the figure, the tooth *H* of the escape-wheel presses with its extreme edge *b* against the oblique surface *bi* of the left pallet, while the print *e* of the right pallet is just in contact with the point of the tooth *J*. If we suppose the clock to be started by swing-

FIG. 700.



ing the pendulum a little to the left, the point b of the tooth H will slide along the surface $b\,i$ of the pallet, for the pull of the escape-wheel will tend to lift the anchor and the pendulum till the point i of the pallet has reached i_1, a point on the circle described through b. At this instant the left pallet lets go of the tooth H, and the wheel would turn to the right if the point d of the tooth L and the point f of the other pallet $E\,G$ (which has moved to the left with the pendulum) did not at the same instant meet at f_1. As a consequence the point f of the pallet $E\,G$ now checks the escape-wheel. During this procedure the anchor has evidently moved through the angle $i\,C\,i_1 = f\,C\,f_1 = \beta$, while the wheel has turned through the angle $b\,A\,i = \alpha_1$.

When the anchor reaches the new position, the pendulum will not generally have completed its motion to the left; now suppose it swings still further to the left through the angle $\gamma_2 = f\,C\,g$, then during this motion the escape-wheel will be stationary, for the surface $f\,g$ of the right pallet is concentric to C and will slide past the point d without exerting any other action than friction on this tooth. We will here neglect this friction. Now if the pendulum has completed its swing to the left and the return swing begins, the cylindrical surface $g\,f$ will slip along the point d of the tooth with an outward motion during which time the escape wheel will again be stationary. But at the instant when d again touches the point f, the inclined surface $f\,e$ will permit the point d to slip past, the escape-wheel in this way again imparting an impulse to the anchor which swings it to the right; this effort is continued till the point d comes in contact with the point e of the pallet. The escape-wheel B during this return motion of the pendulum has turned through the angle $f_1\,A\,e = \alpha_2$, while the anchor during the return swing has described the sum of the angles

$$g\,C\,f + f\,C\,f_1 = \gamma_2 + \beta.$$

The escape-wheel cannot continue its rotation, for in the mean time the point b of the left pallet has again entered the circle B, and the point a of the following tooth K presses against b at the very instant when the point d of the tooth L leaves the right pallet at e. The anchor now swings still further to the right, the wheel remaining stationary, for the surface $b\,h$ concentric to C now slides past the point a and back again. The whole system has now reached its initial position and the whole procedure is repeated. During this operation the wheel has turned through a pitch angle $a\,A\,b = \tau$, first through the angle $\alpha_1 = b\,A\,i$ and, after a pause, through the angle $\alpha_2 = f_1\,A\,e$; after which the wheel rests for a certain time. While the wheel was moving through the angles α_1 and α_2, the anchor swung each way through an angle $\beta = i\,C\,i_1$ or $f_1\,C\,f$. The two periods of rest of the wheel lasted while the anchor was describing the angles $f\,C\,g = \gamma_2$ and $b\,C\,h = \gamma_1$ respectively. A single oscillation of the anchor is therefore given by $\gamma_1 + \beta + \gamma_2$, and if we assume that γ_1 and γ_2 are equal, the swing of the anchor on each side of the middle position will be given by $\gamma + \frac{1}{2}\beta$.

The angles of rotation α_1 and α_2 of the wheel are usually made equal, so that $\alpha_1 = \alpha_2 = \frac{1}{2}\tau$. The number of teeth of the escape-wheel is therefore equal to the number of double oscillations of the pendulum; for example, if an escape-wheel with thirty teeth and a second pendulum were employed, the wheel would need just one minute to make a complete revolution and would therefore be qualified to carry the second hand.

The axis C of the anchor in this escapement is usually placed at the intersection of the two tangents drawn to the outermost circle of the escape-wheel B at the points b and e of the two teeth H and J. In order that the angles α_1 and α_2 may be equal to each other and to the half pitch angle $\frac{1}{2}\tau$, the radial thicknesses $b\,i_1$ and $f_1\,e$ of the pallets must be equal to each other and to the half pitch $\frac{1}{2}t$, for we see from the figure that

$$\alpha_1 = \frac{b\,i_1}{r} \quad \text{and} \quad \alpha_2 = \frac{f_1\,e}{r},$$

where r is the radius $A\,b$ of the escape-wheel. If the pallets were made thinner the escape-wheel would not be caught by one pallet at the same instant that the other pallet left it, and the teeth of the escape-wheel would cause small shocks on the anchor, which, as they are injurious to the whole mechanism, should be avoided. The angle $\beta = i_1\,C\,i$ of the anchor is determined from the thickness $d = b\,i_1$ of the pallet, and from the angle $i_1 i\, b = \delta$ at the point of the pallet, by means of the equation

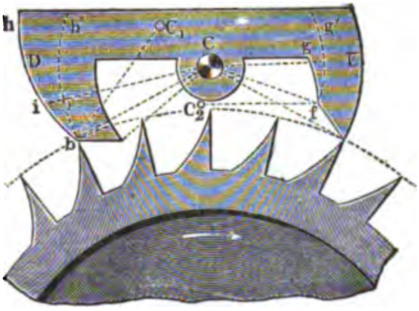
$$B = \frac{i_1 i}{C\,i} = \frac{d \cot \delta}{l},$$

where l is the radius of the inner profile $k\,i_1$ of the pallet. In order that the angle β may be the same on each side, the angles $i\,i\,b$ and $f\,e\,l$ must be equal when the pallets have equal thicknesses. Finally the angles of rest $h\,C\,b$ and $g\,C\,f$ depend on the swing of the pendulum to each side, and principally, therefore, on the magnitude of the impulse received by the anchor from the escape-wheel. The application of a heavier weight would cause a greater angle of deviation of the pendulum. This constitutes a disadvantage of this escapement, for we know from investigations of the ordinary circular pendulum (see Vol. I, § 321) that the duration of its oscillation depends to a certain degree on the magnitude of a semi-oscillation or angle of deviation. Consequently, if variations of driving force occur, the regularity with which the clock runs will be impaired to a certain extent.

This disadvantage has led to the construction of the *recoil-escapement*, which differs from *dead-beat escapement* as follows. In the dead-beat escapement, the escape-wheel is stationary when its teeth are not acting on the oblique surfaces $b\,i$ and $f\,e$ of the anchor, *i.e.*, while the anchor is swinging back and forth through the angle of rest (γ) But in recoil-escapements, the pallets have such forms that the escape-wheel is pressed back a little during the aforesaid interval. The object of this is to limit the swing of the anchor as much as possible by opposing to it the resistance offered by the escape-wheel when pressed back by the anchor and thus to obtain a certain regulation of the swing and consequently of the duration of the oscillation. For example, when the driving force increases, the teeth impart a greater impulse to the anchor and tend to increase its angle of deviation, but this is resisted by the increased resistance offered by the escape-wheel when pressed back by the pallets. However, opinions are divided concerning the advantages and disadvantages of recoil-escapements, and of late their use is largely discontinued.

The manner in which the escape-wheel is pressed back by the pallet is very simple. Let C be the axis of oscillation of the anchor f, Fig. 701, then to make it a recoil-escapement, the outer boundary bh of the left pallet D, instead of being concentric to C, as bh', must be an arc bh concentric to some other centre, as C_1. Therefore, for a semi-oscillation bCi of the anchor, the tooth b will be pressed back a distance ii_1. When the anchor swings back through this angle the escape-wheel will, of course, move forward through the same distance. Neither can the inner limit of the pallet E be concentric to C, as fg', but it must be an arc fg concentric to some other centre, as C_2.

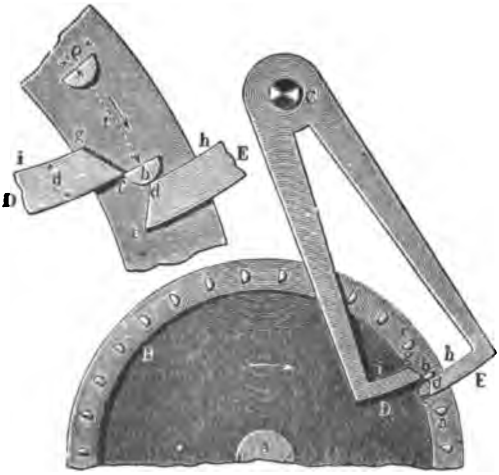
FIG. 701.



The anchor used in watches is generally like *Graham's* anchor, Fig. 700, but it is customary to give it a greater oscillation, the centre of oscillation C, Fig. 701, being nearer the escape-wheel. Greater oscillation is desirable in watches because a *balance-spring* is used instead of a pendulum, and the balance-spring, instead of being placed directly on the axis of oscillation C of the anchor, is placed on a separate axis and acts upon a corresponding arm of the anchor.

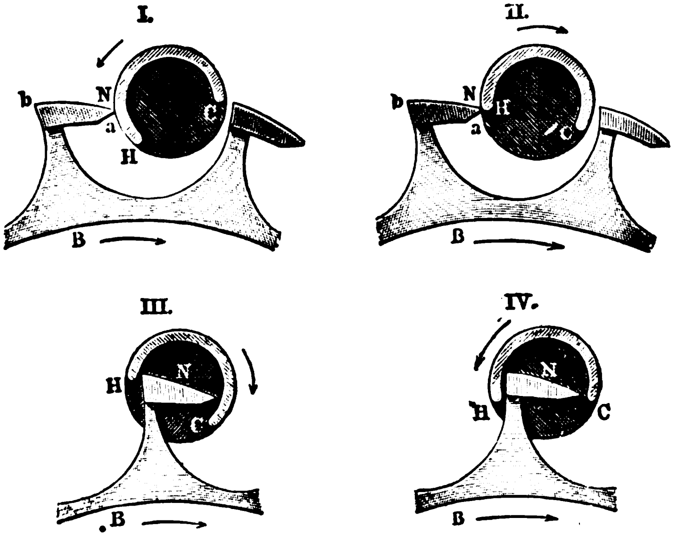
In the anchor-escapement discussed above the teeth of the escape-wheel press upon the pallets of the anchor so that the arbor of the latter is alternately subjected to pressure in *different* directions, which causes a more rapid wear of the bearing than if the direction of pressure were constant. To make it constant, the *pin-escapement* has been devised. Here the arbor A, Fig. 702, carries the escape-wheel whose rim B has semi-cylindrical and parallel pins, a, b, c, . . . The two arms CD and CE act on the same pin and the action of the surfaces de and fg and the surfaces dh and gi is easily explained. The figure represents the instant when the pin *b*, in consequence of the movement of the escape-wheel in the direction of the arrow, is about to leave the surface *gf* of the left pallet at *f*, the anchor having been pressed to the left. Consequently the right arm has also been pushed so far to the left that the pin *b* is caught by the point *d* of the pallet *E* at the instant when *b* leaves *f*. During the further motion of the anchor to the left, the surface *dh* concentric to *C* slides under the pin *b* through the angle of rest and then returns through the same angle. As soon as the point *d* of the bevelled edge of the pallet is again under the pin *b*, the latter will slide down the edge *de*, turning the anchor to the right. At the instant when the right-hand edge of the pin *b* is in contact with the point *e*, the next pin, *c*, will be caught by the point *g* of the left pallet whose surface *gi*, concentric to *C*, will slide under *c* as the anchor swings to the right. On the return swing of the anchor to the left the point *g* is brought under the pin *c*, which then slides down the surface *gf*. This process is continually repeated. Here, too, for every revolution of the escape-wheel the pendulum makes as many double oscillations as there are pins. The pins are semi-cylindrical so as to permit motion of the left pallet *f* to the right when the pins leave *d* of the right-handed acting surface. These pallets are of equal breadth *d*, and we must make $d + \rho = \frac{1}{2} t$, ρ representing the radius of the pin and *t* the pitch. This is a dead-beat escapement and is particularly suitable for large pendulum clocks.

FIG. 702.



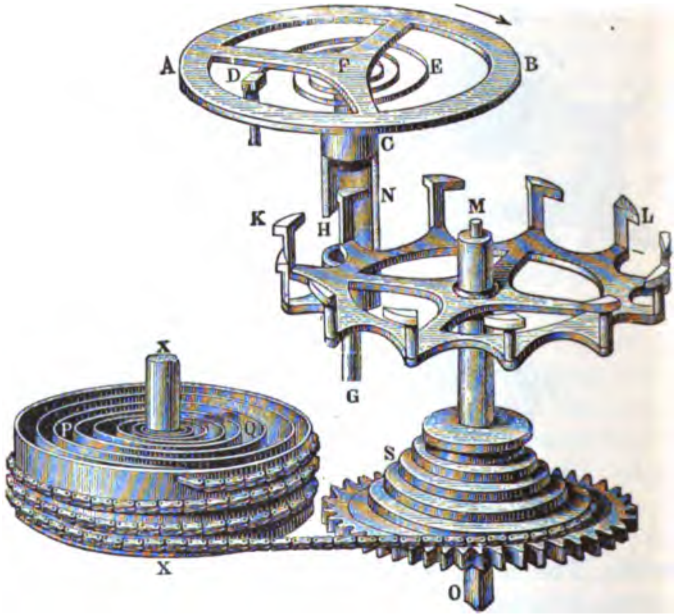
A dead-beat escapement for watches, where the balance-spring acts as regulator, was likewise invented by *Graham* and is known as the *cylinder*-escapement. This may be regarded as developed from the anchor-escapement, the two pallets acting simultaneously on the same tooth, as in the pin-escapement, and contracted so as to form a hollow cylinder which admits a tooth of the escape-wheel through a slot to its inner surface. The action of this escapement is shown in Fig. 703,

Fig. 703.



I to IV. The main spring drives the escape-wheel *B* in the direction of the arrow, and in I the point *a* of its tooth meets the outer circumference *N* of the little steel cylinder representing the anchor, and the wheel is thus prevented from moving while the cylinder, turning to the left, completes its oscillation. When the cylinder swings back the wheel remains stationary till the point *a* of the tooth comes in contact with the *lip H*, as at II. The tooth now enters the cylinder and moves till the point *a*, as at III, strikes the inner surface *N* of the cylinder, which is to be regarded as the second pallet. Here the wheel is again stationary till the cylinder has completed its oscillation to the right and returned to the left to the position in which the point of the tooth meets the second lip *C*, as in IV, after which the tooth can leave the cylinder. The next tooth of the escape-wheel then comes into the position *a b* at I. As the outer surface of the teeth is not concentric to the axis *A* it is evident that the oblique tooth surfaces *a b*, in passing the lips of the cylinder, will impart to it two impulses which will replace the work consumed by the hurtful resistances. Here also each tooth corresponds to the double oscillation of the cylinder.

FIG. 704.



The rotation of the latter can here be considerable, as is desirable when a balance-spring is the regulator. The cylinder receives an angle of oscillation of 180° and more.

Fig. 704 represents the principal parts of a timepiece with cylinder-escapement. The balance mechanism consists of a little fly-wheel *A B* turning about the axis *C*, and of a spiral spring *D E F* whose end *D* is attached to a fixed frame and whose other end is attached to the arbor *C*. If the wheel is turned slightly in the direction of the arrow and is then left to itself, the elasticity of the spring will not only move it back but will impart to it living force which will carry it beyond its position of equilibrium and thus strain the spring in an opposite sense. As a consequence the spring and the balance-wheel oscillate and this oscillation is maintained in the manner described above, by repeated impulses of the escape-wheel (see Vol. I, Appendix, § 5).

The figure also shows the arrangement of the driving mechanism which, in portable timepieces, must of course be based on springs. Here $P Q$ is the steel mainspring, one end of which is fastened to the arbor $X X$, while the other is attached to the circumference of the drum T which incloses the spring and turns loosely on the arbor. The spring is wound up by turning the arbor X with a clock-key, and as a clock-wheel prevents the opposite rotation of X, the drum tends to turn in the direction in which the winding took place (see also Fig. 682). Of late the drum of the timepiece has a toothed wheel which drives a train of wheels whose last and quickest moving member carries the escape-wheel. The arrangement in Fig. 704 is now seldom used. Here the motion of the drum T is transmitted by a *fusee*, *i.e.*, a conical spiral pulley S, whose rotation is increased by toothed wheels and transmitted to the arbor of the escape-wheel. In the figure the escape-wheel, for the sake of clearness, is assumed to be on the arbor $M O$ of the fusee. The object of the fusee is to make the moment of the turning force as constant as possible, the radius of the pull of the chain increasing as the force of the spring decreases. As the driving force has but an indirect influence on the duration of an oscillation, the fusee is often omitted. A heavy fly-wheel and a weak driving spring $Q P$ are used instead, the spring being just sufficient to keep the mechanism in motion, thus rendering the influence of the variations of the driving force as small as possible. The cylinder-escapement allows the watches to be made quite flat, but there is considerable friction between the teeth of the escape-wheel and the surface of the cylinder, so these parts are made of hardened steel and are highly polished.

To reduce this friction as much as possible the *duplex* escapement has been devised, so called because a double escape-wheel is used. The principle of this escapement is shown in Fig. 705; the escape-wheel on the shaft A has two sets of teeth, one set of very slender *spur* teeth *a*, *b*, *c* projecting from the cylindrical surface of the wheel, and another set of teeth *d*, *e*, *f*, . . . projecting from the flat surface of the wheel. The arbor *C* of the balance carries a small steel or ruby cylinder *E*, against which the teeth *a*, *b*, *c* press; it also carries, in the plane of the teeth *d*, *e*, *f*, a cam *D* which receives one impulse from the escape-wheel during every double oscillation of the balance. If the cylinder *E* were everywhere exactly cylindrical it would prevent all rotation of the escape-wheel in the direction of the arrow. At *i*, however, a fine groove in the cylinder permits an intermittent rotation of the escape-wheel. For if we suppose the balance to move in the direction of the arrow α, the groove *i* will pass under the point *b* of the tooth without permitting any motion of the escape-wheel, but on the return swing of the balance the groove will again pass under the point *b* and take it along, thus turning the escape-wheel to the right through one tooth, namely, till the next tooth *c* presses against the cylinder *E*. At the same time that this disengagement of the tooth *b* takes place, the point *h* of the little cam *D* moves to *k*, comes within range of the teeth *d*, *e*, *f*, and receives an impulse from the point *e*. In this escapement, therefore, and in the better timepieces generally, the escape-wheel is checked or freed but once and only one impulse is exerted on the regulator during a double oscillation of the balance.

FIG. 705.

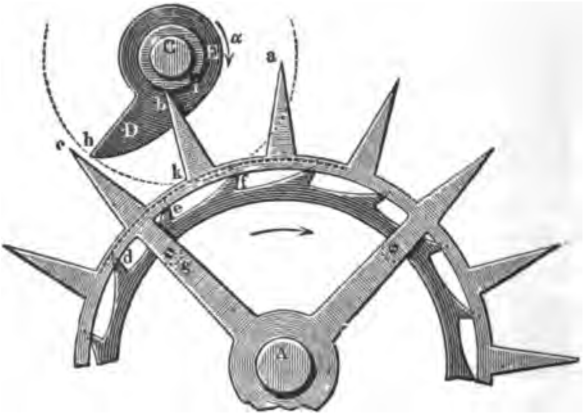
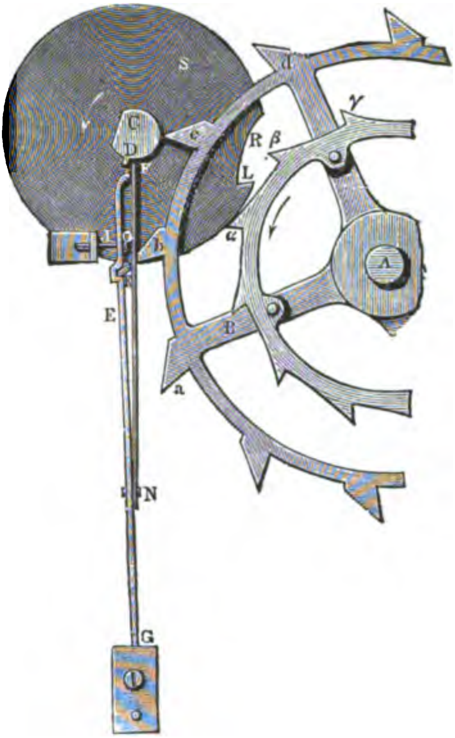


FIG. 706.



In all the escapements hitherto discussed, the regulating part, pendulum or balance spring, is continually connected directly or indirectly with the driving mechanism, *i.e.*, with the escape-wheel. While the acceleration or impulse is imparted to the regulator this connection is of course unavoidable, but as soon as the impulse is imparted, the further action of the escape-wheel can only have a disturbing influence on the regulator by interfering with the freedom of its oscillations. This disturbing influence is greatest in the recoil escapements, where the driving force places a direct hindrance in the path of the regulator, but in dead-beat escapements the sliding friction also acts as a disturbing influence. As the amount of this friction depends upon many external circumstances, as temperature, moisture of the atmosphere, nature of the lubricant, etc., and varies with these, a certain irregularity will be occasioned in the oscillations of the regulator, which is incompatible with an exact timepiece. Much pains has therefore been taken, in exact astronomical and nautical instruments, to construct escapements in which the regulator, after it has received its impulse from the escape-wheel, is free to complete its oscillation independently of the driving mechanism. An example of such a *chronometer escapement* designed by the famous clock-maker *Juergensen* is shown in Fig. 706. Here also the escape-wheel B is a double wheel with an equal number of teeth on each rim (generally twelve). The teeth a, b, c act as checks, while α, β, γ impart the impulses which accelerate the balance whose shaft is represented by C. The arresting of the escape-wheel is effected by the lug K against which, in the drawing, the tooth b of the escape-wheel presses. This log is fastened at G to a slender spring E which presses against the escape-wheel, but is limited in this by the head O of the pin J. The weak spring F serves to free the escape-wheel, *i.e.*, to bend back the spring E with its lug K. This secondary spring F is attached to E at N, and its upper end F receives an impulse from the little tooth D of the balance arbor. If the arbor C turns in the direction of the arrow, the tooth D causes a slight lateral bending (to the right) of the spring F without any further action on the spring E. But on the return swing of the balance, the tooth D presses against the spring F and by means of this presses back the spring E, frees the tooth b and renders the motion of the escape-wheel possible, this motion being interrupted by the immediate return of the spring E. During this motion of the escape-wheel the tooth β of the impulse wheel has entered the notch R on the disk S of the balance arbor C and exerted a pressure at L till the tooth β reached the position α. The disengagement of the spring and the impulse take but a very short time; during the rest of the period the balance is perfectly free to oscillate.

REMARK. The oldest escapement, the *crown escapement*, has not been discussed here on account of its imperfect action. It is only used in inferior watches and in the *alarm gear* of the Black Forest clocks. For the many other escapements proposed and constructed, for instance those with *constant* force and those for electric clocks, we must refer to the special treatises.

An extensive treatise on *ratchet-* and *click-gear* by *Reuleaux* is given in Verhandlungen des Vereins zur Befoerderung des Gewerbfleisses, 1877. Most of the text-books on clock-making treat of escapements; for instance, they are discussed in: *Juergensen's* Die hoehere Uhrmacherkunst, *Moinet's* Nouveau traité général, élémentaire pratique et théorique d'horlogerie, Th. Reid's Clock and Watch Making, and Marten's Beschreibung der Hemmungen der hoeheren Uhrmacherkunst. In this connection see also Willis' Principles of Mechanism, Laboulaye's Traité de cinématique, Ruehlmann's Allgemeine Maschinenlehre, vol. i, and the article on "Uhren" in Prechtl's Technologischer Encyklopaedie.

CHAPTER IX.

REGULATORS.

§ 174. **Regulating in General.**—In order that machinery may work well it is important that it should run as regularly and uniformly as possible. To attain this regularity certain machine parts and mechanisms are employed which may all be grouped under the general head of *Regulators*. These means accomplish their object in very different ways; for example, they may regulate the *velocity* of the machinery, or they may make either the *driving force* or the *resistance* uniform, or they may strive to make the product of these two factors, *i.e.*, the *work*, constant. Such regulation is necessary for many reasons,—excellence of product, economy of power, durability of the machinery and simplicity of the plant. For example, in *spinning machinery* the production of uniform thread requires that the prime mover run with as uniform velocity as possible; and this is also necessary in *grist mills*, that as large a quantity of grain as possible may be ground with a given power. In *looms* the uniformity of the cloth depends principally on the uniform tension of the warp and the energy of the beats of the lay. In rolling mills and hoisting gear, which act intermittently, and have periods of running empty, the use of a fly-wheel or accumulator enables the driving machinery to work uniformly and thus allows it to be smaller and simpler.

These few examples suffice to show that the action and arrangement of regulators differ greatly according to the object to be attained.

When we inquire into the cause of irregularity in the running of any machine we at once find that it is due to the inequality between the moments of the driving force and of the resistance to be overcome, for every inequality of this kind is accompanied by an acceleration or retardation of the machine, and therefore by a variation of its velocity. It follows that these inequalities must be avoided or remedied by establishing equilibrium between the resistance and driving force. As this condition can be fulfilled by varying either the resistance or the driving force, all regulators are naturally divided into two large groups according as the aforesaid equilibrium is effected by varying the driving force or resistance. To a certain degree there is a natural regulating-action due to the fact that the intensity of all known driving forces diminishes as the velocity of their point of application increases, while the resistances increase with the velocity. For example, if a water wheel which is used to drive a grist mill is suddenly acted upon by the full force of the water while the stones are out of gear, its velocity will increase, not indefinitely but till the diminished power of the water is in equilibrium with that of the increased frictional resistances. Under all circumstances the number of revolutions of the water wheel can become so great that the velocity of the floats will equal the velocity with which the water moves toward them. But this kind of regulating never suffices in practice, for the velocity limits thus established are too far apart for machine work, and would be accompanied by accelerations which, in most cases, would endanger the strength of the machine parts.

In examining the regulators which establish uniformity by *varying the resistance* we must remember that there are both *useful* and *hurtful* resistances, and must distinguish between the corresponding devices. *Brakes* are the most important of the regulators for varying the hurtful frictional resistances. They are employed in hoisting apparatus for lowering loads, and on wagons when descending hills. In these cases the driving force due to the weight of the sinking load exceeds the ordinary frictional resistances, and acceleration of motion is prevented by adding brake-friction sufficient to equalize the forces and thus produce uniform motion. Under this head come the *flies* used in the *striking gear* of clocks for effecting a uniform sinking of the weight. To regulate the tension of the *warp yarns* in *looms*, the friction at the *warp beam* is uniformly diminished so as to correspond to the smaller lever arms of the warp yarns accompanying the gradual paying out of the yarn. Of course, all these arrangements involve a corresponding loss of work and the further disadvantage that this work is consumed in wearing away the acting parts of the brake. Hence these means are only employed where, as in hoisting apparatus, the lost work could not be usefully employed, or where, as in looms, the loss of work is insignificant in comparison with the other considerations of manufacture. It would, however, be utterly out of the question to regulate the running of the prime mover of any establishment by introducing artificial resistances.

When the regulation is effected by introducing *useful resistances*, the action is such that the excess of driving force is expended in storing up work in a certain piece, which work is given out when the driving force becomes less than the average resistance. *Counter-weights* belong to this class; they are lifted to a certain height by the excess of power on hand in order that they may aid the motion by giving up this power when they descend. The *accumulators* which drive machinery that works intermittently may be regarded as a particular kind of counter-weight, and in principle it is immaterial whether the resistance of the accumulator is an actual weight resting on the piston, or whether it is the pressure of compressed air in an air chamber. Fly wheels also belong to this class of regulators, the living force being stored by accelerating the masses instead of lifting the load. Such a storing up and giving out of work is only possible in a fly-wheel when the latter alternately assumes greater and smaller velocities. Variations of velocity therefore can never be completely removed by the application of fly wheels, *i.e.*, the motion can never be made a perfectly uniform one by this means. This perfect condition can be approximately attained by employing large masses to reduce the variations of velocity due to the fluctuations of force. Fly wheels are preferred for rotary motions and counter-weights for reciprocating motions. When counter-weights are employed with rotary shafts, as in steam engines, the object generally is to balance other machine parts, as cranks, connecting rods, etc.

By effecting a more uniform distribution of the masses they prevent shocks and cause the machine to run quietly.

This kind of regulator involves loss of work only so far as its arrangement causes certain hurtful resistances, as friction; but if we neglect these resistances, the work expended in lifting the weights or accelerating the masses is all given back again and usefully applied when the weights descend or the masses are retarded.

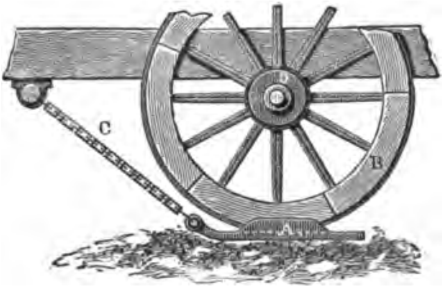
The second group of regulators, better known as *governors*, are mechanisms which strive to keep the machines running regularly by varying the driving force as the resistances vary. Under this head come the arrangements employed in steam engines and water wheels. These are directly connected with the machine and produce the desired equilibrium between resistance and driving force by varying the intensity of the motor, steam or water, flowing into the prime mover.

Moreover we may say generally of all kinds of regulators that they can never effect a perfectly uniform motion of a machine, but can only bring about a more or less close approximation to this ideal condition. In judging, therefore, of the excellence of any such arrangement, we must, neglecting such considerations as simplicity, look principally to the degree of irregularity still existing in the machine.

§ **175. Brakes.**—Under this head come all the devices for moderating or stopping the motion of a machine by an artificially produced *frictional resistance*. To generate sufficient friction, one part of the brake must be pressed with a force R against another part having a motion relatively to the first, so that the friction $F = \phi R$, occurring between the two bodies, may resist the motion. In most cases the machine part to be controlled has rotary motion, for example the drum of a hoisting gear or the wheels of a wagon, but occasionally there are cases where friction must be overcome along a rectilinear path, we may instance the *wagon-drag* and some of the safety devices used in hoisting apparatus. When the brake is applied principally to keep the motion uniform, while the driving force exceeds the resistance, the aforesaid frictional resistance F must be made equal to P, where P represents the excess of force reduced to the same point as F. This occurs in hoisting gear when loads are lowered with uniform speed by means of brakes. It also often happens, for example, in the running of trains, that the living force of a machine must be wholly or partially destroyed, and then, by means of the brake, the work of friction must be exerted, equal to the living force L, so that $L = Fs$, where s is the path traversed relatively to the brake by the piece which is to be controlled during the time that the brake is applied.

If the brake is to act as a regulator, it is necessary to so arrange it that the aforesaid pressure R can be regulated at any time according to circumstances. From this point of view the wagon-drag is a very imperfect arrangement, as it does not admit of any such regulation of the pressure. The wagon-drag A, Fig. 707, is allowed to fall so that the wheel B rolls on to it, whereupon the rolling motion is stopped by the chain C becoming taut and pulling the drag along with the velocity of the wagon and axle D. The resistance thus developed is the friction which is due to the load on the wheel B, and takes place between the road and the drag. When the chain C is so short that only a portion R_1 of the wheel rests on the drag, while the other part, $R_2 = R - R_1$, rests on the road, the wheel can still turn and the frictional resistance at the brake will be $(\phi_1 + \phi_2)\, R_1$, where ϕ_1 represents the coefficient of friction between the drag and the road, and ϕ_2 that between the wheel and the drag; but in practice this action cannot be depended upon.

FIG. 707.



Brakes which worked like a wagon-drag and thus rendered a certain regulating of the pressure possible have also been applied to the tenders of locomotives, but were rejected from practical considerations. In this case the driver by means of a screw pressed a shoe, placed between the wheels, against the rails, thus causing sliding friction. Brakes whose rubbing parts have a rectilinear motion seldom occur except in safety devices, and probably are never used as regulating gear. They occur, for example, in hoists, so that if a rope or chain breaks, the catch arms connected with the cage will be pressed by springs against the guides, thus generating friction at the latter and hindering the fall of the cage. The fire-escape devices, intended to let persons down from an upper story by a rope, may also be classed with brakes. All these devices depend on the principle that the rope which is fastened above is so wound about a spool that when the spool slides down, the friction between it and the rope will be sufficient to properly moderate the falling motion.

All other brakes are arranged to restrain rotating pieces; and according to the manner in which the rubber or shoe is applied to the drum of the brake, we may distinguish between block-brakes, chain-brakes, band-brakes, eccentric-brakes, and cone-brakes.

§ **176. Block-brakes.**—Here the frictional resistance producing the retardation at the surface of the brake-drum is called forth by *blocks* or *cheeks* pressed against the drum. To generate the necessary pressure a lever like DE, Fig. 708, is generally used, the force K being applied at the end E of the longer arm so that a greater force may be exerted at the block B. If we assume that the pressure of the block B against the drum A is uniformly distributed over the surfaces in contact, the resultant R will pass through the axis C; and if we neglect the journal friction at D, we shall have for the condition of equilibrium

$$Ka = Rb - \phi Rc,$$

from which we have

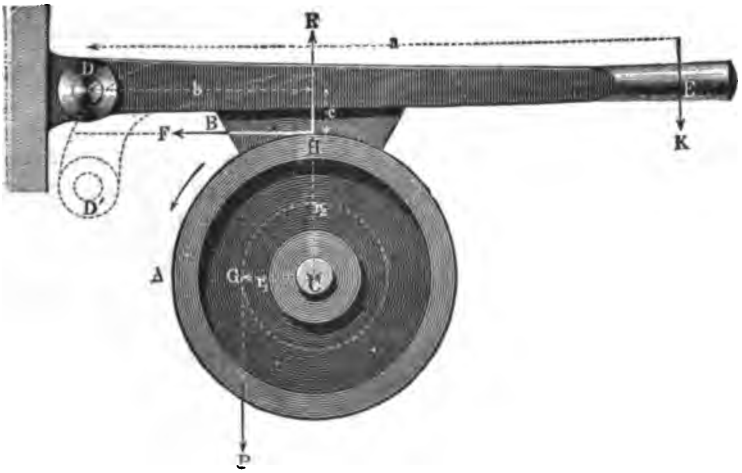
$$K = R\frac{b - \phi c}{a}.$$

It is here assumed that the brake drum turns in the direction of the arrow, in consequence of which the friction ϕR, acting on the brake-block, tends to turn the lever DE in the same direction as the force K. If the brake-drum turned in the opposite direction, the equation of equilibrium would be $K a = R b + \phi R c$. Therefore, for both directions of rotation, we have the general equation

$$K = R \frac{b \mp \phi c}{a}.$$

Now if P is the force acting with the radius $C G = r_1$, which is in equilibrium with the friction $F = \phi R$ acting with the radius $C H = r_2$, we have

FIG. 708.



$$P r_1 = \phi R r_2; \quad R = P \frac{r_1}{\phi r_2},$$

and therefore the force needed at the lever end E to work the brake will be

$$K = P \frac{r_1}{\phi r_2} \frac{b \mp \phi c}{a}.$$

In order that the desired object may be attained as easily and conveniently as possible by the brake, the force K, needed to work the brake, must be made as small as possible. The above formula shows that for a given moment $P r_1$, the force K diminishes as the radius r_2 of the drum and the coefficient of friction ϕ increase. On this account the brakes are never allowed to act directly against the shaft, but against large disks or drums whose circumferences have considerable velocity during the motion. If a machine has several shafts which, as in a windlass, rotate with different velocities, it is best, for the same reason, to place the brake drum on the most rapidly moving shaft, for which the moment Pr_1 of the force is least. To exert considerable friction F when the pressure R at the brake is comparatively large, it is customary to make the block or shoe of wood because of its greater coefficient of friction, and occasionally the drums are of the same material.

From the equation

$$K = P\frac{r_1}{\phi r_2}\frac{b \mp \phi c}{a}$$

developed above, we see that K diminishes as the lever arm a increases, but the length of this arm is limited by the condition that the shoe B must move away from the drum through a certain distance, not too small, which will allow the drum to turn freely even when it is not perfectly round. On the other hand, the path traversed by the lever end E is limited by the length of arm of the workman. This circumstance must be especially considered in those cases where a large pressure is to be exerted at the brake by means of a series of levers.

Finally, the above value for K shows that, other things being equal, this force depends on the direction of rotation of the brake drum. In the case considered in Fig. 708 the drum assumed to have a left-handed rotation and the force K is given by

$$K = R\frac{b - \phi c}{a},$$

and is therefore smaller than when the opposite rotation obtains, in which case

$$K = R\frac{b + \phi c}{a}.$$

From this we may deduce the general rule that in order to diminish K as much as possible it is best to so choose the arrangement that *the friction F acting on the shoe of the brake will tend to turn the lever in the same direction as the force K.*

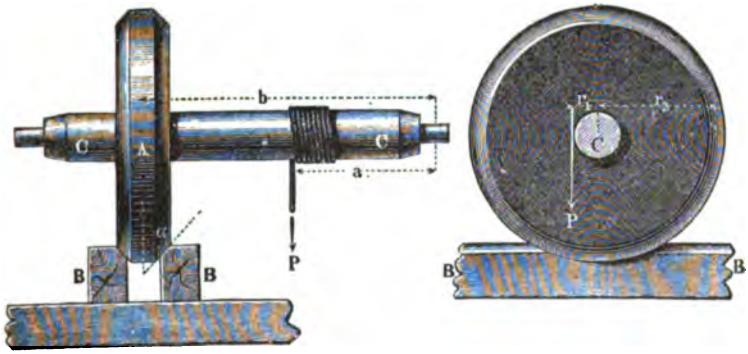
This is to be investigated in each case; for example, if the axis of the lever, instead of being at D, were at D' on the other side of the direction of F, the negative sign would apply only when the drum turned in a direction opposite to the arrow.

From the expression for the lever force

$$K = R\frac{b - \phi c}{a}$$

we see that $K = 0$ when $\frac{b}{c} = \phi = \tan\rho$, where ρ is the angle of friction. This condition is fulfilled when the connecting line DH makes with the radius CH, *i.e.*, with the pressure R on the brake, an angle DHR equal to the angle of friction ρ, and in such a case the slightest force exerted upon the lever DE, for example its own weight, will suffice to control the motion of the drum. If the angle DHR were smaller than ρ we should have the case of a frictional click-gear (see § 172).

FIG. 709.



We must class with block-brakes the arrangement in Fig. 709, used in the hoisting apparatus of grist-mills. The shaft C supports the load P and carries a brake-drum K which is pressed against the two fixed shoes B by its own weight G and the component $P\frac{a}{b}$ of the load. The friction F at the two bevelled edges of the brake-drum, according to what has preceded, is given by

$$F = \frac{\phi R}{sin\,(\alpha + \rho)},$$

where α is half the angle formed by the bevelled edges. When the hoist is at rest we have

$$R = P\frac{a}{b} + G,$$

and the radius r_2 of the brake-drum must be taken sufficiently large to make

$$\phi\frac{P\frac{a}{b} + G}{sin\,(\alpha + \rho)}r_2 = P r_1.$$

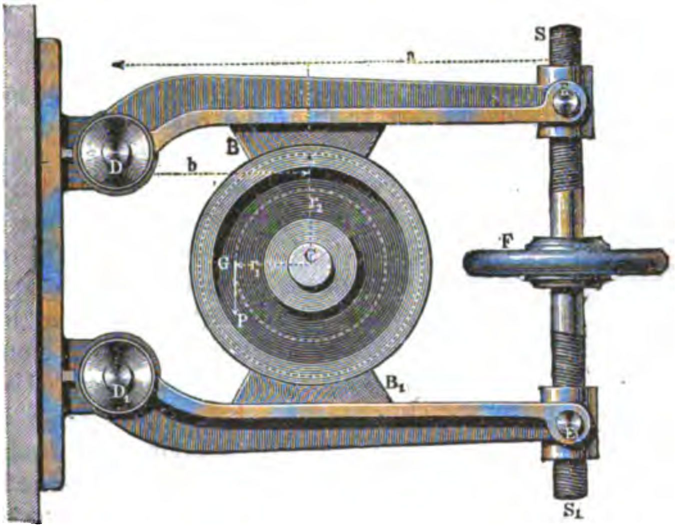
If the load is to be lowered uniformly, the hauling rope (see Fig. 676) must be drawn down sufficiently to lift the brake-drum till it presses against the shoes with a pressure R, which may be found from

$$P r_1 = \phi\frac{R}{sin\,(\alpha + \rho)}r_2.$$

Here all other hurtful resistances, as journal friction, have been neglected.

In the simple block-brake, Fig. 708, the pressure R exerted by the shoe on the drum is directly sustained by the shaft C and its bearings. Although the friction thus produced in the bearings, acts favorably so far as it helps to resist the motion, nevertheless under some circumstances the considerable pressure R to which the shaft is subjected may become dangerous to the strength of the latter, particularly if the brake-drum is not placed close to a bearing. To avoid such a lateral pressure on the shaft, two shoes are frequently placed on opposite sides of the brake-drum. For example, in *drilling machines* where the drill is fed with constant pressure the arrangement shown in Fig. 710 is employed. Here the two levers $D E$ and $D_1 E_1$ are drawn together by means of a screw-spindle $S S_1$ having right- and left-handed threads. Retaining the designations hitherto employed, we have for any direction of rotation of the shaft C, $Ka = Rb$, where K is the pull exerted by the screw on each lever; in this formula the term ϕRc disappears because it has opposite signs for the two levers. If this arrangement is to exert a constant pressure P at the end of the radius $CG = r_1$, we must determine R and K from the equation

FIG. 710.



$Pr_1 = 2\phi Rr_2$, and the force that must be exerted on the hand-wheel F can then be ascertained from the formulas for screws contained in the fifth chapter.

§ **177. Wagon-brakes.**—The brakes employed on wagons and cars also come under the head of block-brakes. In road wagons the brake is principally used in descending hills, to neutralize the excess of the driving component of the weight of the wagon, while with cars, brakes are principally used to partially or wholly destroy the velocity of a train in a short time. In the first case let α represent the angle of inclination of the road to the horizon, G the weight of the wagon, and P_1 the pulling force necessary to give a uniform motion to the wagon. As the inclination of the road is always slight, this force will differ but little from that needed on a horizontal road, and the excess, which must be neutralized by the brake, will be given by

$$P = G \sin \alpha - P_1 .$$

As the friction F at the wheels is overcome through a distance equal to the travel of the wagon, we have

$$\phi R = G \sin \alpha - P_1 .$$

FIG. 711.

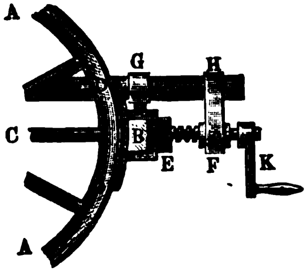
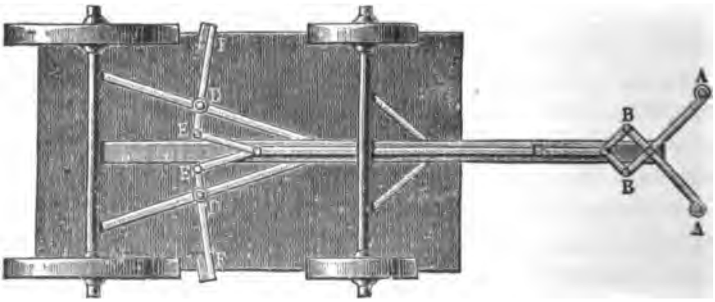


Fig. 711 represents an ordinary brake for road wagons. Here B is the brake bar back of the hind wheels A A. By means of the screw $E F$ and the crank K, the iron or wooden shoes $D D$ are pressed against the wheel tires A. A carriage brake differs from this in that the screw is placed near the driver's seat so that he can conveniently apply it, the motion of the nut being communicated to the shoe or rubber by suitable levers. Instead of applying the brakes by hand, the holding back of the horses in descending a hill is applied to the levers A, Fig. 712, at the end of the carriage tongue, and this causes the rod C to turn

FIG. 712.



the two levers EF about D and press the shoes F against the wheels.*

By pressing the brakes against the hind wheels (the brakes are not applied to the fore wheels because the fore-gear is swivelled) friction is generated which increases with the brake pressure R, the wheels at first retaining their rotary motion. But this motion only lasts till the friction between the rubber and tire and the journal friction together equal the sliding friction between the locked hind wheels and the road. In the brake examined in the preceding article the resistance could be made as large as desired, but in wagon brakes the limit of the resistance to the brake is established beforehand. If G_1 represents the load on the axle to which the brake is applied, and ϕ_1 the coefficient of sliding friction between the wheel tire and the road, we have

$$\phi_1 G_1 = F$$

as the maximum resistance attainable by the brake. Hence if brakes were applied to *all* the wheels of a wagon or train, their acceleration could not be avoided if the inclination of the road exceeded the corresponding angle of friction. From this we see that the action of the brake with railway trains depends principally on the number of axles to which brakes are applied, and on the load carried by those axles. We also see that, for a given inclination of the road, uniform motion can only be attained when the cars and their load have a certain weight, equal at least to that found from the equation $\phi_1 G_1 = F$.

It is often necessary on railways to stop a swiftly moving train, or reduce its velocity as quickly as possible. This case may be examined as follows: Let M be the total moving mass of the train, and v its velocity, which is to be reduced to c in the time t; let P represent the excess of the pulling force over ordinary train resistances, and F the friction developed by the brake; then the retardation of the train per second will be

$$p = \frac{F - P}{M}.$$

---

* See *Knight's* American Mechanical Dictionary.

Therefore, in order that the velocity may experience the change $v - c$ in t seconds, the equation

$$v - c = pt = \frac{F - P}{M} t$$

must be satisfied. Here P is the excess of pulling force over average resistances, journal friction, etc. When the steam is cut off from the engine, P has a negative value P_0 which is equal to this resistance to the ordinary motion. For this case

$$v - c = pt = \frac{F + P_0}{M} t.$$

The distance traversed by the car while the brake is applied is determined according to the laws of uniformly retarded motion (see Vol. I, § 14) from

$$s = \frac{v + c}{2} t,$$

and the work performed at the brake while this distance is being traversed is

$$Fs = \phi R \frac{v + c}{2} t.$$

The energy lost by the train while the brake is applied is

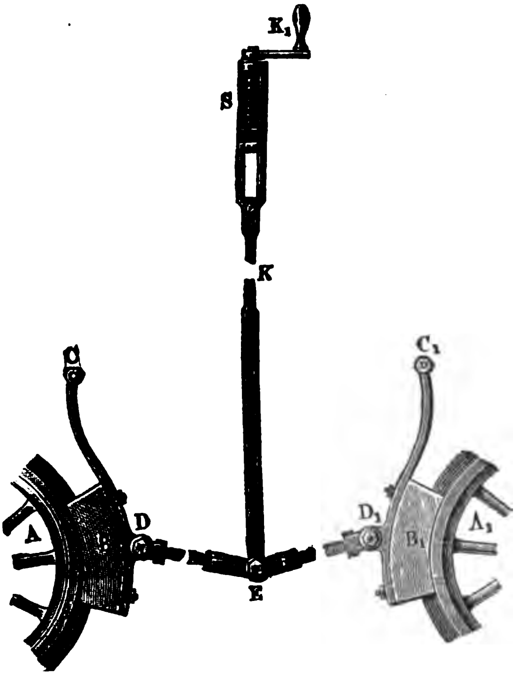
$$M \frac{v^2 - c^2}{2} = (F + P_0) \frac{v + c}{2} t.$$

That the axles of railway carriages may not be subjected to the brake pressure, it is customary to apply two diametrically opposite rubbers to each wheel. When the brakes are worked by hand the rubbers are so connected, by levers and rods, with the screw operated by the brakeman, that all the brake shoes are applied simultaneously. The lever arrangements for this purpose are very different, a very common one being shown in Fig. 713. Here the pulling force K, exerted by the screw S on its nut, is transmitted through the toggle joint $D E D_1$ to the brake shoes B and B_1 suspended at C and C_1, so that the force K exerted by the connecting rod at E generates a pressure R in each link $E\ D$ and $E\ D_1$ of the toggle joint, which pressure is expressed by

$$R = \frac{K}{2 \cos \beta},$$

provided the journal friction of the joint is neglected, and provided $2\ \beta = D E D_1$.

FIG. 713.



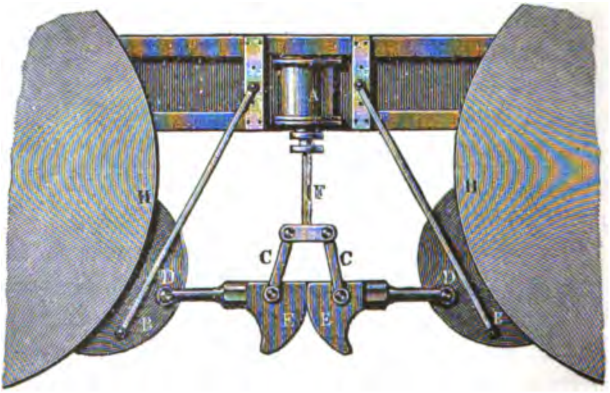
The great speeds employed of late on railways have caused much attention to be paid to brakes in which all the shoes of the train are simultaneously applied by steam or air. To accomplish this many arrangements have been proposed and applied. The living force of the train itself has been utilized to apply the brakes. For example, in the *Heberlein* brake occasionally used in Germany, an axle of the brake van is provided with two wooden friction disks; these are pressed upon by two other friction disks, which are turned by the friction generated, and thus cause a chain to apply the brakes. On account of the severe shocks experienced with this arrangement when the brakes are suddenly put on, other more elastic means are preferred. Thus in England the *air-brakes* of *Steele & Westinghouse* and *Smith's vacuum-brake* are much used.* In the former each car has a cylinder whose piston is subjected on each side to highly compressed air (8 atmospheres—about 120 lbs.), which pressure is produced by a steam pump attached to the locomotive. Now when the engine driver opens a cock, the pressure in the main pipe and consequently on one side of the pistons is diminished, and the excess of pressure on the other side of the pistons causes them to move and apply the brakes. This is a quick and reliable method of applying brakes by the engine driver, and has the additional advantage of automatically applying them when a car coupling breaks, for the breakage of the main pipe produces the diminution of pressure needed for putting on the brakes.

---

* See article by *Schneider*, "Ueber continuirliche Bremsen," Zeitschrift des Vereins deutscher Ingenieure, 1878, p. 353.

The principal difference between *Smith's* vacuum-brake and the one just described consists in having, in place of the brake-cylinder with piston, a stiffened, cylindrical, rubber vessel under each car, whose upper end is attached to the car frame and its lower end to the brake-rods. Now if an *ejector*, attached to the locomotive and worked by steam, sucks the air out of the main pipe and the rubber chambers, the excess of atmospheric pressure will cause them to contract and thus apply the brake-shoes.

FIG. 714.



The Westinghouse brake is shown in Fig. 714. Here *A* is the compressed-air cylinder whose piston rod *F* when it rises causes the links DE to rise and press the shoes B against the wheels H.

An attempt has also been made to retard railway trains by contracting the exit areas of the exhaust steam. A more frequently applied means, in emergencies, is to reverse the engine, *i.e.*, to so admit steam to the cylinders that it would drive the engine backward if its living force did not cause it to move forward. How much can be effected by this means may be learned from the following discussion: By preventing the steam from leaving the cylinder, the most that can be done is to prevent the *driving wheels* from turning. As a consequence the stationary wheels will slide or *skid* along the rails with the velocity v with which the whole train moves. Therefore if G represents the weight of the locomotive that comes on the driving wheels, the loss of energy experienced by the moving train in each second is equal to the work $\phi G v$ consumed by the friction. The retarding action cannot be increased beyond this amount by reversing the engine. To be sure the driving wheels can be made to rotate backward with a velocity v_1 in spite of the train's forward motion v, and the friction of the driving wheels must accordingly be overcome through a distance $v + v_1$, which means that the work $\phi G(v + v_1)$ must be consumed by the friction in every second. But of this work only the portion $\phi G v$ is performed by the living force of the train, the other portion, $\phi G v_1$, being exerted by the *counteracting* steam. The action of this steam can therefore be regarded as equivalent to rendering the driving wheels *stationary*, *i.e.*, equivalent to *skidding* these wheels. However, this mode of using steam is rarely applied on account of the disadvantages connected with it.

EXAMPLE. What brake-pressure is needed for a railway train whose whole weight is 300 tons of 1000 kg. each [about 300 gross tons], when the inclination of the road is represented by 0.02 and when the velocity of the train is not to exceed 10 m. [33 ft.] per second, the resistance of the air and journal friction with this velocity being assumed at ½ per cent of the weight?

Here the pulling force is the component of the weight diminished by the resistance of the air and journal friction, *i.e.*, equal to (0.02–0.005) 300000 = 4500 kg. [9920 lbs.]. If we assume the coefficient of friction between the brake-shoe and the wheel tire to be $\phi = 0.25$, we shall have for the necessary brake-pressure

$$\frac{4500}{0.25} = 18000 \text{ kg. } [39700 \text{ lbs.}].$$

When the gross load on each axle is 7500 kg. [16,530 lbs.], and the coefficient of sliding friction between rails and wheels is 0.2, the greatest brake resistance that can be generated at an axle is

$$7500 \times 0.2 \doteq 1500 \text{ kg. } [3310 \text{ lbs.}];$$

consequently there must be at least

$$\tfrac{4500}{1500} = 3 \text{ axles}$$

provided with brakes. For the sake of greater safety it is customary to have brakes on a greater number of axles or cars for such inclinations of road.

2) Let us suppose the train just considered to have a velocity of 16 *m* [52.5 ft.] per second, and that it is to be brought to rest in 250 *m* [820 ft.]: what brake-pressure must be exerted by all the shoes when the average resistance of air and journal friction is taken at ½ per cent of the weight and steam is cut off when the brakes are applied?

If we place the living force of the train equal to the work done in overcoming all resistances through 250 *m*, we can find the brake-pressure R from

$$\frac{300000 \times 16^2}{2 \times 9.81} = 250\,(0.005 \times 300000 + 0.25\,R),$$

or

$$R = 56632 \text{ kg. } [124850 \text{ lbs.}].$$

As each axle sustains a load of 7500 kg. [16530 lbs.], and can exert at most only a resistance at the brake of $0.2 \times 7500 = 1500$ kg. [3310 lbs.], the effective brake-pressure cannot exceed

$$\frac{1500}{0.25} = 6000 \text{ kg. } [13230 \text{ lbs.}],$$

and it follows that in the present case there must at least be

$$\frac{56632}{6000} = 10 \text{ axles},$$

i.e., there must be at least 5 cars provided with brakes. The time during which the brakes are applied may be found from

$$\tfrac{16}{2}\,t = 250, \qquad \text{or} \qquad t = 31.25 \text{ seconds.}$$

§ 178. Band-brakes.—These are often employed on account of their simplicity and efficiency in hoisting and winding machinery when the brake-pressure is comparatively small. Here the resistance at the circumference of the brake-drum is caused by a steel or iron band E, Fig. 715, which is passed around the drum A and is made taut by forces pulling at the ends B and B_1. The tightening is usually effected by a lever turning about D and operated by a force K exerted at its end.

FIG. 715.

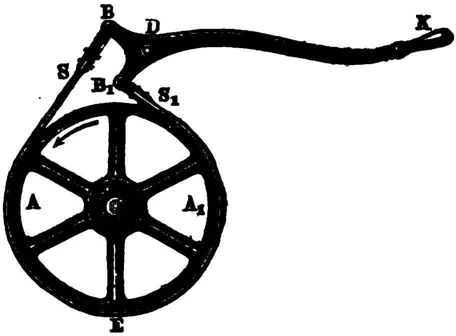
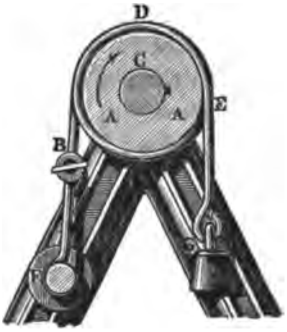


FIG. 716.



Generally the brake drum A has on its circumference a groove whose width is sufficient to receive the band. The wire rope shown in Fig. 716 depends on the same principle, a wire rope D being used instead of a flat band. The rope is wound in several coils about the shaft A, and one end is fastened at F to the frame of the windlass, while a weight G sufficient to produce the desired tension is hung from the other end of the rope. In all cases the friction between the circumference and the band must be overcome when the shaft or drum A rotates. Therefore by suitably regulating this friction the motion may be moderated or stopped as may be desired. An action similar to that of the wire-rope brake in Fig. 716 occurs in cases where heavy loads, for example barrels, are lowered into cellars on an inclined plane; a rope connected with the load is passed a few times about a round cross-bar, and the sinking of the load is regulated at the free end of the rope. The friction between the rope and the fixed cross-bar regulates the motion.

The friction of band-brakes is determined, like that of belts, according to the rules laid down in Vol. I, § 194, for the friction of cords. Assuming the drum A to rotate in the direction of the arrow, Fig. 715, let S and S_1 represent the tensions in the portions of the band attached at B and B_1. Then in case the pulley slips in the band, we have the relation

$$S = S_1 e^{\phi\gamma},$$

where γ is the arc of contact of the band measured on a circle whose radius is 1, and $e = 2.71828 \ldots$ is the base of the hyperbolic system of logarithms. In a certain sense the portion BA of the band corresponds to the tight, and the portion $B_1 A_1$ to the slack, side of a belt. Therefore the condition of equilibrium will give the friction at the circumference of the drum, and we have

$$F = S - S_1 = S_1 (e^{\phi\gamma} - 1).$$

Hence if P again represents the force tending to turn the brake-drum at the radius r_1, and if r_2 represents the radius of the brake-drum, we shall have for uniform motion the equation

$$Pr_1 = Fr_2 = S_1 r_2 (e^{\phi\gamma} - 1),$$

or

$$S_1 = P\frac{r_1}{r_2}\frac{1}{e^{\phi\gamma} - 1}.$$

If we assume, for example, for the coefficient of friction of iron upon iron, $\phi = 0.18$, and assume that in the band-brake, Fig. 715, the band is in contact with 0.7 of the circumference of the drum, we shall have

$$e^{\phi\gamma} = 2.71828^{0.18 \times 0.7 \times 2 \times 3.14} = 2.21;$$

hence

$$S_1 = \frac{1}{2.21 - 1} P\frac{r_1}{r_2} = 0.826\, P\frac{r_1}{r_2}.$$

On the other hand, for the wire-rope brake, Fig. 716, when the rope is wound one and a half times about the shaft, we have

$$e^{\phi\gamma} = 2.71828^{0.18 \times 1.5 \times 2 \times 3.14} = 5.45,$$

and hence

$$S_1 = \frac{1}{5.45 - 1} P\frac{r_1}{r_2} = 0.225\, P\frac{r_1}{r_2}.$$

We see from this how rapidly friction increases with the arc of contact, and that in the case of the rope-brake, Fig. 716, a comparatively small weight $G = 0.225\, P\frac{r_1}{r_2}$ suffices for efficient brake action when the arrangement is such that the force P tends to turn the drum in the direction of the arrow. For an opposite rotation the weight G must be made equal to the tension

$$S = S_1 e^{\phi\gamma} = 5.45 \times 0.225\, P\frac{r_1}{r_2} = 1.225\, P\frac{r_1}{r_2}.$$

Hence follows the rule, that the brake must be worked ***with*** the motion and not ***opposite*** to it, *i.e.*, ***the brake-force must be exerted so as to produce the tension S_1 in the driven or slack portion of the band, and not the tension S in the driving or tight portion.***

When the band, Fig. 715, is tightened, a force K must be exerted at the lever, which, journal friction at B, B_1 and D being neglected, can be found from

$$K a = S b + S_1 b_1,$$

where a, b and b_1 represent the lever arms, *i.e.*, the distances of the axis D from the respective forces. Hence for this arrangement we have

$$K = \frac{S b + S_1 b_1}{a} = S_1 \frac{b e^{\phi\gamma} + b_1}{a};$$

or introducing

$$S_1 = P\frac{r_1}{r_2}\frac{1}{e^{\phi\gamma} - 1},$$

we get

$$K = P\frac{r_1}{r_2}\frac{b e^{\phi\gamma} + b_1}{a(e^{\phi\gamma} - 1)},$$

which expression when $b = b_1$ becomes

$$K = P\frac{r_1}{r_2}\frac{b}{a}\frac{e^{\phi\gamma} + 1}{e^{\phi\gamma} - 1}.$$

Now in order that the brake may be applied conveniently, *i.e.*, in order that the brake-force K may be as small as possible, the arrangement must be so chosen that one of the terms of the sum in the expression

$$K = \frac{Sb + S_1 b_1}{a}$$

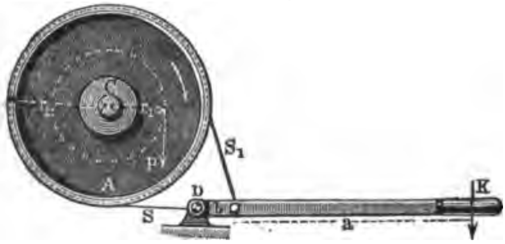
becomes equal to zero by making b or b_1 equal to zero; in other words, by making one or the other force pass through the fixed axis D. Fig. 717 shows a brake in which $b = 0$, *i.e.*, in which the greater tension S of the driving or tight band passes through the axis D of the lever. In this case, therefore, we have the simple formula

$$K = \frac{S_1 b_1}{a},$$

or

$$K = P \frac{r_1}{r^2} \frac{b_1}{a} \frac{1}{e^{\phi\gamma} - 1}.$$

FIG. 717.



But the force K needed to work the band-brake can be reduced considerably, even to zero, as is evident from the equation of moments,

$$Ka = Sb + S_1 b_1.$$

For this it is only necessary to make $Sb = - S_1 b_1$, *i.e.*, the arrangement must be so chosen that the tensions of the two portions of the band will tend to turn the lever DK in *opposite directions* with *equal moments.* It is easy to see that the brakes shown in Fig. 718 and Fig. 719 satisfy these conditions when

$$\frac{b_1}{b} = \frac{S}{S_1} = e^{\phi\gamma}.$$

Theoretically, the least force K at the lever suffices to work the brake, however great the moment $P r_1$. In reality there is only one particular position of the lever in which the equation $S b - S_1 b_1 = 0$ is satisfied. If we suppose the lever DK to be turned slightly in the direction of the force K out of the position shown in Fig. 718 and Fig. 719, we see that b_1 becomes greater and b smaller, the difference $S b - S_1 b_1$ assuming a certain value. At any rate the force K, whose moment $K a$ is always equal to the difference $S b - S_1 b_1$, will always be small, and the weight of the lever DK itself will suffice for applying the brake, the arrangement being so chosen that the brake is thrown out of action by lifting the lever. This brake was invented by *Napier*, and is known as the *differential brake* (see Civil-Ingenieur, 1863, p. 223).

FIG. 718.

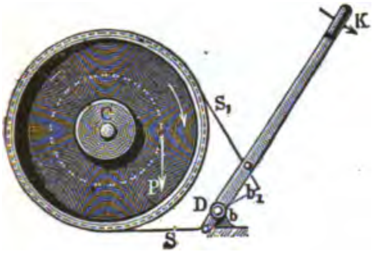
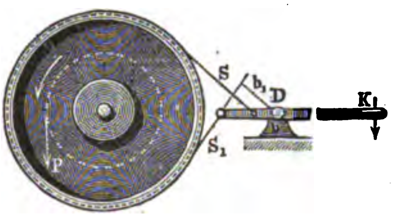
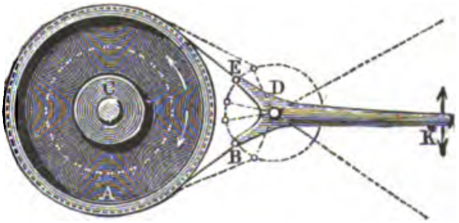


FIG. 719.



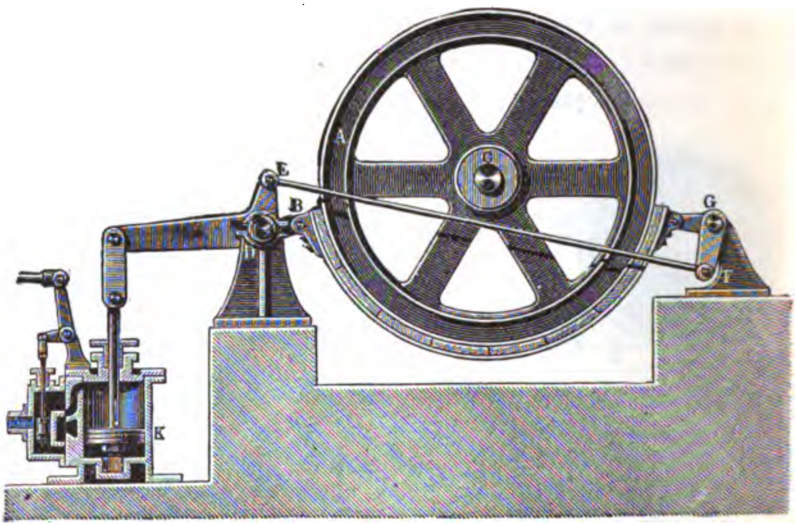
The brakes shown in Figs. 717 to 719 act most favorably when the rotation of the shaft C is in the direction of the arrows, for with an opposite rotation the pressure K to be exerted on the lever becomes considerably greater on account of the exchange of S and S_1. In ordinary hoisting gear the motion is always retarded in one direction, but in winding machinery and lifts it is necessary to retard it in each direction. For these cases the arrangement of brake-lever, Fig. 720, given by *Reuleaux* is particularly suited (see Civil-Ingenieur, vol. ix. p. 220).

FIG. 720.



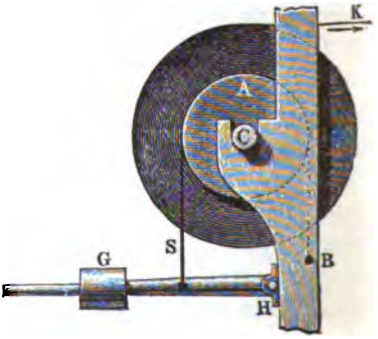
To exert a great force on the brake it is customary in winding engines to press the shoe or band against the drum by means of a special steam piston, Fig. 721.

FIG. 721.



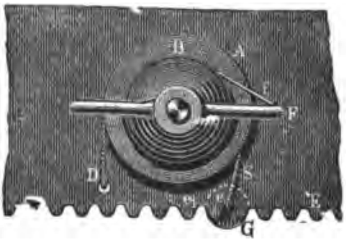
Among band-brakes we must also class the arrangement employed in *looms* for keeping the warp K which is wound on the warp-beam A, Fig. 722, in proper tension. For this purpose a cord S is wound around each end of the warp-beam, one end of the cord being fastened at B, while the free end carries a weight G, so that the warp K, as it gradually unwinds during the weaving, is kept taut by the friction generated. As the radius r of the warp-beam becomes smaller, self-acting regulators have been devised for moving the weight G toward the axis H of the lever, thus diminishing the friction of the cord S.

FIG. 722.



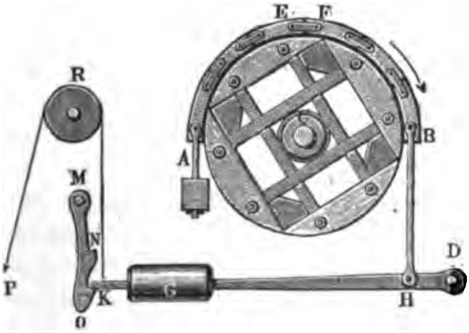
In the water-frame used in flax-spinning, the proper tension of the thread *f*, Fig. 723, is generated by providing the bobbin *B*, on which the flier *F* winds the thread, with a grooved pulley *A* on which runs a cord *S* fastened at *D* and loaded at *G*. As the thread is wound on the bobbin the radius r_1 of the body of the thread increases, and with the constant load *G* the friction of the cord may be varied by increasing its arc of contact, the cord being shifted from the notch *e* of the rack *E* to the next notch, e_1.

FIG. 723.



§ **179. Other Brakes.**—There are other brakes which occur less frequently but still deserve mention. We will call them the *link-*, *cone-*, and *eccentric-brakes*. The *link-brake* is used in *wind-mills* to control the sail-shaft, and differs from the band-brake principally in that the band is composed of a half ring *A B*, Fig. 724, made up of separate links like a chain. The separate wooden shoes which constitute this ring are united by wrought-iron links and pins *E F*. One end of the ring is fastened to a beam at *A*, and sustains the greater tension, *S*, while the smaller tension, S_1, is exerted at the other end of the ring by means of the connecting rod *B H* and the loaded lever *D K*. Ordinarily this lever is held at *N* by the click *M O* so that there is no contact between the brake-drum and the ring. To apply the brake, a jerk-like pull must be given to the rope *R P*. This causes the lever end *K* to push back the click so that the lever can fall into the hook *K*. The throwing off of the brake is effected by

FIG. 724.



pulling uniformly on the rope and then letting it back slowly. The brake-pressure may be regulated by shifting the weight *G* on the lever.

The resistance at the circumference of the brake-drum is determined like the friction of a chain sliding over a cylinder, Vol. I, § 198. If S_1 is the tension in the rod *BH*, the tension at the point *A* will be

$$S = S_1 \left(1 + 2\,\phi\,sin\,\frac{\gamma}{2\,n}\right)^n,$$

where γ is the arc of contact measured on a circle whose radius is unity, n the number of links, and $\frac{\gamma}{2n}$ half the angle at the centre subtending each link. The friction generated is therefore

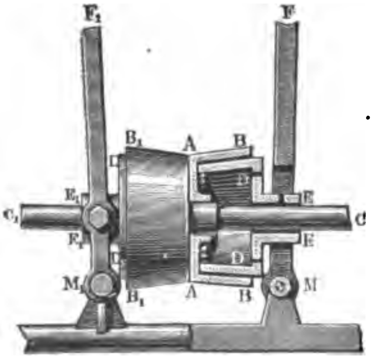
$$F = S - S_1 = S_1 \left[\left(1 + 2\,\phi\,sin\,\frac{\gamma}{2n}\right)^n - 1\right],$$

in which

$$S_1 = G\,\frac{a}{b},$$

where *G* is the load, *a* the distance *DG* of its centre of gravity from *D*, and *b* the arm *DH*. The brake-drum in wind-mills is usually formed by the rim of a toothed wheel on the sail-shaft.

FIG. 725.



In the *cone-brake*, Fig. 725, the shaft *C* to be controlled has keyed to it a double cone $B\,B_1$ turned on the inside so as to fit the two cones *D* and D_1 which enter it from opposite sides, these cones sliding loosely along the shaft *C*. By means of the disengaging levers *MF* and $M_1\,F_1$ the cones *D* and D_1 can be simultaneously pressed against the double cone $B\,B_1$, the ends *F* and F_1 being made to approach, by means of a screw, as in Fig. 710, or in some other manner. As D and D_1 are prevented from turning by bolts passing through the disengaging lever into the muffs E and E_1, there must be friction at the inner surface of the double cone when the shaft turns, which friction retards or stops the motion. The arrangement of two pairs of conical surfaces is chosen to avoid end thrusts along the shaft C and to double the force of the brake.

The friction at the conical surfaces can be determined as in conical friction wheels and friction couplings (see § 31 and § 52). Therefore if T represents the force with which each of the cones D, D_1 is pressed against the hollow cone BB_1 in the direction of the axis, the normal pressure which generates the friction at the conical surfaces is expressed by

$$R = \frac{T}{\sin\alpha + \phi\cos\alpha},$$

where α is half the angle at the vertex of the cones. Hence the friction at the two cones can be found from

$$2\phi R = \frac{2\phi T}{\sin\alpha + \phi\cos\alpha} = \frac{2T\sin\rho}{\sin(\alpha+\rho)},$$

where ρ is the angle of friction. The pressure T can be obtained from the brake-force K exerted on each lever at the distance a from M and M_1, thus:

$$T = K\frac{a}{b},$$

where b is the distance of the axis C from the fulcrums M and M_1. Therefore if an excess of force P acts at a distance r_1 from the shaft C, we have

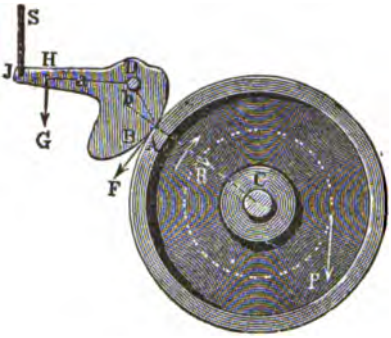
$$Pr_1 = 2K\frac{a}{b}r_2\frac{\sin\rho}{\sin(\alpha+\rho)},$$

where r_2 is the average radius of the conical surfaces in contact, and from this formula we obtain the brake-force to be exerted on each of the levers MF, namely,

$$K = P\frac{r_1}{r_2}\cdot\frac{b}{2a}\frac{\sin(\alpha+\rho)}{\sin\rho}.$$

In the *eccentric-brake*, Fig. 726, the desired friction F is developed at the brake-drum by an eccentric sector DB turning about D. To throw the brake out of action the cord JS is lifted, and to throw it into action the rope is lowered, thus allowing the weight G to press the sector against the drum A with a certain normal pressure R producing the friction ϕR which resists the rotation of the drum. Neglecting journal friction at D and C, we can determine the necessary weight G from

FIG. 726.

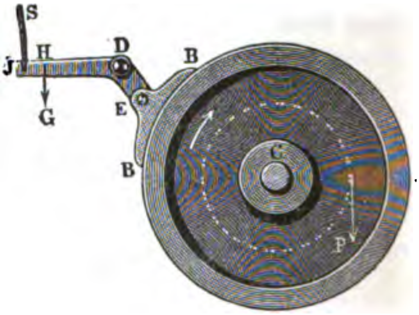


$$Ga = Rb,$$

provided a and b represent the lever arms of G and R with respect to the axis D of the eccentric.

This arrangement has the disadvantage of concentrating the pressure R at one point A, thus causing rapid wear of the eccentric and a change of form which must alter the action of the eccentric. To remedy this evil the eccentric may be replaced by the lever DE, Fig. 727, and a shoe jointed to the latter at E.*

FIG. 727.



In special cases brake action may be brought about by other means, but always by introducing an artificial resistance. For example, in ***hydraulic*** or ***pneumatic lifting apparatus*** the load is lowered uniformly by regulating the orifice of exit of the water or steam, and by completely closing this orifice the load can be brought to rest. In the same way the duration of the pauses in the strokes of a pumping engine is regulated by the cataracts. Further details concerning these and similar devices will be given later.

---

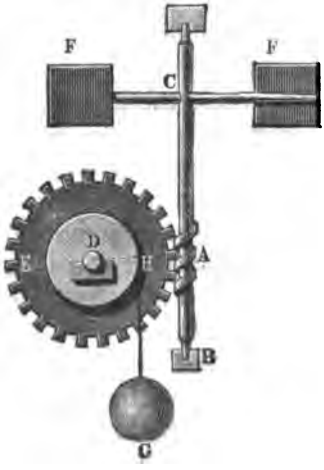
* For further information concerning these devices, see an article by *Reuleaux:* Ueber die Sperrwerke und ihre Anwendungen. Verhandlungen des Vereins zur Befoerderung des Gewerbfleisses, 1877, p. 17.

We may say generally of brakes, that there is always a loss of work connected with them, and their application is therefore undesirable from the standpoint of economy of power. Nevertheless they are indispensable in many cases, and many propositions have been made for preventing this loss of power which have, thus far, no practical importance. For example, it has been proposed to utilize the work of railway trains in rolling down inclined planes to accelerate large rotating masses, to compress air, etc., in order that this stored-up power may afterwards be expended in doing useful work. All brakes must be arranged to act as quickly and easily as possible, for safe running often depends largely on their prompt action. But if a powerful action of the brake is required, it is equally important that this action shall not take place suddenly, for then shocks would inevitably occur which would endanger the machine parts. These shocks occur principally when great living forces are to be destroyed by the brake, and less often when the brake serves to neutralize an excess of driving force. For this reason the elastic character of air-brakes renders them preferable for railway trains.

That a brake may be as effective as possible with a slight brake-pressure, it is generally advisable to apply it to machines at places where the velocity is great, *i.e.*, where the distance is great through which friction must be overcome, for in brake applications it is always a question of consuming large quantities of *work*. Therefore brakes are not applied directly to the shaft of a hoisting drum, but usually to a more rapidly moving shaft in the machine. Moreover, it is desirable that there be as few machine parts, shafts, wheels, etc., as possible between the brake and the point of application of the force to be destroyed or the mass to be retarded. For when the brake is applied, these parts are subjected to the moments introduced by the application of the brake, and the probability that breakages will occur increases with the number of the intermediate parts. For this reason the shafts of steam engines are retarded by brakes applied to the fly-wheels, and the shafts of water-wheels by brakes applied to their crowns or to brake-drums connected with them. In each case the best method depends on circumstances.

§ **180. The Fly** is an excellent means for generating uniform motion, its action, like that of the brakes, depending on the destruction of work. Therefore it is not applied in machines which continually perform great quantities of work, but only in cases where it is a question of producing as uniform motion as possible during a short period of time. It is often employed in the *striking gear of clocks* and in *Morse's recording telegraph*, in which the motion of the paper must be as uniform as possible. It is also used to advantage as a regulator in *chronometric apparatus;* in any case it exerts but a slight force. The fly consists principally of an arbor or spindle *B C*, Fig. 728, carrying two flat vanes *F* which are put in rapid rotation by a pair of gears, usually screw-wheels. The motion is communicated to the driving arbor *D* by a weight *G* or a spring, either directly as in the figure, or by means of intermediate gear. While the vanes rotate, the resistance of the air to their rotation increases as the square of their velocity, which is finally in equilibrium with the driving force of the arbor *D*, and from this instant on, the originally accelerated motion becomes uniform. The relations are here entirely different from those of clock escapements, for with the fly the velocity depends principally on the magnitude of the driving force, while the velocity of clocks depends on the duration of the oscillations of the pendulum or spring, which duration is but slightly influenced by the magnitude of the driving effort. For this reason the motion of the fly does not remain perfectly uniform, for it varies with the frictional condition of the apparatus and with the state of the atmosphere.

FIG. 728.



Let F represent the area of the two vanes or their projection on to a plane perpendicular to their motion when the vanes are inclined to the axis; let γ represent the specific weight, ζ the coefficient of resistance of the air, and v the velocity of the centres of the vanes; then, according to Vol. I, § 512, the resistance Q of the air is

$$Q = \zeta \frac{v^2}{2g} F \gamma.$$

Now if P is the driving force exerted by the wheel A at the radius r when all hurtful resistances have been subtracted, and if l is the distance of the centres of the vanes from the axis, we have

$$Q l = \zeta \frac{v^2}{2g} F \gamma l = P r;$$

hence

$$v = \sqrt{\frac{2g}{\zeta} \frac{Pr}{Fl\gamma}}.$$

Therefore, other things being equal, the velocity of the vane increases with the square root of the turning force. The coefficient of resistance ζ is not constant for all circumstances, and for small rectangular surfaces F it is given by the expression

$$\zeta = 1.254 \left(1 + 1.295 \frac{\sqrt{F}}{l}\right).$$

To recognize the influence of a slight variation of the turning force P, we differentiate the above formula

$$v = \sqrt{\frac{2grP}{\zeta F l \gamma}} = k \sqrt{P},$$

and then get

$$\delta v = k \frac{\delta P}{2\sqrt{P}}.$$

If we divide this lower equation by the upper one, we obtain

$$\frac{\delta v}{v} = \frac{\delta P}{2P},$$

from which we see that a certain percentage of variation of P involves only half that percentage of variation in v.

EXAMPLE. What weight G is needed to turn the vanes in Fig. 728 with a velocity of 10 m. [32.8 ft.] when the weight acts with a drum radius $DH = 0.12$ m. [4.72 ins.], while the average radius of the vane is $l = 0.20$ m. [7.87 ins.] and the length of each side of the square vanes is 0.10 m. [3.94 ins.], the toothed wheel DE having $z = 30$ teeth and the screw being double-threaded? If we assume that the loss by friction in the apparatus is 35 per cent, we obtain the turning moment about the arbor of the fly equal to

$$Pr = (1 - 0.35)\tfrac{2}{30}\, G \times 0.12 = 0.0052\, G. \quad [Pr = 0.205\, G.]$$

Moreover, as the area of each vane is 0.01 m² [15.5 sq. ins.], we have

$$\zeta = 1.254\left(1 + \frac{1.295\sqrt{0.01}}{0.20}\right) = 2.066.$$

The weight of a cubic meter of air is $\gamma = 1.25$ kilograms [weight of 1 cubic foot of air is 0.0807 lb.]; hence from

$$0.0052\, G = \zeta\,\frac{v^2}{2g}\,F\gamma\, l = 2.066\,\frac{100}{2 \times 9.81}\,2 \times 0.01 \times 1.25 \times 0.20 = 0.0527$$

we get

$$G = 10.14 \text{ kg. } [22.35 \text{ lbs.}].$$

The number of revolutions of the vane is given by

$$n = \frac{10 \times 60}{2 \times 3.14 \times 0.2} = 478.$$

If we suppose the hurtful resistances to vary between 30 and 40 per cent—in other words, if Pr can become $\tfrac{5}{65} = 0.077$ greater or less than the value calculated above, the number of revolutions would vary only $\tfrac{1}{2} \times 0.077 = 0.0395$, *i.e.*, the revolutions would vary between the limits

$$478 \times 1.0395 = 497 \quad \text{and} \quad 478 \times 0.9605 = 459.$$

As the weight falls with a velocity

$$w = \frac{2}{30}\,\frac{0.12}{0.20}\,10 = 0.4 \text{ m } [13.12 \text{ ft.}],$$

the work needed per second to move the fly is

$$Gw = 10.14 \times 0.4 = 4.056 \text{ meterkilograms } [29.34 \text{ ft. lbs.}].$$

§ **181. Counterweights.**—Counterweights are an excellent means for regulating force. In general they are *actual weights* which by their rise and fall regulate the intermittent or variable action of a force, or help to overcome a variable resistance. But the weights may be replaced by the pressure of water or air, in which case we may speak of *hydraulic* or *pneumatic* counterweights. When the motion to be regulated is a continuous circular one, the counterweight is rigidly attached to the rotating shaft, and when the motion is a reciprocating rectilinear or circular one, the action of the counterweight is trans mitted by levers or rollers to the machine part which is to be regulated.

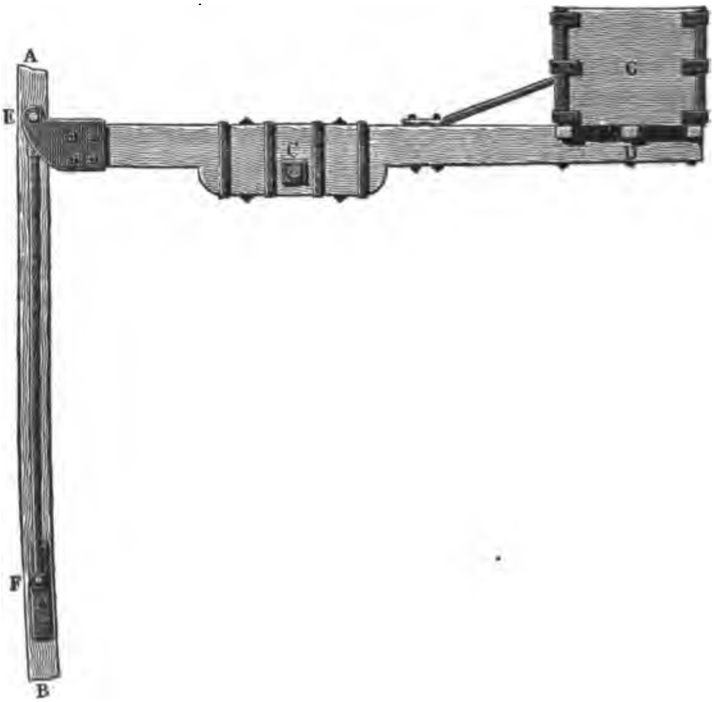
In continuous circular motion, for example the motion of a double crank, each revolution completes a period of motion, and within this period the counterweight must alternately rise and fall, being lifted by the excess of driving force and assisting the latter as it (the counterweight) sinks. In intermittent motion there is a similar rise and fall during a period of motion, but there may be *pauses* of any desired length between these up and down motions. In very many cases the counterweight simply serves to balance the weight of certain machine parts, in which case their motion must always be exactly opposite to that of the counterweight so that the latter sinks when the former rise, and *vice versa*. This explains why double-armed levers or rollers are used with intermittent motions. If the force to be balanced is a very variable one, we must subject the moment of the counterweight to corresponding variations. This may be done by varying the weight or the lever-arm. Sometimes the resistance to be overcome varies; an example of this is found in the hoisting gear of the shafts of mines, where the resistance is varied by the gradual winding of the rope.

In coupled and in double-acting engines or pumps the weights may be balanced by simply coupling the machines without any special counterweight, if the arrangement is so chosen that the parts of the two machines balance each other.

A simple application of the counterweight often seen at mines is shown in Fig. 729. A rod $A\ B$ needs a greater force on the up stroke than on the down stroke, principally on account of its own weight, consequently a counterweight G is employed which, by means of the beam $E\ C\ D$ swinging about

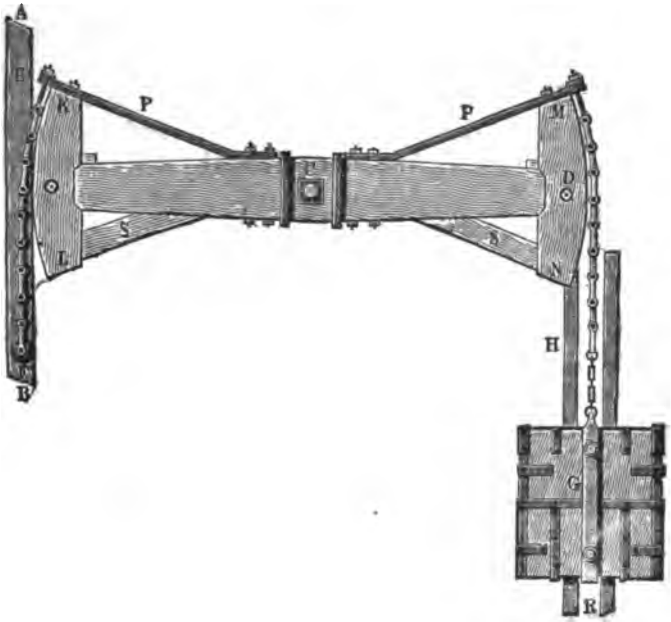
C, so acts on the rod that the up stroke is effected with the same force as the down stroke. As the lever turns about *C* it cannot act directly in the direction of the axis of the rods and must therefore be connected with the latter by means of the connecting rod *E F*. In order that the deviation of the path of the end *E* of the beam may differ as little as possible from the direction of the stroke, it is advisable to take the angle of deviation of the beam as small and the rod *E F* as long as convenient. Instead of a rod a linked chain *E F*, Fig. 730, is often employed, the chain keeping in contact with the sector *K L* while the beam oscillates. In like manner the counterweight *G* can be attached to the sector *M N* by means of chains, the lateral swinging of the weight being prevented by guide-rollers which move between the vertical guides *H R*. The struts *S* and the tie-rods *P* serve to stiffen the beam.

Fig. 729.



The angle of oscillation 2β of the beam can be determined from the stroke s of the main rod and the arm $a = CE$ of the beam. Whether the main rod is vertical or inclined, the arrangement in Fig. 729 is always so chosen that the perpendicular dropped from the centre of the beam on to the stroke bisects the angle of oscillation 2β of the beam, the latter deviating the same amount β to each side of its middle position. Hence when the main rod's motion is vertical, the middle position of the beam is horizontal, while an inclination α of the main rod to the vertical causes the middle position of the beam to deviate from the horizontal by the angle α. As the counterweight is always guided in a vertical direction, the middle position of its arm of the beam is always horizontal. Therefore when the main rod has a vertical motion the beam is a straight lever, and when the main rod's motion is inclined the beam becomes a bell-crank lever. Moreover the arrangement is so chosen that the stroke of the main rod bisects the rise of the

FIG. 730.



arc described by the end E of the beam, and this causes the connecting rod EF in its extreme positions to make with the stroke on one side the same angle that it makes when it is at the other side of the stroke and in its middle position. We have accordingly, for the suspension of the rods in Fig. 729,

$$s = 2a \sin \beta,$$

and for Fig. 730 simply,

$$s = 2a\beta.$$

The counterweight can easily be determined from the condition that the force P needed for the up and down stroke shall be the same. Let Q_1 represent the resistance of the main rod during the up stroke, and Q_2 the resistance for the down stroke. These forces act in the direction of the main rod and are composed of the *useful* (pump) resistances and the *hurtful* (frictional) resistances and of the weight of the rods. With vertical main rod, therefore, we have

$$Q_1 = W_1 + G_0 \quad \text{and} \quad Q_2 = W_2 - G_0,$$

where W_1 and W_2 each represent the useful and hurful resistances, and G_0 the weight of the main rod. Let us place $CE = a$ and the counterweight arm $CD = b$, and though these arms are frequently equal we will assume that they are unequal in order to make the case more general. Then, neglecting the friction of the beam, we have for the up stroke

$$P + \frac{b}{a} G = Q_1,$$

and for the down stroke

$$P - \frac{b}{a} G = Q_2;$$

adding these, we get

$$P = \frac{Q_1 + Q_2}{2},$$

and by subtracting them we obtain

$$G = \frac{a}{b} \frac{Q_1 - Q_2}{2}.$$

If we substitute

$$Q_1 = W_1 + G_0 \quad \text{and} \quad Q_2 = W_2 - G_0,$$

we get

$$P = \frac{W_1 + W_2}{2},$$

and

$$G = \frac{a}{b}\left(G_0 + \frac{W_1 - W_2}{2}\right).$$

For the case in which W_1 and W_2 are equal to each other and to W we have

$$P = W; \quad G = \frac{a}{b} G_0;$$

i.e., the counterweight simply serves to balance the weight of the rods.

We see from the expressions for P that the driving force needed for the motion depends only on the resistances W, the counterweight and weight of the rods having no direct influence on the value of P. Only so far as these weights generate friction at the journals do they increase P. In the above calculation the beam's own weight G_1 is not considered, for it is not needed for the determination of G when the beam is symmetrical to the axis of rotation, *i.e.*, when its centre of gravity coincides with the latter. But when the centre of gravity lies at the distance c from C we must in the above expressions increase the counterweight G by the value $\pm \frac{c}{b} G_1$.

The influence of the counterweight G on the journal friction is determined as follows: The pressure on the journals on the up stroke is

$$R_1 = G_0 + G_1 + G + W_1 - P = G_0 + G_1 + G + \frac{W_1 - W_2}{2},$$

and on the down stroke

$$R_2 = G_0 + G_1 + G - W_2 + P = G_0 + G_1 + G + \frac{W_1 - W_2}{2}.$$

Therefore not only is the pressure on the journal the same during both strokes, $R_1 = R_2 = R$, but its direction is also the same (downward). Consequently the work consumed by the friction at the middle journals with radius r during each stroke s is

$$\phi R \frac{r}{a} s = \phi \left(G_0 + G_1 + G + \frac{W_1 - W_2}{2}\right)\frac{r}{a} s.$$

Reduced to the end of the beam the journal friction becomes

$$\phi R \frac{r}{a}.$$

As this may be regarded as a part of W_1 and W_2 received from the above values for P and G, P is increased by the amount $\phi R \frac{r}{a}$, while the value of the counterweight is not influenced by it. The same is true of the friction at the journals of the connecting rod or at the links of the chain EF and MG, Fig. 730.

The beam and the counterweight increase the inertia of the machine, the mass M_1 of the beam and counterweight when reduced to the main rod, according to Vol. I, § 283, becoming

$$M_1 = \frac{T + Mb^2}{a^2},$$

where T is the moment of inertia of the beam with reference to its axis of rotation, and $M = \frac{G}{g}$ the mass of the counterweight. This mass influences the running and the velocity of the machine, but has no influence on the work performed, for the work expended in accelerating the masses at the beginning of the stroke is given back again towards the end of the motion when retardation occurs.

REMARK. In what has preceded it has been assumed that the driving force P is the same for the up as for the down stroke, as is usually the case in double-acting steam engines. But if the force P_1 for the up stroke differs from P_2 for the down stroke, as occurs, for example, in engines with comparatively thick piston rods, investigation must be carried on in the same way with P_1 and P_2, the ratio of P_1 to P_2 being ascertained according to the circumstances and introduced into the calculation. Here we will only consider the frequently occurring case of single-acting engines, where $P_2 = 0$. For this we have as above

$$P_1 + \frac{b}{a} G = Q_1$$

and

$$0 - \frac{b}{a} G = Q_2$$

by addition and subtraction we get

$$P_1 = Q_1 + Q_2 = W_1 + W_2$$

and

$$G = \frac{a}{b} \frac{Q_1 - Q_2 - P_1}{2} = -\frac{a}{b} Q_2 = \frac{a}{b} (G_0 - W_2).$$

For $G_0 = W_2$ we have $G = 0$, *i.e.*, a counterweight is not needed in this case.

EXAMPLE. If we suppose the rods to have the weight $G_0 = 10{,}000$ kg. [22,046 lbs.], and the resistance of the pumps on the up stroke to be 20,000 kg. [44.092 lbs.] and 4000 kg. [8820 lbs.] on the down stroke, what counterweight is needed for its equalization?

If the beam is symmetrical ($a = b$) and the counterweight suspended, and if the stroke $s = 1.5$ m. [59.06 ins.] and the whole angle of oscillation $2\beta = 50°$, we shall get the length $a = b$ of an arm of the beam from

$$a \frac{50}{360} \pi = 1.5, \quad \text{or} \quad a = 1.719 \text{ m. [67.68 ins.]};$$

the driving force needed is

$$P = \frac{20000 + 4000}{2} = 12{,}000 \text{ kg. [26,460 lbs.]},$$

and the counterweight needed is

$$G = 10000 + \frac{20000 - 4000}{2} = 18{,}000 \text{ kg. [39,680 lbs.]}$$

If the unloaded beam weighs $G_1 = 2000$ kg. [4410 lbs.], the pressure on the beam axle becomes

$$R = 10000 + 2000 + 18000 + \frac{20000 - 4000}{2} = 38{,}000 \text{ kg. [83,770 lbs.]};$$

consequently each journal sustains 19,000 kg. [41,885 lbs.]. Hence, when, for wrought-iron journals, the ratio

$$\frac{l}{d} = 1.5,$$

we have, according to § 3, for the diameter of the journal

$$d = 1.13 \sqrt{19000} = 155.7 \text{ mm. [6.13 ins.]}.$$

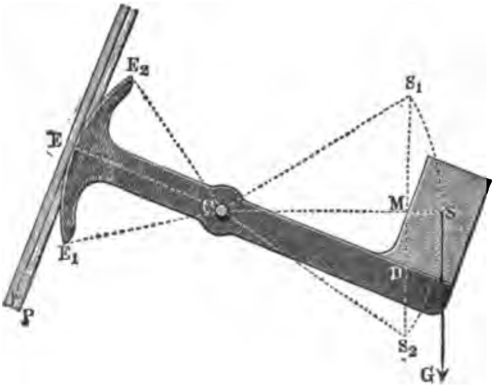
When the coefficient of friction $\phi = 0.08$, the work of friction at the beam centre during each stroke becomes

$$0.08 \times 38000 \frac{0.160}{2 \times 1.719} 1.5 = 212.2 \text{ mkg. } [1535 \text{ ft. lbs.}];$$

and when there are ten single strokes per minute, the work per second becomes

$$\frac{10}{60}\, 212.2 = 35.4 \text{ mkg.} = 0.47 \text{ horse-power.}$$

Fig. 731.



§ **182.** The preceding determination of the counterweight G was based on the supposition that the main rod and the counterweight were suspended from the sector-shaped ends of the beam by means of chains, as in Fig. 730. The formulas must be modified for the arrangement shown in Fig. 729. If we assume that the centre of gravity S of the loaded beam DCE, Fig. 731, is on the same level as the beam centre C when the rods are in their middle position, and that S swings to each side of the horizontal line CS through the angle $SCS_1 = SCS_2 = \beta$, then, if the distance $CS = b$, the counterweight will traverse the vertical path $S_1S_2 = 2\, b \sin \beta$, and therefore the work performed by the weight will be $2\, G b \sin \beta$. The path described by the main rod in the direction of its motion during the same time is $s = 2\, a\, \beta$ when the beam terminates in an arc, as in Fig. 731. Consequently the average value of the force exerted by the counterweight on the ascending rod is

$$p = \frac{2\, G b \sin \beta}{s} = \frac{G b \sin \beta}{a\, \beta};$$

and since

$$\sin \beta = \beta - \tfrac{1}{6}\beta^3,$$

approximately, we have by substitution

$$p = \left(1 - \frac{\beta^2}{6}\right) \frac{b}{a} G.$$

This force p, exerted by the counterweight, helps the rod on the up stroke and resists it on the down stroke; moreover, this force at the middle position CS becomes

$$p_1 = \frac{b}{a} G,$$

and in the extreme positions CS_1 and CS_2

$$p_2 = \frac{b}{a} G \cos \beta.$$

Consequently in the expression

$$G = \frac{a}{b} \frac{Q_1 - Q_2}{2}$$

deduced in the preceding article, we must substitute for the ratio $\frac{b}{a}$, which is now only correct for the middle position, the average value

$$\left(1 - \frac{\beta^2}{6}\right) \frac{b}{a},$$

and hence for $\frac{a}{b}$ the expression

$$\left(1 + \frac{\beta^2}{6}\right) \frac{a}{b},$$

in order to find the counterweight from

$$G = \left(1 + \frac{\beta^2}{6}\right) \frac{a}{b} \frac{Q_1 - Q_2}{2} = \left(1 + \frac{s^2}{24 a^2}\right) \frac{a}{b} \frac{Q_1 - Q_2}{2}.$$

For $2\beta = 60°$ or $\beta = \frac{\pi}{6} = 0.5236$ the counterweight becomes

$$G = 1.0457 \frac{a}{b} \frac{Q_1 - Q_2}{2},$$

i.e., about $4\frac{1}{2}$ per cent greater than when the counterweight is suspended. For the case where the rod is not suspended by chains but is connected to the beam by a long rod, as in Fig. 729, the stroke of the main rod may be very accurately obtained from

$$s = 2\,a\,\sin\beta,$$

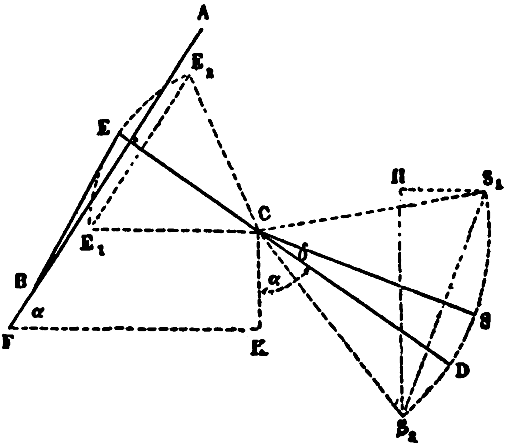
and the formula first found,

$$G = \frac{a}{b}\,\frac{Q_1 - Q_2}{2},$$

then retains its validity.

Remark. In the above investigation it was assumed that the centre of gravity of the loaded beam was on the same level as the beam centre when the beam occupied its middle position. When this is not the case the formulas may be developed as follows: Let the perfectly general assumption be made that the main rod AB, Fig. 732, makes any angle $AFK = \alpha$ with the horizontal, and hence that the longitudinal axis of the beam ED in its middle position makes the same angle $KCD = \alpha$ with the vertical. Moreover, let b again represent the distance CS of the centre of gravity from the beam centre, and δ the angle DCS made by this distance with the longitudinal axis of the beam ED. Then, if β again represents half the angle of oscillation, we have

Fig. 732.



$$S_1 S_2 = 2\,b\,\sin\beta.$$

Now the angle $HS_1S_2 = KCS = \alpha + \delta$; hence the vertical path traversed by the centre of gravity S during each stroke is

$$HS_2 = 2\,b\,\sin\beta\,.\,\sin(\alpha + \delta),$$

and the average force with which the counterweight acts on the main rod is as above:

$$p = \frac{2\,G\,b\,\sin\beta\,\sin(\alpha + \delta)}{s} = \left(1 - \frac{\beta^2}{6}\right)\frac{b}{a}\,G\,\sin(\alpha + \delta).$$

If we make $\alpha + \delta = 90°$, *i.e.*, draw CS horizontal, p reaches its greatest value, and we have as above

$$p = \left(1 - \frac{\beta^2}{6}\right)\frac{b}{a}\,G.$$

Therefore, for this most general arrangement of the weight, the counterweight is determined from

$$G = \left(1 + \frac{s^2}{24\,a^2}\right)\frac{a}{b}\,\frac{Q_1 - Q_2}{2\,sin\,(\alpha + \delta)}$$

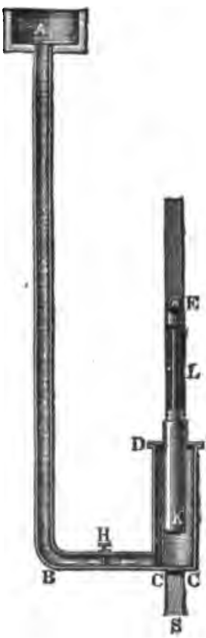
when the rod, as in Fig. 730, is connected with the sector by chains, and from

$$G = \frac{a}{b}\,\frac{Q_1 - Q_2}{2\,sin\,(\alpha + \delta)}$$

when the rod is connected with the beam by connecting rods, as in Fig. 729.

§ **183. Hydraulic Counterweight.**—Instead of balancing the pump rods by a beam loaded with a weight, hydrostatic pressure may be employed. An example of such an application of water pressure has been given in Vol. II, in connection with the valve gear of water-pressure engines. The arrangement of such an hydraulic equalizing device is shown in Fig. 733. Here the rod $E\,S$ is attached to a plunger $K\,L$ which works in the cylinder $D\,C$, whose lower surface is subjected to the vertical pressure exerted by the water in the pipe $A\,B\,C$. To avoid one-sided action of the piston K on the rod, the cylinder may be placed in the axis of the latter, which is made fork-shape for this purpose as in the figure.

FIG. 733.



If in this arrangement $F = \frac{\pi\,d^2}{4}$ represents the cross-section of the plunger and h the height of the water level above the end of the piston K in its middle position, the upward action on the piston will be given by $P = 1000\,F\,h$ kilograms (when d and h are expressed in meters) [$P = 62.5\,F\,h$ lbs. when d and h are expressed in feet]. The motion of the piston is accompanied by certain hydraulic resistances, such as the friction of the water in the pipe $A\,B$ and at the regulating valve H, which resistances may be determined according to the rules laid down in Vol. I, § 7, but in most cases the velocity v of the piston is so small and the pipes are so large that these resistances may be neglected. The friction of the piston at the stuffing box is much greater, and can be determined in the same way as the piston friction in Vol. II, thus: $K = \phi \pi d b h \gamma = 4 \phi F \frac{b}{d} h \gamma$, where b is the length of the stuffing box and ϕ the coefficient of friction. Consequently the upward action of the piston on the rod is given by

$$P_1 = P - K = \left(1 - 4 \phi \frac{b}{d}\right) F h \gamma,$$

and its resistance to the down stroke of the rod by

$$P_2 = P + K = \left(1 + 4 \phi \frac{b}{d}\right) F h \gamma.$$

From this follows that for every double stroke of the rod there is a loss of work

$$L = 4 \phi \frac{b}{d} F h \gamma \,.\, 2 s$$

which is due to the counterweight apparatus, s representing the stroke of the rod. This loss of work is a great disadvantage of the hydraulic arrangement, the journal friction of the ordinary counterweight being much less than the hydraulic resistance. On the other hand, the hydraulic balance requires little space and can be readily set up. An important advantage of the ordinary counterweight is that its action can be easily regulated by adding or subtracting weight, while the action of the hydraulic counterweight is varied with difficulty, for it must be done by varying the diameter d of the piston or the head h. It is easy to see that the position of the valve H will not accomplish this object, for the partial closing of this valve introduces a new resistance W which must be overcome by the rods as they descend, and must be subtracted from the upward action of the water column on the up stroke. This resistance W consumes, therefore, the work Ws during each stroke, acting in this respect like the friction of the piston rod. If ζ represents the coefficient of resistance for a certain position of the valve H (see Vol. I, § 443), d_1 the diameter of the pipe, and v the velocity of the piston of diameter d, then the passage of the water through the valve is accompanied by a loss of head h_1 given by

$$h_1 = \zeta \left(\frac{d}{d_1}\right)^4 \frac{v^2}{2g},$$

and the consequent loss of work during each stroke is

$$L_1 = F h_1 \gamma s = \frac{\pi}{4} \zeta \frac{d^6}{d_1^4} \frac{v^2}{2g} \gamma s.$$

Remark. The pressure on the piston may be exerted by compressed air instead of a column of water, the air being enclosed in an *air-chamber* communicating with the cylinder CD and the unavoidable loss of air being made good by a small air-pump. The force with which the air acts on the piston is, to be sure, variable, but these variations may be reduced as much as desired by giving a correspondingly large volume to the air-chamber.

If V represents the total volume of the air-chamber and supply-pipe, p the pressure of the air when the piston is in its lowest position, F the area and s the stroke of the piston, the tension of the air, when the piston is in its highest position, will be

$$p_1 = \frac{V}{V + Fs} p,$$

when we assume that the expansion takes place according to *Mariotte's* law, which is equivalent to neglecting the heat that has been generated and has disappeared.

The work performed by the expansion of the air during the up stroke of the piston is (Vol. I, § 388)

$$Vp \text{ hyp log } \frac{V + Fs}{V};$$

and as a portion $Fp_0 s$ is expended in overcoming the atmospheric pressure p_0, the work communicated to the pump-rod is

$$L = Vp \text{ hyp log } \frac{V + Fs}{V} - Fp_0 s.$$

Consequently the average force exerted by the piston on the pump-rod is

$$P = \frac{L}{s} = \frac{Vp}{s} \text{ hyp log } \frac{V + Fs}{V} - Fp_0.$$

If we take into account the piston friction

$$4 \phi \frac{b}{d} F (p - p_0),$$

the action of the piston on the rod becomes

$$P = \frac{Vp}{s}\, hyp\ log\, \frac{V+Fs}{V} - Fp_0 \mp 4\,\phi\, \frac{b}{d}\, F(p - p_0),$$

where the minus sign refers to the up stroke and the plus sign to the down stroke.

If $\frac{Fs}{V}$ is small, we can place

$$hyp\ log\, \frac{V+Fs}{V} = \frac{Fs}{V} - \frac{1}{2}\left(\frac{Fs}{V}\right)^2,$$

and then get

$$P = \left(1 \mp 4\,\phi\,\frac{b}{d}\right) F(p - p_0) - \frac{1}{2}\,\frac{Fs}{V}\,Fp.$$

EXAMPLE. The example discussed in § 181 may be solved as follows when an hydraulic or pneumatic counterweight is employed:

If we assume the disposable head of water to be 40 m. [131.2 ft.], the area F of the piston for an upward force of 18,000 kg. [39,680 lbs.] will be

$$F = \frac{18000}{40 \times 1000} = 0.45 \text{ square meter } [697.51 \text{ sq. ins.}],$$

which corresponds to a piston diameter

$$d = \sqrt{\frac{4 \times 0.45}{3.14}} = 0.757 \text{ m. } [29.8 \text{ ins.}].$$

If we assume $4\,\phi\,\frac{b}{d} = 0.1$, the work lost by piston friction during each stroke will be

$$4\,\phi\,\frac{b}{d}\, F h \gamma s = 0.1 \times 18000 \times 1.5 = 2700 \text{ mkg. } [19,530 \text{ ft. lbs.}],$$

or the work lost per second will be

$$2700\,\frac{10}{60} = 450 \text{ mkg.} = 6.0 \text{ horse-powers.}$$

On the other hand, if we employ a pneumatic counterweight with air of ten atmospheres pressure, and make the condition that, during the up stroke, the tension of the air be diminished by $\frac{1}{8}$ of its initial value, we get the area F of the piston from

$$\frac{V}{V+Fs} = \frac{7}{8} \quad \text{or} \quad F = \frac{V}{7s}.$$

When the stroke $s = 1.5$ m. [58.06 ins.] and $p_0 = 0.1\,p = 10{,}336$ kg. per m.^{2} [14.7 lbs. per sq. in.], the volume V of the air-chamber may be found from

$$P = \frac{Vp}{s}\,hyp\ log\,\frac{p}{p_1} - Fp_0 = \frac{Vp}{1.5}\left(hyp\ log\,\frac{8}{7} - \frac{0.1}{7}\right) = 0.0795\ Vp$$

[$P = 3.5\ Vp$, when V is in cu. ft. and p in lbs. per sq. in.].

or

$$V = \frac{18000}{0.0795 \times 10 \times 10336} = 2.191 \text{ cubic meters [77.38 cu. ft.]}.$$

Accordingly the area of the piston is

$$F = \frac{V}{7s} = \frac{2.191}{7 \times 1.5} = 0.209 \text{ sq. meter [324 sq. ins.]},$$

and the diameter

$$d = \sqrt{\frac{4 \times 0.209}{\pi}} = 0.516 \text{ meter [20.31 ins.]}.$$

Hence the loss of work by piston friction during each stroke is

$$4\,\phi\,\frac{b}{d}\,F(p - p_0)\,s = 0.1 \times 0.209 \times 9 \times 10336 \times 1.5 = 2917 \text{ mkg. [21,100 ft. lbs.]},$$

or the work thus lost per second is

$$\frac{10}{60} \times 2917 = 486 \text{ mkg.} = 6.5 \text{ horse-powers.}$$

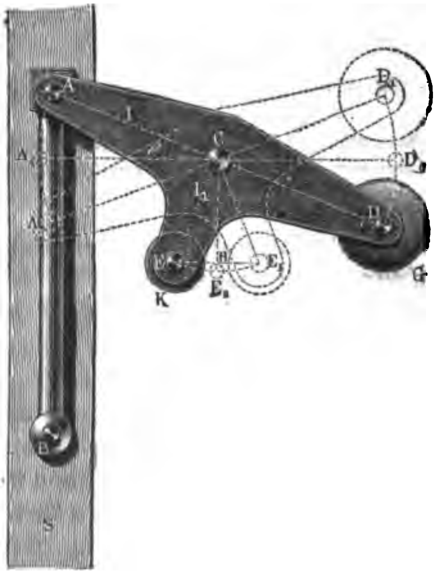
By comparing this result with that obtained in § 181 it is found that the hurtful resistances of the ordinary counterweight are only one twelfth of those of the hydraulic and pneumatic counterweights.

§ 184. Force Regenerator.—An interesting counterweight has been invented by *Bochkoltz* for drainage engines * which opens the delivery valves of the pumps at the beginning of the down stroke of the rods. Experience shows that to open a valve which is pressed against its seat by a head of water, there is needed a greater pressure per unit of area on its lower surface than it sustains on its upper surface, for the seating surface required by the valve makes one of the surfaces acted upon by pressure larger than the other. The lifting of the valve therefore requires a certain specific pressure which exceeds by 10 or 12 per cent that of the head of water resting on the upper valve surface. When, as is usually the case, this excess of pressure is obtained by increasing the load on the rods, there is the attendant disadvantage that when the valve is opened the greater pressure is no longer needed and the excess over the head of water is expended in accelerating the downward motion of the rods. As a consequence, shocks would occur towards the end of the down stroke were it not that the living force of the rods is destroyed by the introduction of an artificial resistance, for example by increasing the back pressure on the steam piston by contracting the exhaust passages. This causes loss of the work expended in lifting the aforesaid excess of load, and this loss is repeated during every double stroke of the piston. To avoid these losses as well as the shocks, *Bochkoltz* applied the counterweight shown in Fig. 734. Here the rod *S* is so far balanced by the weight *G* on the beam *A D* that the excess of weight of this rod just suffices to lift the head of water and overcome the hurtful resistances in the pipes, but not to open the delivery valves. To open these, a special weight *K* is attached to the arm *CE* of the beam; this arm is at right angles to the beam *A C D* and oscillates like a pendulum about its middle, vertical, position CE_0, deviating to each side of the latter by the angle β, so that, the weight alternately rises and falls through the distance $HE_0 = L_1(1 - \cos\beta)$. Now if *K* is so chosen that when it is in the extreme position *C E*, its weight just suffices to open the valves, the downward motion of the rod can begin. As this resistance to opening disappears with the beginning of the motion, the further action of *K* will be to accelerate the motion

---

* See Zeitschr. deutsch. Ing., 1872, p. 1; 1873, pp. 1, 79, 141; 1874, p. 449.

FIG. 734.



of the rod till the weight K has reached its middle, lowest, position E_0. During the further descent of the rod K again rises to the height E_1 from which it fell, thus storing up work equal to that which it performed during its fall. During this second half of the motion, therefore, the weight retards the rod, which reaches its lowest position with a velocity zero. Consequently there is no danger of shock and no need of destroying work by throttling the exhaust steam, the work performed by K being always stored up so that, when it returns to E from E_1 through E_0, it can assist in the lifting of the rod. On account of this property of utilizing the work performed in falling, the inventor has given the name of *force regenerator* to this arrangement.

This device not only economizes power and avoids hurtful shocks, but is said to have a favorable influence on the velocity of the main rod, which gradually increases its velocity up to the middle of its stroke and then gradually loses it till it is again zero. It is also evident that the two arms CD and CE with their weights G and K can be replaced by a single arm with a single weight equal to $G + K$ which is placed at the centre of gravity of the two loads. In this case the beam takes the form of an obtuse-angled bell-crank lever.

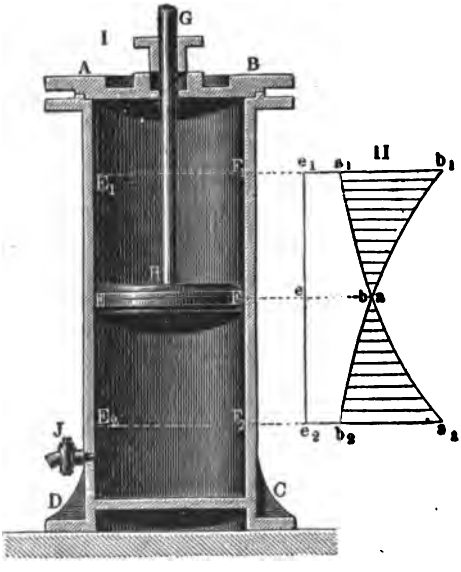
REMARK. In *Laboulaye's* Traité de Cinématique there is given a counterweight for the beam of a steam engine working with expansion, the counterweight being attached to an arm projecting above the centre of the beam. This counterweight is therefore at its *lowest* position when the piston is at the end of its stroke, and is lifted out of this position by the full pressure of the steam at the beginning of the admission. The work thus stored up is given out again by the sinking of the weight during the second half of the stroke, thus aiding the work performed during expansion and rendering the driving force more uniform.

The inventor has also substituted a pneumatic apparatus, Fig. 735, for the counterweight in the force regenerator. This apparatus consists of a cylinder $ABCD$ in which slides a tight-fitting piston EF whose rod GH is attached to the beam. The spaces AF_1 and CE_2 between the extreme piston positions and the cylinder covers are each made equal to V_0. If the whole cylinder is first filled with air of the ordinary atmospheric pressure when the piston is in its lowest position E_2F_2

and the piston is then brought into its highest position by the up stroke of the main rod, the volume $V + V_0$ of the air of atmospheric pressure p_0 contained in the space $A\,B\,F_2\,E_2$ will be compressed to the volume V_0 and will have the pressure

$$p = p_0 \frac{V + V_0}{V_0},$$

FIG. 735.



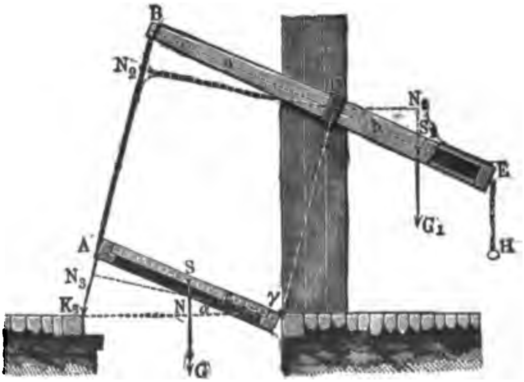
this compression requiring a certain expenditure of work by the main rod. During the first up stroke of the piston the air-cock J is open, and it is closed when the piston reaches the highest position $E_1\,F_1$, and after that is kept closed; the space $D\,E\,F_1\,C_1$ is thus also filled with air of the ordinary atmospheric pressure p_0. During the following down stroke of the main rod and the piston $E\,F$, the latter is acted upon by the downward pressure $F(p - p_0)$ and the values of F and p must be so chosen that this force will open the delivery valves. After this has been done the tension p of the air above the piston will diminish with further expansion and the tension p_0 below the piston will increase till the piston reaches its middle position $E\,F$, when the pressure will be equal on both sides. Up to this point therefore the air above the piston has performed a certain quantity of work L which has been expended in accelerating the rods. During the second half of the down stroke of the piston from E to E_2, the compression of the air below it will absorb exactly this amount of work L

from the main rod, so that the latter will reach its lowest point with the velocity *O*, provided the counterweight on the beam so far balances the main rod that the excess of weight just suffices to overcome the head of water and the hydraulic hurtful resistances. In the following up stroke of the main rod the same operation is repeated but in inverse order, the air under the piston performing the work of expansion *L* and assisting the main rod during the first half, and receiving the same quantity of work from the main rod for the compression of the air during the second half, of the stroke.

The diagram Fig. 735, II, shows the variations of pressure during the two strokes, and we see that the distances of the curves $a_1 a a_2$ and $b_1 b b_2$ from the straight line $e_1 e_2$ represent the pressures of the air above and below the piston respectively, and that each of the equal areas $a_1 b_1 a$ and $a_2 b_2 a$ represents the aforesaid work *L*. The resemblance between the pneumatic apparatus and the counterweight of Fig. 734 is clear from what has preceded. That the force regenerator introduces new hurtful resistances by increasing the piston friction and by increasing the journal friction is self-evident, but this disadvantage is more than counterbalanced by the advantages of the apparatus.

§ **185. Draw-bridges.**—There are many counterweight devices used in draw-bridges, their object being to balance the bridge or leaf in all positions so that it will only be necessary to overcome the hurtful resistances in lifting the bridge.

FIG. 736.



The simplest arrangement is that shown in Fig. 736, where the leaf *A C* is hinged at *C* and suspended from the beam *B D E* by the chains *A B*. The beam is so loaded by a counterweight G_1 that it is in equilibrium with the weight G of the bridge, the leaf being moved by workmen at H. If the axes C and D and the points of suspension A and B constitute the vertices of a parallelogram in which

$$CA = DB = a,$$

then, when the counterweight G_1 balances the bridge in any one position, it also balances it in every other position. To prove this let G be the weight of the leaf and $CS = C$ the distance of the centre of gravity S from the fulcrum C. When the leaf makes any angle $\alpha = ACN$ with the horizon the tension K will be generated in the chain AB and may be determined from

$$G c \cos \alpha = K a \sin \gamma,$$

γ representing the angle ACD of the parallelogram. Now if $DS_1 = b$ represents the distance of the centre of gravity of the loaded beam from the axis D, then, since the beam is always parallel to the leaf of the bridge, we have for the equilibrium of the beam

$$G_1 b \cos \alpha = K a \sin \gamma.$$

Hence

$$G c \cos \alpha = G_1 b \cos \alpha,$$

or

$$G_1 = \frac{c}{b} G$$

for every angular position α of the leaf.

If a beam is not used, say from considerations of space, the leaf of the bridge can be balanced by a counterweight acting on the chains AB, Fig. 737, which are fastened to the bridge at A and pass over fixed guide pulleys B. If $a = AC$ again represents the length of the leaf between the chain and the fulcrum C, and $c = SC$ the distance of the centre of gravity of the leaf from its fulcrum, the pull on the chain K for any inclination α of the bridge will be given by

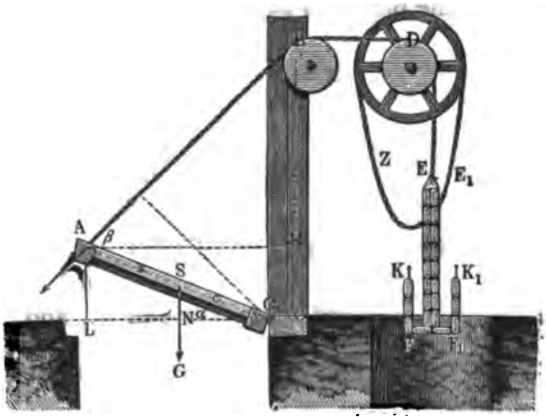
$$G c \cos \alpha = K a \sin (\alpha + \beta),$$

or

$$K = G \frac{c \cos \alpha}{a \sin (\alpha + \beta)},$$

where $\beta = BAM$ is the angle made by the chains with the horizon. This tension K varies with the angles α and β and becomes equal to zero when the leaf is vertical. Therefore if the bridge is to be in equilibrium in all positions, the action of the counterweight must vary with this pull on the chain. This may be accomplished in various ways, either by varying the counterweight acting directly on the chain, or by applying a constant counterweight and varying its lever-arm, or by so guiding a constant counterweight in a curved path that the component of the force transmitted to the chain will always equal the pull of the bridge. In *Poncelet's* device, Fig. 737, a variable counterweight ABD is directly attached to the chain ABD of the leaf, the chain being led over the guide pulleys B and D and loaded by the link chain EFK representing the counterweight. This chain, as the figure shows, is double and is suspended by its free ends from the fixed points K and K_1. These fixed points only sustain the weight of the portions FK of the chain directly below them, the weight of the other portions FE serving to balance the leaf. It is evident that the length of the portion FE diminishes in proportion as the end E sinks with the gradual lifting of the leaf of the bridge. The closed chain Z passed over the pulley R enables the attendants to turn the shaft of D in either direction and thus raise or lower the bridge.

Fig. 737.



To determine the relations of this balancing arrangement, we must find from the dimensions of the construction the value of the angle β contained in the formula

$$K = G \frac{c \cos \alpha}{a \sin (\alpha + \beta)}$$

found above for the pull on the chain. If the vertical distance of the axis C from the running-on point B of the pulley is represented by $OB = h$ and the horizontal distance by $CO = b$,* we have

$$\tan \beta = \frac{BM}{AM} = \frac{h - a \sin \alpha}{b + a \cos \alpha}.$$

When the bridge is down,

$$\alpha = 0 \quad \text{and} \quad \tan \beta = \frac{h}{a + b};$$

hence

$$K_1 = G \frac{c}{a \sin \beta} = G \frac{c}{a} \frac{\sqrt{(a + b)^2 + h^2}}{h};$$

and when the bridge is drawn up,

$$\alpha = 90°, \quad \tan \beta = \frac{h - a}{b}, \quad \text{and} \quad K_2 = 0.$$

Now if the counterweight chains are to establish equilibrium in the extreme positions, their length must equal

$$l = EF + FK = \sqrt{(a + b)^2 + h^2} - \sqrt{b^2 + (h - a)^2},$$

and their weight

$$G_1 = K_1 = G \frac{c}{ah} \sqrt{(a + b)^2 + h^2};$$

i.e., each unit of length of the double chain must have a weight $\frac{G_1}{l}$ if the chain is to be uniformly heavy.

---

* As the point B at which the chain runs on to the pulley varies with the position of the leaf, an average position must be chosen for this point, and average values must be assumed for h and b.

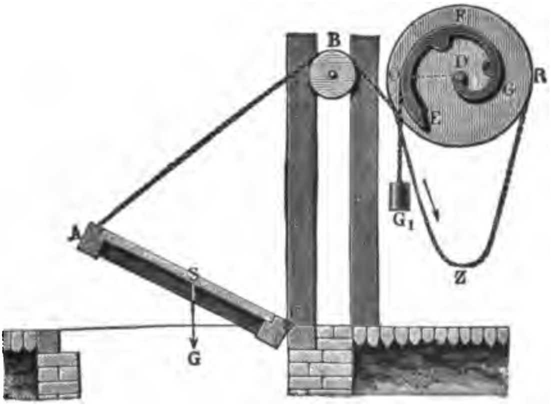
If equilibrium is also required at the intermediate positions, the links must be of unequal weight. To find the weight of the separate links in this case we must calculate by means of the preceding formula, for a series of values of $\alpha = 0°$, $10°$, $20°$, $30°$, . . . the corresponding values of β_1, and then from $\alpha + \beta$ find the corresponding pulls K, K_1, K_2, . . . on the chain. If at the same time we calculate the lengths l, l_1, l_2, l_3, . . . of the portion AB of the chain corresponding to these values of α, from

$$l = \sqrt{\overline{A M}^2 + \overline{B M}^2} = \sqrt{(b + a \cos \alpha)^2 + (h - a \sin \alpha)^2},$$

then the portions $l - l_1$, $l_1 - l_2$, $l_2 - l_3$, . . . of the counterweight chain must be given the respective weights $K - K_1$, $K_1 - K_2$, $K_2 - K_3$, . . .

In *Derché's* bridge, Fig. 738, a constant weight acts at the end of a variable lever-arm. Here each of the supporting chains $A B E$ of the leaf winds itself on a pulley R which is turned by the closed chain C when the bridge is to be raised or lowered. Attached to this is a spiral cam $O F G D$ on which the chain of the counterweight G_1 can be wound. When the leaf is raised G_1 is gradually lowered and the lever-arm $D O$ constantly diminishes, so that the turning moment of the coun-

FIG. 738.



terweight is in all positions in equilibrium with the moment of the weight G with respect to the axis C.

The formulas may be developed here as with *Poncelet's* balancing arrangement. If r is the radius of the pulley R, and z the variable radius of the spiral, we have $G_1 z = K r$ and hence

$$z = \frac{G\, r\, c \cos \alpha}{G_1\, a \sin (\alpha + \beta)}.$$

Therefore when the bridge is down, *i.e.*, when $\alpha = 0$, we have

$$z = \frac{G\, r\, c}{G_1\, a \sin \beta} = \frac{G\, r\, c}{G_1\, a\, h} \sqrt{(a + b)^2 + h^2};$$

on the other hand, when the bridge is drawn up or when $\alpha = 90°$; we have $z = 0$, for which, on account of friction, a radius equal, say, to that of the shaft D is assumed. If the spiral cam is to make only one complete revolution, the circumference of the pulley R must equal the length of the chain to be wound up, *i.e.*, we must have

$$2 \pi r = \sqrt{(a + b)^2 + h^2} - \sqrt{b^2 + (h - a)^2}.$$

We can again find the values $B_0, B_1, B_2, B_3, \ldots$ corresponding to a series of values of the angle of inclination of the leaf, $0, \alpha_1, \alpha_2, \alpha_3, \ldots$ and then, by means of the above formula, determine the radius-vectors $z_0, z_1, z_2, z_3, \ldots$ of the spiral. If we also calculate the corresponding lengths $l_0, l_1, l_2, l_3, \ldots$ of the portion of the chain between A and B and then subtract from these lengths the portion $\sqrt{b^2 + (h - a)^2}$, the remainders divided by $\frac{2 \pi r}{360°}$ will give the corresponding angles of rotation of the spiral cam. By means of these angles and the corresponding radii z, the spiral itself may be laid off.

Example. A hoisting arrangement with spiral cam is to be constructed for a draw-bridge in which the weight of the leaf is $G = 3000$ kilograms [6610 lbs.], the length of the leaf $a = 4$ meters [13.12 ft], the height $h = 5$ meters [16.4 ft.], the distance $b = 1\frac{1}{3}$ meters [4.37 ft.], and the centre of gravity of the leaf at a distance $c = 2$ meters [6.56 ft.] from the axis.

For the values

$$\alpha = 0^\circ, \quad 18^\circ, \quad 36^\circ, \quad 54^\circ, \quad 72^\circ, \quad 90^\circ,$$

we have, from the formula $\tan \beta = \frac{5 - 4 \sin \alpha}{\frac{4}{3} + 4 \cos \alpha}$,

$$\beta = 43^\circ\, 9', \quad 36^\circ\, 14', \quad 30^\circ\, 6', \quad 25^\circ\, 34', \quad 24^\circ\, 57', \quad 36^\circ\, 52';$$

hence

$$\alpha + \beta = 43^\circ\, 9', \quad 54^\circ\, 14', \quad 66^\circ\, 6', \quad 79^\circ\, 34', \quad 96^\circ\, 57', \quad 126^\circ\, 52'.$$

Now the length of chain to be wound on the pulley is

$$l = \sqrt{(1\tfrac{1}{3} + 4)^2 + 5^2} - \sqrt{(\tfrac{4}{3})^2 + (5 - 4)^2} = 5.644 \text{ m. } [18.5 \text{ ft.}];$$

therefore the radius r of the chain pulley must be

$$r = \frac{5.644}{2\pi} = 0.898 \text{ meter } [35.35 \text{ ins.}]$$

if it is to wind up the chain in one revolution.

If we make the largest radius vector of the spiral equal to this radius r, we get for the counterweight

$$G_1 = \frac{Gc}{a \sin \beta} = \frac{3000 \times 2}{4 \sin 43^\circ\, 9'} = 2193 \text{ kg. } [4835 \text{ lbs.}].$$

Now the variable radius-vector z may be found from

$$z = \frac{G c r \cos \alpha}{G_1 a \sin (\alpha + \beta)} = r \sin 43^\circ\, 9' \, \frac{\cos \alpha}{\sin (\alpha + \beta)} = 0.614 \frac{\cos \alpha}{\sin (\alpha + \beta)},$$

$$z = 0.898, \quad 0.720, \quad 0.544, \quad 0.367, \quad 0.191, \quad 0.00 \text{ meter}$$

$$[z = 35.35, \quad 28.34, \quad 21.42, \quad 14.45, \quad 7.52, \quad 0.00 \text{ inches}];$$

and for the length of the chain above the bridge,

$$l = \sqrt{(1.33 + 4 \cos \alpha)^2 + (5 - 4 \sin \alpha)^2} = \frac{1.33 + 4 \cos \alpha}{\cos \beta},$$

$$l = 7.31. \quad 6.33, \quad 5.00, \quad 3.75, \quad 2.59, \quad 167$$

$$[l = 287.8, \quad 249.2, \quad 196.9, \quad 147.6, \quad 102.0, \quad 65.7].$$

If the last of these values is subtracted from all the preceding ones we obtain the arcs of chain to be wound on the pulleys:

$$s = 5.64, \quad 4.66, \quad 3.33, \quad 2.08, \quad 0.92, \quad 0.00$$

$$[s = 222.1, \quad 183.5, \quad 131.2, \quad 81.9, \quad 36.3, \quad 00.0].$$

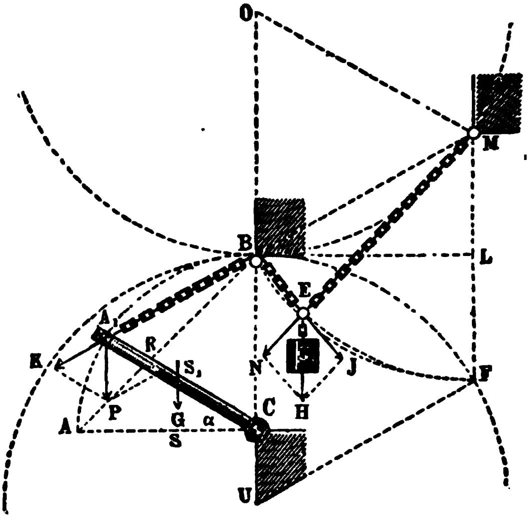
From this we finally get angles at the centre corresponding to the above radius-vectors

$$\phi = s\frac{360^\circ}{2\pi r} = \frac{57^\circ.296}{0.898}s = 63.78\,s,$$

$$\phi = 360^\circ,\quad 297^\circ.4,\quad 212^\circ.4,\quad 132^\circ.6,\quad 58.^\circ 9,\quad 0^\circ.$$

REMARK. The leaf of the bridge may be balanced by a constant counterweight which is guided along a certain curvilinear path. To illustrate this suppose the leaf CA, Fig. 739, to make any angle $A\,C\,A_1 = \alpha$ with its horizontal position. The weight G will cause a tension K in the chain passing over the guide pulley B, which can be ascertained graphically by resolving the vertical component $P = G\frac{c}{a}$ of G at A_1 into the forces R and K acting in the directions $A_1\,C$ and $A_1\,B$. The thrust R is sustained by the journals C of the bridge, and K produces tension in the chain. Now if the weight G_1 attached to the other end E of the chain $A_1\,B\,E$ is guided along the fixed guide curve $B\,E\,F$, only the component $E\,J$ of the weight $G_1 = E\,H$ will cause tension in the chain $E\,B$, this component being obtained by resolving $E\,H$ into forces acting in the direction $B\,E$ of the chain and of the normal $E\,N$ to the guide curve. That the bridge may be perfectly balanced the form of the guide curve $B\,E\,F$ must be so determined that in every position of the bridge the component $Z = E\,J$ of the counterweight shall equal the corresponding force K on the chain. The draw-bridges of *Belodor* and *Delile* depend on this principle. *J. Bernoulli* has

FIG. 739.



shown that the curve of equilibrium satisfying the above condition is an epicycloid when the point A of the bridge describes the circular path $A\ A_1\ B$.*

This curve can be easily constructed as follows: First place the weight G_1, equal to the tension

$$K = G \frac{c}{a \sin \beta}$$

when the leaf is in its horizontal position. When the leaf has turned through the angle α its centre of gravity will have risen the distance $h = c \sin \alpha$, and, according to the principle of virtual velocities, the counterweight will have fallen a distance $h_1 = \frac{G}{G_1} h$. The counterweight can therefore be found somewhere along a horizontal line drawn at the distance h_1 below the highest position of the counterweight. Moreover, as the distance of the counterweight from the pulley B is given by the known length of the free portion $B\ E$ of the chain, the position of the counterweight at any instant may be found at the intersection of the aforesaid horizontal with an arc described from B as a centre and the chain length $B\ E$ as a radius. In this way any number of points of the guide curve can be constructed.†

In an article in Der Civil-Ingenieur, 1861, p. 65, it was shown by *J. Noeggerath* that the fixed guide-curve could be replaced with great accuracy by a circular arc, the counterweight G_1 being suspended like a pendulum. If the distance from the axis c of the point at which the chain runs on to the guide pulley B is equal to the distance $A\ c = a$, the length of the chain between A and B will be $s = a \sqrt{2}$. The exact curve of equilibrium is represented by the epicycloid $B\ E\ F$ generated by rolling a circle U with radius $U\ B = s$ on the equally large circle of radius $O\ B$. If the equilateral triangle $B\ O\ M$ is drawn in this latter circle, we shall get at M the point of suspension of a pendulum whose length $M\ B = s = a \sqrt{2}$ and whose path not only passes through the point B of the epicycloid, but also touches the latter at its lowest point F, and between B and F passes sufficiently near to the exact curve of equilibrium. As the point A rises to the height $C\ B = a$ during a quarter-turn of the leaf, and the point B of the pendulum to the height

$$L\ F = B\ M \sin 30^\circ = \tfrac{1}{2} a \sqrt{2},$$

we find the necessary counterweight from the relation

$$P\ a = G\ c = G_1 \tfrac{1}{2} a \sqrt{2} \quad \text{or} \quad G_1 = \frac{c}{a} G \sqrt{2}.$$

The theory of the spiral drum and the equalizing carriage will be treated under the head of Lifting Apparatus.

§ 186. Counterweighting Cranks.—Counterweights are frequently needed with cranks to balance the reciprocating masses. This is evidently unnecessary when there are two diametrically opposite cranks of equal length r subjected to equal resistances Q_1 on the up stroke and Q_2 on the down stroke. For the weights G_0 of the equal rods and crank arms are in equilibrium in every position, the only influence of these weights being to increase the pressure on the crank-shaft by $2\ G_0$ and thus increase the journal friction of this shaft. If the resistance experienced by each of the cranks $A\,B$ and $A\,B_1$, Fig. 740, is expressed by $Q_1 = W_1 + G_0$ for the up stroke and by $Q_2 = W_2 - G_0$ for the down stroke, the pressure on the journals A, when the influence of the circumferential force P is neglected, is constantly

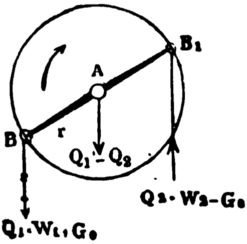
---

* See *Grunert's* Lehrbuch der Statik fester Koerper and *Poncelet's* Cours de mécanique app. aux machines.

† See also *A. Ritter's* Lehrbuch der analytischen Mechanik, p. 170.

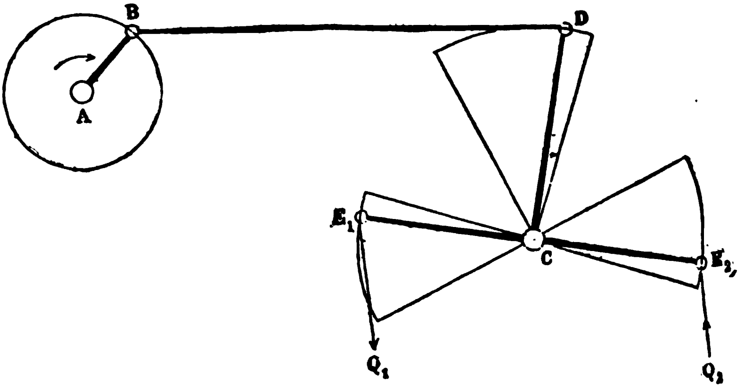
$$R = W_1 - W_2 + 2\ G_0 .$$

FIG. 740.



On the other hand, the journal friction at the crank pins B and B_1 is increased at one of the pins by the weight G_0 of the rods, as much as the friction is diminished on the other pin by the same influence; the weight of the rod therefore increases the crank-pin friction only when the crank-pins are made larger, by reason of G_0, than they would be if the rods were without

FIG. 741.



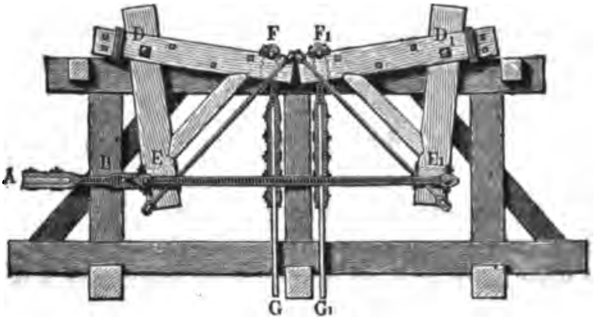
weight. This happens when Q_1 is greater than Q_2, for in the opposite case, in which $Q_2 > Q_1$, the diameter of the pin is calculated from the pressure $Q_2 = W_2 - G_0$, from which follows that, as the load on the crank pin is diminished by G_0, its diameter may be smaller.

In the present case the circumferential force P needed in the crank circle may be found from $P r \pi = Q_1 2 r + Q_2 2 r$ or $P = \frac{2}{\pi}(W_1 + W_2)$, exactly as in a single double-acting crank with piston force $Q = W_1 + W_2$.

These relations still hold when a single crank $A B$, Fig. 741, communicates reciprocating motion to two equal rods by means of the lever $C D E$, and also when for want of space the rods are moved by two bobs $D E F$ and $D_1 E_1 F_1$, Fig. 742, connected by a rod $E E_1$ which receives its motion from the connecting rod $A B$ of a crank.

On the other hand, when a crank is connected with only one rod, as in all steam engines and many pumps, the application of a counterweight for balancing the reciprocating masses is necessary for various reasons.

FIG. 742.



Let us first examine the case where the crank moves heavy pump rods for which the resistances $Q_1 = W_1 + G_0$ and $Q_2 = W_2 - G_0$ are very different, either on account of the weight G_0 of the rods or on account of the difference in W_1 and W_2. In this case the want of uniformity in the motion of the crank will be greater the more the values of Q_1 and Q_2 differ from each other. The formulas established in § 144 for *constant* piston force Q must be so far modified for the present case that on the up stroke we may substitute Q_1 and on the down stroke Q_2 in place of Q, and the circumferential force P must be expressed by $P = \frac{Q_1 + Q_2}{\pi}$ instead of by $P = \frac{2\,Q}{\pi}$. Then, as in § 144, the velocity of the crank pin for any angle of rotation α, estimated from the dead point, is given by

$$v = v_1 \left[1 + \frac{Q_1 r}{m_1 v_1^2} \left(1 - \cos \alpha - \frac{Q_1 + Q_2}{Q_1 \pi} \alpha \right) \right]$$

for the up stroke, and

$$v = v_2 \left[1 + \frac{Q_2 r}{m_1 v_2^2} \left(1 - \cos \alpha - \frac{Q_1 + Q_2}{Q_2 \pi} \alpha \right) \right]$$

for the down stroke, when m_1 represents the rotating mass reduced to the crank pin, v_1 the velocity at the lower and v_2 the velocity at the upper dead point. These velocities v_1 and v_2 are, of course, unequal because Q_1 and Q_2 are unequal, and it is evident that, other things being equal, there will now be less uniformity than in the case where $Q_1 = Q_2$, § 145, where

$$\delta = 0.4210 \frac{Q r}{m_1 v_1^2}$$

was found for very long connecting rods. Therefore in order to obtain a sufficiently uniform motion, the denominator $m_1 v_1^2$ must be very large, and as pumps only work with a moderate average speed v, very great rotating masses would be needed. To avoid this a counterweight should be applied so as to equalize the resistances for the up and down stroke, after which, since $Q_1 = Q_2 = Q$, the formulas of § 144 are directly applicable. It has been explained in § 181 how this equalization may be directly affected by a counterweight beam. But when such a beam is not employed the equalization may be effected by a counterweight attached to the crank shaft. A direct balancing of the rods has some advantages: it spares the rods and the crank pin by lessening the load on them, while the counterweight connected with the crank shaft increases the load on the shaft and the friction at its already large journals. In this connection it seems advisable to place the counterweight as far from the shaft as convenient, say by fastening it to the rim of the fly-wheel or water-wheel carried by the crank shaft.

If we represent the distance $A\,C$ of the counterweight G from the axis by b, we obtain, as in § 181, the counterweight

$$G = \frac{r}{b}\,\frac{Q_1 - Q_2}{2} = \frac{r}{b}\left(G_0 + \frac{W_1 - W_2}{2}\right)$$

needed to equalize the resistance.

For double cranks where the cranks $A\,B$ and $A\,B_1$, Fig. 744, are at right angles to each other, we can determine from the above formula the counterweight G_1 for each crank, and can then suppose the two weights acting at C and C_1 to be combined so as to form a resultant force $2\,G_1$ acting at D half way between C and C_1. This weight $2\,G_1$ acting at D can be statically replaced by a weight

FIG. 743.

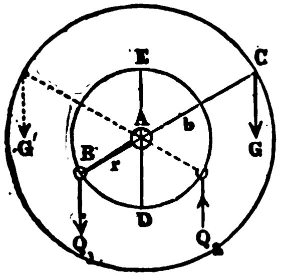
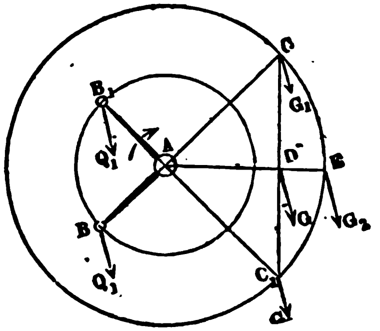


FIG. 744.



$$G = 2\,G_1\,\frac{A\,D}{A\,E} = 2\,G_1\,\sqrt{\tfrac{1}{2}} = G_1\,\sqrt{2}$$

acting at the distance $A\,E = b$, so that we have

$$G = \frac{r}{b}\,\sqrt{2}\left(G_0 + \frac{W_1 - W_2}{2}\right).$$

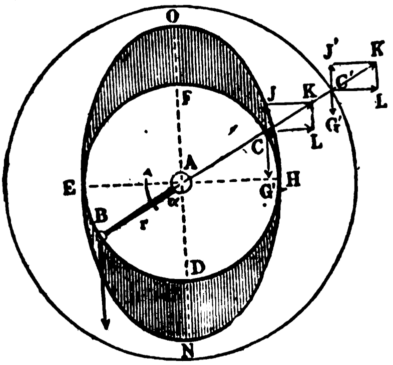
§ 187. Inertia of the Counterweight.—In the preceding paragraph only the statical action of the counterweight was discussed. So long as the statical actions are most prominent, *i.e.*, so long as the velocities of the mechanism are moderate as in pumps, the counterweight may be ascertained from the formulas which have been established. With quick rotary motions such as occur in most steam engines, the *inertia* of the counterweight and the reciprocating masses are most prominent, the *weights* of the latter being usually neglected because they are small in comparison with the other forces. In these cases, therefore, the counterweights are not intended to balance the *weights* of the reciprocating masses, but to balance their *living forces*. The following considerations will show to what extent this can be accomplished.

Attention is called in § 151 to the manner in which the forces of inertia of the reciprocating masses m_2 act in the crank train, and it is there shown that the pressure needed to accelerate or retard the reciprocating masses m_2 when the connecting rod is very long is expressed by

$$M = \cos \alpha \frac{m_2 v^2}{r},$$

where v is the average velocity of the crank pin and α the angle through which the crank has turned beyond a dead point. Between the angles of rotation $\alpha = 0$ and $\alpha = 90^\circ$ this force varies within the limits $\frac{m_2 v^2}{r}$ and 0. It was further shown that this accelerating pressure *resists* the crank motion during the *first* part of the stroke and *assists* the motion during the *second* part, and that it is immaterial whether the crank or piston is the driver. In the crank, Fig. 745, the resistance M due to inertia is laid off downward in the lower half HDE and upward in the upper half EFH, and the increase and decrease of the accelerating pressure are represented by the section-lined area, whose vertical ordinates represent the accelerating pressure M. It is also evident that this variable force of inertia renders the pressure on the crank pin B and that on the crank

FIG. 745.



shaft A variable, for the force P exerted by the piston and alternately increased and decreased by the pressure M is transmitted through the connecting rod to the crank. Now as long as these forces of inertia M are small because of slow motion, this circumstance is unimportant; but when the velocity v of the crank is great, the value of M increases rapidly, and it may happen that the accelerating pressure M is greater than the piston force. In this case there are certain positions of the crank pin outside of the dead points at which a reversal of pressure takes place, causing noisy and unsteady running; this is particularly true of steam engines working with great expansion and velocity. To avoid this a counterweight is attached to the crank shaft diametrically opposite to the crank. Suppose a counterweight G to be placed at the point C on the crank circle diametrically opposite to the crank pin B; this weight will exert a centrifugal force

$$CK = \frac{G}{g}\frac{v^2}{r},$$

whose vertical component,

$$CJ = \frac{G}{g}\frac{v^2}{r} \cos \alpha,$$

is opposed to the accelerating force

$$M = \frac{m_1 v^2}{r} \cos \alpha,$$

and is equal to the latter when $\frac{G}{g} = m_1$, *i.e.*, when the counterweight is taken equal to the weight of the reciprocating mass

m_2. To be sure the centrifugal force of this counterweight G introduces a new component,

$$CL = \frac{G}{g}\frac{v^2}{r} \sin \alpha,$$

perpendicular to the stroke, which by its change of direction tends to press the crank shaft from one side to the other, perpendicularly to the direction of the stroke. In *horizontal* engines this component is usually harmless, for it is directly taken up by the foundations, which can exert a much greater resistance in a vertical than in a horizontal direction. But in *vertical* engines this component

$$\frac{G}{g}\frac{v^2}{r} \sin \alpha,$$

on account of its horizontal direction, is more injurious to the stability of the high frame than is the inertia of the reciprocating mass m_2. The pressure due to the inertia of this mass is favorably sustained by the frame on account of its vertical direction. For this reason it is not customary to employ counterweights in vertical engines. Moreover in horizontal engines the counterweight G is usually taken smaller than the weight of the reciprocating masses m_2, a value of G equal to 0.5 to 0.8 $m_2 g$ sufficing, according to *Radinger*, for ordinary cases. If we assume $G = \nu\, m_2\, g$, where ν is smaller than unity, the force of inertia will be equal to the difference

$$m_2 \frac{v^2}{r} \cos \alpha - \frac{G}{g}\frac{v^2}{r} \cos \alpha = (1 - \nu)\, m_2 \frac{v^2}{r} \cos \alpha,$$

which is sustained by the crank shaft, the component perpendicular to the direction of the stroke being

$$\nu\, m_2 \frac{v^2}{r} \sin \alpha.$$

If the counterweight is placed outside of the circle at C' at a distance $A C' = b$, we must, to obtain the same centrifugal force, make its weight equal to

$$G' = \frac{r}{b} G.$$

It is tacitly assumed in the foregoing investigations that the connecting rod is very long. But as it has only a short length l in steam engines, a more exact determination of the force of inertia may be found from the formula

$$M = \left(\cos \alpha \mp \frac{r}{l} \cos 2 \alpha\right) \frac{m_2 v^2}{r}$$

given in § 151. Hence for a counterweight $G = \nu m_2 g$, the action exerted by the inertia of the masses will be

$$W' = \left[(1 - \nu \cos \alpha) + \frac{r}{l} \cos 2 \alpha\right] \frac{m_2 v^2}{r}$$

in the direction of the stroke, and

$$W'' = \nu \frac{m_2 v^2}{r} \sin \alpha$$

perpendicular to the stroke. The expression for W' shows that for a finite length l of the connecting rod there is no value of ν which will make the force W' equal to zero in all positions of the crank. The reciprocating parts could be perfectly balanced by placing opposite to the main crank a similar and equally heavy crank which, by a connecting rod equal to the first, gives a reciprocating motion, parallel to that of the piston, to a weight as heavy as the piston-rod, piston, and cross-head of the crank proper. It hardly needs to be said that such a clumsy construction is not employed in practice.*

Remark. The balancing of masses when the cranks are coupled at right angles is discussed under the head of Locomotives.

---

* An interesting method of counterbalancing engines is given by Prof. *S. W. Robinson* in Trans. Am. Soc. Mech'l Eng'rs, vol. ii.

§ **188. Fly Wheels.**—When machines *act normally* they have either a *uniform* or a *variable motion*, and we may then speak of them as having a uniform or variable *régime* (Ger. Beharrungszustand). A uniform régime is established in a machine only when there is continuous motion, particularly continuous circular motion, and then only when the moment of the force and that of the load do not vary during the motion. Consequently this condition of motion occurs principally in wheel work, for example in water wheels. On the other hand, variable motion occurs in all machines with *interrupted motion*, for these within short intervals of time pass from rest to motion and from motion to rest, and are therefore alternately accelerated and retarded. This condition of motion is found in all piston machinery, whether water-pressure engines or steam engines, on account of the reciprocating rectilinear motion of the driving piston. If in addition to this the piston force is variable, as in engines working with expansion, the motion of course becomes still more variable, but the state of motion depends not only on the prime mover, but also upon the machinery driven by it. If a machine performs its work with an interrupted, for example a reciprocating, motion, the machine will of course have variable motion and we may speak of it as having a variable régime. For this reason a pump or an ordinary gang saw is always in a state of variable motion even when driven by a water-wheel. When the moment of resistance of a machine is variable or when its useful resistance acts intermittently, it has a variable régime even though it possesses continuous motion. In this case, of course, there can be no statical equilibrium between force and load, for sometimes one and sometimes the other will be in excess and a machine will therefore be alternately accelerated and retarded. Consequently in trip hammers and rolling-mill trains we can only speak of a variable régime, that is, a variable state of motion. For example, a rolling-mill train is retarded during the rolling and again accelerated after the metal has passed through the rolls, when these are running empty.

Accordingly we have the following cases in which a machine possesses a variable régime:

1) The prime mover acts with a continuous circular motion and the machine driven by it with an interrupted, usually rectilinear, motion (water-wheel driving a pump).

2) The prime mover has an interrupted rectilinear motion and the machine driven by it a continuous, usually circular, motion (steam engine turning a hoisting drum).

3) Both the prime mover and the machine driven by it have a continuous circular motion, but the moment of the force or load, usually the latter, is variable (water-wheel driving a rolling-mill train).

4) Both the prime mover and the machine driven by it move with an interrupted, usually rectilinear, motion (steam engine driving a pump).

Whatever the motion of a machine may be and however it may perform its work, we can always demand that

1) those mechanisms which have a continuous (circular) motion shall run as uniformly as possible, and that

2) those mechanisms which have an interrupted (rectilinear) motion shall, during each period of motion (double stroke), pass from rest to motion and from motion to rest, gradually and without shocks.

As a rotary prime mover only works advantageously with a certain number of revolutions, it will be less efficient if it runs with alternately greater and less speed. For example, in the case of an overshot water-wheel, some of the buckets will contain too little and others too much water and the latter will begin to empty too soon, thus depriving the wheel of work. The same may be said of other rotary machinery; there is a certain velocity with which a machine works advantageously, and its efficiency is diminished or a poor product delivered when the number of revolutions varies greatly. It is equally necessary that a reciprocating machine, whether a prime mover or a machine driven by it, during each double stroke shall pass from rest to motion with a gradually increasing velocity and from motion to rest with a gradually diminishing velocity, but be subjected to no additional, greater, variations of speed. The shocks connected with sudden variations of velocity cause not only loss of efficiency in machinery, but also great wear and rapid destruction, and if the velocity of a machine, during a period of its motion, is subjected to great variations, the hurtful resistances and particularly the hydraulic resistances become greater than if the velocity did not vary greatly from its average value.

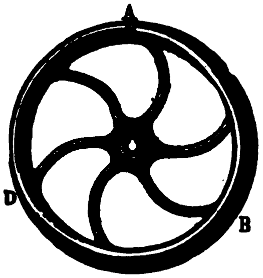
The best way to obtain as uniform motion as possible is to increase the masses, particularly the rotating masses. As the acceleration $p = \frac{P}{M}$ increases directly as the force P and inversely as the mass M, a certain excess of force or load must, of course, bring about a greater increase or decrease of velocity the smaller the mass M of the machine. If the mass of a machine is very small, it may not be able to assume any permanent régime; a moderate excess of driving force may suffice in a short time to give the machine an excessive velocity, and a moderate excess of the moment of resistance over the driving moment may quickly and completely deprive the machine of its living force, *i.e.*, bring it to rest. It is therefore necessary that the machine possess a certain mass and that this mass be greater the more regular and uniform the motion is to be.

The theory of crank motion showed that the revolution of a crank becomes more uniform the greater the masses connected with it.

In many cases the machine proper possesses so great mass or so great moment of inertia that its number of revolutions varies within very narrow limits, consequently its *degree* or *coefficient of fluctuation* is very small. But in other cases it is necessary to increase the rotating mass of a machine, in order that its fluctuations may be reduced to a certain degree. Rotating masses thus used solely to reduce the fluctuations of a machine are given the form of a wheel and are then called *fly wheels*. Therefore fly wheels are very important parts of machines with a variable régime, without which parts the machines either could not perform their work at all, or at best very imperfectly. Large and rapidly running water wheels, large drums and toothed wheels, rapidly running mill- and grind-stones, etc., of course, act as fly wheels and often take their place. For example, saw mills, pumps, etc., would need fly wheels when working, were it not that they are driven by water wheels whose inertia keeps the variations of velocity within narrow limits in spite of the great variations which here occur in the moment of the load.

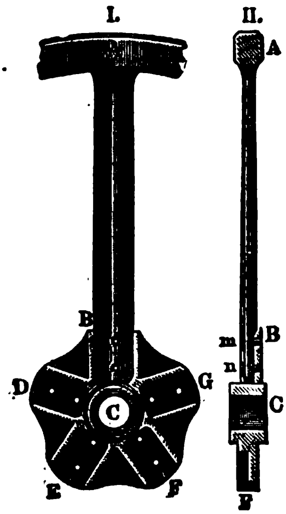
§ **189.** Fly wheels are generally of cast iron and consist of *rim*, *arms*, and *nave* or *boss*.

Fig. 746.



A small fly-wheel cast in one piece is shown in Fig. 746. *A B D* is the rim, *C* the nave, and *B C*, *D C*, etc., the arms of the wheel. These arms are often curved in order that they may not separate from the rim nor be subjected to injurious tension by the cooling that takes place after they are cast. To reduce the resistance of the air as much as possible, both arms and rim are rounded or both portions have an elliptical cross-section. Larger fly wheels are cast in two pieces in order to avoid unequal cooling, the boss being separate from the rim and arms. Fig. 747 shows a part of such a fly-wheel, I. representing a front view and II. a section. *A* is a piece of the rim, *A B* an arm of the wheel, and *B D E F G* the boss with its eye *C*. The boss is made to receive five arms, each of these being attached to it by two bolts *m*, *n*, but the nave may also be cast in one piece with the arms, the rim being cast separately. Large fly wheels 5 meters [16 ft.] in diameter are almost always composed of several pieces. Each segment of the rim may be cast with an arm, or the arms may be cast in one piece with the nave, or finally, and this is necessary in wheels with very great diameter, the arms may be cast separately and then connected with the rim and boss by means of bolts, keys, etc. The segments of a fly-wheel rim may be united to each other by

Fig. 747.



lapping and by bolts: but the ends of the segments may also be butted against each other, two recesses being cast, one on each side of the ends of the segments and dovetailed wrought-iron plates fitted into each and bolted, or holes are cast in the ends of each segment and fitted with rectangular wrought-iron bars which are keyed to the segments. The first method of connecting segments is shown in Fig. 748. *A* and *B* are the two ends that overlap, *D D* and *E E* the bolts drawn through them, and *F F* and *G G* the inserted splines. The second method is shown in Fig. 749; *A* and *B* are the two ends of the segments that butt against each other, *D D* and *E E* the plates inserted in the recesses and united by the bolts *D E*, *D E*, and *F* an intermediate spline. Fig. 750 shows the method of connecting segments by inserting wrought-iron bars; *A* and *B* are the ends of the segments, *C D* is the inserted bar, not visible from the outside, and *E* and *F* the keys by means of which the ends of the segments are driven up against each other.

FIG. 748.

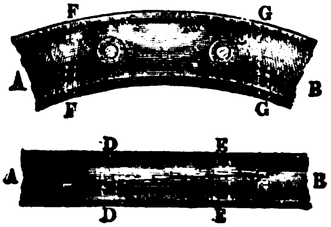


FIG. 749.

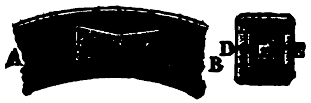


FIG. 750.

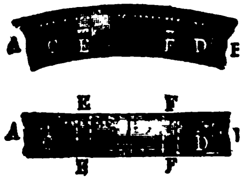
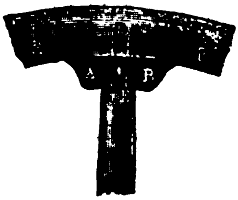


FIG. 751.



The simplest connection between rim and arms is made by keying the dovetailed ends of the arms to corresponding recesses on the inner surface of the rim. If the radial spline *D E*, Fig. 751, is also added, the connection between the dovetail *A B* and the rim *R* will be strengthened. Instead of dovetails, lugs are sometimes used, as in Fig. 752, which figure also shows the method of connecting the arms with the boss; *A B* is the arm, *D D* are the lugs in the rim, *E E* the lugs in the boss, and *F F* . . . and *G G* . . . the bolts which press the ends of the arms against their bearing surfaces. The segment connections either lie between the arms or at them; the latter arrangement is shown in Fig. 753. Here the arms of the wheel form one piece with the boss *A B* keyed to the shaft *C*, and are connected with the rim by dovetails *D* and bolts *F F*. If the shaft carrying the fly wheel needs a counterweight to equalize the weights, it may be attached to the fly wheel by placing a certain amount of lead opposite the centre of gravity of the weight to be balanced, the lead being poured into a hole cast in the rim for this purpose.

Fig. 752.

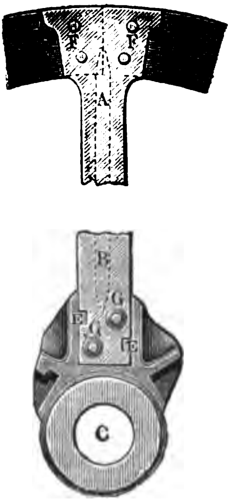
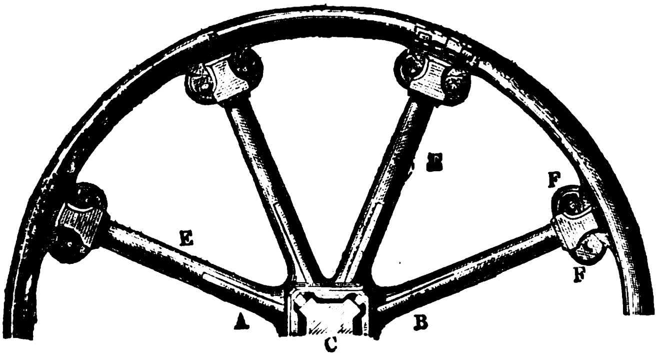


Fig. 753.



§ 190. Action of Fly Wheels.—The action of fly wheels can be explained as follows: If in any machine the work performed by the driving force in a certain small interval of time is exactly equal to the work consumed by all the useful and hurtful resistances in the same time, the velocity of the machine will not vary during this interval. But when no such equality exists between the quantities of work performed by the driving force and load, there will be a change in the speed of the machine, the velocity becoming greater or less according as the work performed by the driving force is greater or less than that performed by the resistances. If A represents the difference between these quantities of work during a certain time and M the total mass of the moving parts reduced to a certain point (see Vol. I, § 283), the velocity v of the point at the beginning of the interval will change to v_1 at the end of the interval, the relations between these quantities being given by the equation

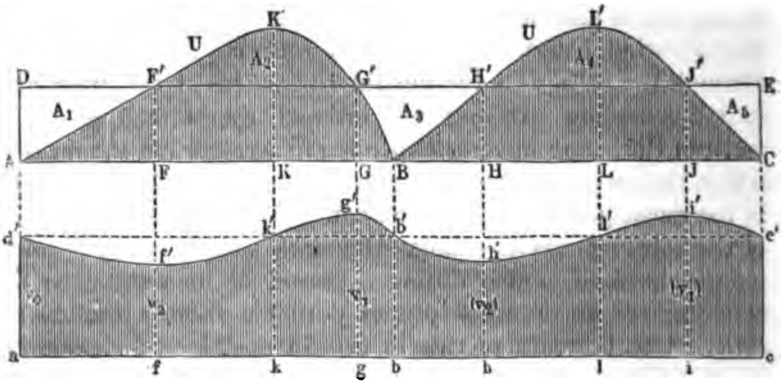
$$A = M\frac{v_1^2 - v^2}{2},$$

i.e., the increase of the living force of the machine is equal to the excess of work. The final velocity v_1 will be greater than the initial velocity v if A is positive, *i.e.*, if the work of the driving force is in excess. In the inverse case the velocity of the machine will be less. The mass M of the moving machine parts can be regarded as a storehouse of energy which is taken up or given off according as there is an excess or a deficiency of work in the machine. There is, of course, a limit to the taking up and the giving off of work, for with continued storing up of living force, the velocity would very soon exceed its practically permissible value, while a continual giving off of work would quickly stop the machine. Such a continual acceleration always occurs in starting a machine, the acceleration continuing till the machine runs with the velocity needed for its normal action. The *stopping* of a machine is characterized as a short period of uninterrupted work by the moving masses, which period is ended when the machine comes to rest. When a machine attains the state of motion between starting and stopping, which corresponds to its proper, normal, action, we may say that it has established its *régime* (Ger. Beharrungszustand). In this state of motion the velocity is, to be sure, subject to certain fluctuations caused by the difference between driving force and resistance; nevertheless regular periods continually recur within which the work of the driving force is exactly equal to that of the resistances. For example, such a period in a steam engine is represented by one revolution of the crank, as was shown in the investigations in Chapter VI. But cases do occur where the duration of a period depends, not on the rotation of a crank, but on the nature of the work. For instance, in *rolling mills* the resistance experienced by the bar in passing through the rolls is very great, but during the following pause, before the bar returns and again passes through the rolls, only the hurtful resistances of the train are to be overcome. During this interval, therefore, the whole work of the prime mover is expended in accelerating the masses which thus store up living force that greatly assists the machine in the subsequent rolling of the bar. Consequently the action of the rotating masses enables the work performed by a comparatively small engine during a period of its motion to be concentrated on the work proper during a shorter interval of time. Use is made of this in certain machines, for example *punching machines* and *coining presses*, where the resistance to be overcome during a very short time is so great that to overcome it directly, the steam engine would need to be of extraordinary dimensions, while by the use of a fly wheel having sufficient mass and velocity a moderately large engine accomplishes the same object.

A clear idea of the action of fly wheels, for example in steam engines, can be obtained from diagrams like those used to illustrate crank motion in § 153. For this purpose let the base line $A\ C$, Fig. 754, be made equal to the circumference $2\,\pi\,r$ of the crank circle of a double-acting steam engine, the ends, A and C, corresponding to one dead point and the middle, B, to the other. Also let the curve $A\ F'\ K'\ B\ L'\ C$ be so drawn that its ordinates will represent the tangential turning forces exerted by the piston at the circumference of the crank pin circle. Then if $A\,D = C\,E$ represents the constant resistance P of the engine, reduced to the crank pin, the rectangle $A\ E$ will represent the work consumed by the resistance during one revolution and the area included between the curve

$A K' B L' C$ and the axis $A C$ will represent the work of the steam during the same period. When the machine has established its régime, these two areas are equal. Now if we assume that the crank pin has the velocity v_0 at the dead point A, and if we make $a d'$ equal to it and then lay it off from another base $a c$, we see that the crank starting from the point A will at first be subject to a retardation which will continue till the driving force or the ordinate of the curve U is smaller than the resistance $A D = P$. The velocity v_0 of the crank at A will therefore assume at F a smaller value v_2 which can be determined from the equation

FIG. 754.



$$A_1 = M\frac{v_0^2 - v_2^2}{2},$$

where M is the mass of the moving parts reduced to the crank pin, and A_1 the work performed by the excess of resistance, which work is represented by the area $A D F'$. The velocity $v_2 = f f'$ at the point F is relatively a minimum, for with further motion an acceleration takes place which continues up to the point G. During the motion from F to G the work a_2 represented by the area $F' K' G'$ and performed by the excess of driving force of the steam has brought the velocity from v_2 to v_1, for which we have

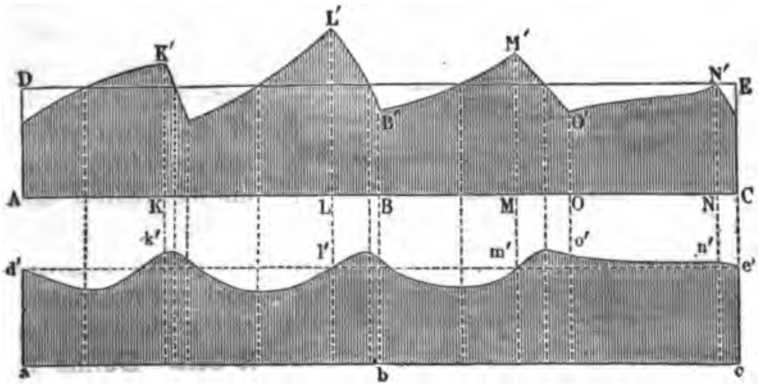
$$A_2 = M\frac{v_1^2 - v_2^2}{2}.$$

The velocity v_1 at G is relatively a maximum, for another retardation now begins; this lasts till the dead point B is passed and the point H reached, where the velocity assumes a new minimum (v_2), etc. It is plain that the velocity of the crank at C is again v_0 as at A, for when the engine's régime is established we have the relation

$$A_1 + A_3 + A_5 = A_2 + A_4,$$

i.e., the algebraic sum of all the work successively expended in retardation and acceleration during a revolution of the crank is equal to zero. As this last relation is perfectly general even though some other than the dead point is chosen for the starting point, it follows that when the crank is at one and the same point it is always in the same state of motion, *i.e.*, the period of the variable motion is one whole revolution. Moreover as regards the curve $d' f' k' g'$ for the velocities, we see that the

FIG. 755.



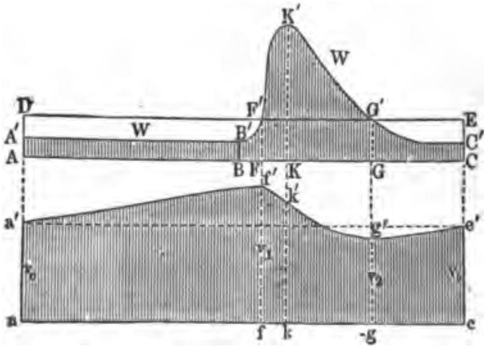
point k' corresponding to one of the maximum values KK' of the turning force must be a point of inflection, for at this point the accelerating force $p = \frac{P}{M}$ is a maximum, *i.e.*, $\frac{\delta p}{\delta t} = 0$; and since $p = \frac{\delta v}{\delta t}$, the condition

$$\frac{\delta p}{\delta t} = \frac{\delta^2 v}{\delta t^2} = 0,$$

for a point of inflection, is satisfied for the point f' of the velocity curve.

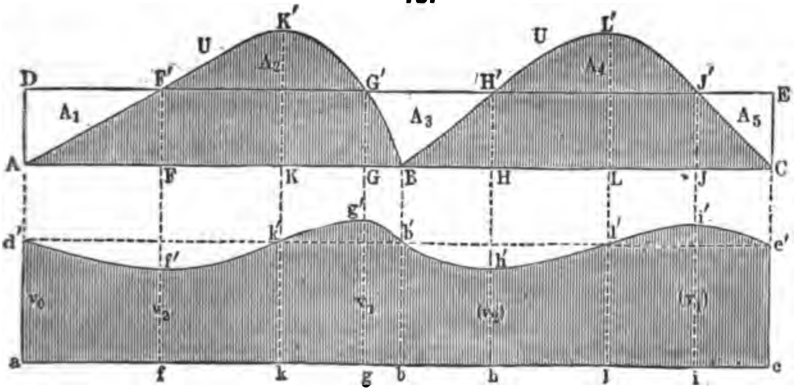
In like manner Fig. 755 is a diagram in which the base line $A\,C = 2\pi r$ and the ordinates of the line $A\,K'\,L'\,B'\,M'\,O'\,N'\,C$ represent the circumferential pressures U for the forward and return stroke of the double-cylinder engine working with expansion, for which the diagram was constructed in Fig 578, on the stroke $2r$ as a base, the line $d'\,k'\,l'\,m'\,o'\,n'\,e'$ representing the corresponding velocities. It is evident from this that such a diagram gives a clear picture not only of the comparative magnitude of the individual fluctuations of velocity, but also of their positions relatively to each other.

FIG. 756.



The diagram may be similarly constructed for other cases, for example in machines in which a great resistance occurs suddenly. Fig. 756 represents the motion of a *punching machine.* Here the slide carrying the punch is slowly moved up and down by a crank or an eccentric and has to overcome a considerable resistance during a portion of its down stroke, the resistance attaining its greatest value at the instant when the punch begins to push out the material of the plate on which it acts. During the shearing off of the material this pressure becomes gradually smaller, and ceases when the sheared-off piece is completely separated from the plate. When this is done the punch has not yet reached its lowest position, but continues to move a short distance through the punched hole, overcoming only the hurtful resistances during this motion. The same occurs on the up stroke of the punch. This process is illustrated by the line $A'\,B'\,F'\,K'\,G'\,C'$ drawn over the base $A\,C$, the ordinates of the line representing the resistance acting *tangentially* to the crank circle. We see that during the up stroke AB and a portion BF of the down stroke, corresponding to a rotation of 20° to 30°, this resistance is comparatively small and then passes almost instantaneously from BB' to KK', after which it gradually diminishes, and just before the return stroke is ended becomes equal to the original value $AA' = CC'$ of the hurtful resistances. Now if the crank shaft is turned by a constant circumferential force, for example the pressure of a toothed wheel keyed to it, and if this force is represented by $AD = CE$, the rectangle AE will represent the work done by the driving force during an up and down stroke of the punch. This rectangle must equal the area included between the base AB and the line W of the resistance, and the fluctuations of velocity can be determined as in the preceding cases. In the figure, $aa' = ce'$ again represents the velocity v_0 at the lower dead point, the greatest velocity, v_1, occurring at the point F where the punch already exerts a certain pressure on the plate. The smallest velocity, v_2, occurs at the point G, where the total useful and hurtful resistance becomes equal to the pressure GG' exerted by the wheel or belt. That the rapid increase of the resistance must cause a rapid diminution of velocity along the portion fg of the stroke is self-evident.

FIG. 757.



§ 191. Weight of the Rotating Mass.

—There is now no difficulty in calculating the rotating mass m in any particular case. We see from the diagrams, say that shown in Fig. 757, for a one-cylinder engine, that the fluctuations of velocity between a minimum value $ff' = v_2$ and the following maximum, $gg' = v_1$, depends on the size of the area $F'\,K'\,G' = A_2$. In like manner $G'\,H'\,B = A_3$ below the line $D\,E$ of resistance determines the decrease of velocity from $gg' = v_1$ to $h\,h' = (v_2)$, and the area $H'\,L'\,J' = A_4$ the increase of velocity $i\,i' - h\,h'$. Finally, the two areas

$$J'\,E\,C + A\,D\,F' = A_5 + A_1,$$

which we must regard as continuous areas, determine the diminution of velocity from $i\,i' = (v_1)$ to $ff' = v_2$.

Now if A represents the greatest of these areas A_1, A_2, $A_3 \ldots$, we shall have for the corresponding maximum and minimum velocities, v_1 and v_2, the relation

$$A = m\,\frac{v_1^2 - v_2^2}{2}$$

or if $v = \frac{v_1 + v_2}{2}$ represents the average velocity and $\frac{v_1 - v_2}{v} = \delta$ the coefficient of fluctuation (see § 145), we shall have

$$A = m\,v^2\,\delta.$$

Accordingly the coefficient of fluctuation δ increases directly as the work A and inversely as the mass m. Hence for a given average velocity v and for a desired coefficient of fluctuation δ we can determine from the formula

$$\delta = \frac{A}{m\,v^2}$$

the mass m reduced to the crank pin when A is known. Now in every case the work A can be determined from the corresponding area in the diagram, and we obtain a numerical result in meterkilograms [ft. lbs.] when the scale of the drawing is so chosen that the units of length along the base $A\,C$ represent meters [feet], and those of the ordinates, kilograms [pounds]. Such a graphical determination will always lead to the result and is generally preferable; but in certain cases use may be made of special formulas, obtained as follows: The work done during a revolution of the crank, represented by the area $A\,D\,E\,C$, is given by

$$L = P\,2\,\pi\,r = \frac{60 \times 75 \times N}{n}\text{ mkg.,} \left[L = \frac{60 \times 550\,N}{n}\text{ ft. lbs.}\right]$$

where N represents the number of horse-powers and n the number of revolutions per minute. Placing this equal to A, we get

$$A = \mu L = \mu P\,2\,\pi\,r = \mu\,\frac{4500\,N}{n}, \left[A = \mu\,\frac{33000\,N}{n}\text{ ft. lbs.,}\right]$$

where μ is a small value which depends in steam engines on the ratio of expansion and the ratio $\frac{r}{l}$ of the crank to the connecting rod. The above equation can also be written

$$\delta = \frac{A}{m\,v^2} = \mu\,2\,\pi\,\frac{P\,r}{m\,v^2} = 4500\,\mu\,\frac{N}{n\,m\,v^2}.$$

$$\left[\delta = 33000\,\mu\,\frac{N}{n\,m\,v^2}.\right]$$

Now in Chapter VI the coefficient of fluctuation δ was determined for different proportions of the crank train, the expression taking the form

$$\delta = k\,\frac{P\,r}{m_1\,v^2}$$

when the reciprocating masses m_2 were neglected, m_1 representing the rotating mass reduced to the crank pin, and k a constant number depending on the particular relations of the crank train. Consequently if we are content with the approximation obtained by neglecting the reciprocating mass m_2, we can employ the values k found in Chapter VI, to find in every case the value of μ, *i.e.*, that part of the whole work performed by the engine during one revolution, which causes the fluctuations of velocity $v_1 - v_2$ in the engine. By equating the value here found for δ with that found in Chapter VI, we have

$$k = \mu 2\pi, \quad \text{or} \quad \mu = \frac{k}{2\pi},$$

and therefore in a steam engine in which the horse powers N and the number of revolutions n are given we have

$$\delta = 4500 \frac{k}{2\pi} \frac{N}{n m v^2} = 716.6\, k \frac{N}{n m v^2}. \quad \left[\delta = 5255\, k \frac{N}{n m v^2}.\right]$$

If we introduce the weight $G = mg$ instead of the mass m, we shall have for the weight of the rotating mass reduced to the crank pin, the expression

$$G = 716.6\, g k \frac{N}{\delta n v^2} = 7030\, k \frac{N}{\delta n v^2} = \alpha \frac{N}{\delta n v^2}.$$

$$[G = 169000\, k \frac{N}{\delta n v^2} = \alpha \frac{N}{\delta n v^2}$$

when v is expressed in feet per second.]

If we here introduce n instead of v, using

$$v = \frac{2\pi r n}{60} = 0.1047\, r n,$$

[$v = 0.00873\, r n$ when v is in feet and r in inches,]

we shall obtain

$$G = 7030\, k \frac{N}{0.1047^2\, \delta r^2 n^3} = 641300\, k \frac{N}{\delta r^2 n^3} = \alpha_1 \frac{N}{\delta r^2 n^3}.$$

$$\left[G = 2{,}220{,}000{,}000\, k \frac{N}{\delta r^2 n^3} = \alpha_1 \frac{N}{\delta r^2 n^3}\right.$$

when r is expressed in inches.]

The following table contains the values of α and α_1 which belong to the crank trains discussed in articles 145 to 150.

where the values of k are given. It may here be remarked that the values of k are directly obtained only in § 150 for the engine working with expansion, in accordance with the formula

$$\delta = k \frac{Pr}{m_1 v^2},$$

while in articles 145 to 148 the formula

$$\delta = k_1 \frac{Qr}{m_1 v^2}$$

is employed, Q representing the constant piston force. But the value k here used can easily be obtained from k_1 and the ratio $\frac{P}{Q}$, which ratio for a constant piston force becomes equal to

$$\frac{2}{\pi}, \ \frac{4}{\pi} \ \text{and} \ \frac{6}{\pi}$$

for the single, double and triple crank respectively, the corresponding values of k used in the determination of α and α_1 being

$$\frac{\pi}{2} k_1, \ \frac{\pi}{4} k_1 \ \text{and} \ \frac{\pi}{6} k_1.$$

TABLE FOR THE VALUES OF α AND α_1

FOR CALCULATING THE FLY WHEELS OF STEAM ENGINES.

Without Expansion.

$\frac{l}{r}$	Single Crank.		Double Crank.		Triple Crank.	
	α	α_1	α	α_1	α	α_1
	4647	423900	466	42500	133	12150
∞ *	[111800	146860	11210	14725	3200	4210]
	5494	501100	1290	117700	—	—
6	[132180	173610	31037	40780	—	—]
	5683	518900	1577	143000	427	38900
5	[136730	179780	37942	49540	10274	13480]
	5999	547300	1756	160300	—	—
4	[144330	189620	42250	55540	—	—]

* The bracketed values of α_1 must be multiplied by 10,000 in the above table.

With Expansion and $l = 5r$.

Expansion e	2	3	4	5	6
α	9174 [220500]	9659 [232100]	9989 [240100]	10214 [245500]	10383 [249500]
α_1	836900 [2895000000]	881100 [3048000000]	911300 [3152000000]	931800 [3223000000]	947200 [3277000000]

The expressions found for G give the weight of the rotating mass reduced to the crank pin, provided v represents the velocity of the crank pin. The same formulas and coefficients will give the weight reduced to any other point, for example to the circumference of the fly wheel, when v is the velocity and r the radius of rotation of this point, the reduction of the masses being effected according to the well-known rules.

According to the investigations on the crank train, G represents not only the reduced weight of the rotating mass m_1, but also that of half the reciprocating mass m_2 (see § 148). In using the table and the formulas for G we must assume certain values for the permissible coefficient of fluctuation δ, and for this purpose may use the following values ordinarily employed in practice:

For hammer work, ... $\delta = \frac{1}{5}$
" pumps and gang saws, ... $\delta = \frac{1}{20}$ to $\frac{1}{30}$
" grist mills, ... $\delta = \frac{1}{25}$ to $\frac{1}{35}$
" looms and paper-machinery, . . $\delta = \frac{1}{30}$ to $\frac{1}{40}$
" spinning machinery, ... $\delta = \frac{1}{40}$ to $\frac{1}{100}$.

In constructing steam engines it is customary to assume with *Watt* an average coefficient of fluctuation of $\frac{1}{35}$.

EXAMPLE. For a double-acting engine with a ratio of expansion equal to 2, there was found in § 150 the equation

$$\delta = 1.305 \frac{P r}{m v^2} \text{ (where } r \text{ and } v \text{ are expressed in the same linear units);}$$

consequently for a coefficient of fluctuation $\delta = \frac{1}{25}$ the reduced mass becomes

$$m = 39.15\frac{Pr}{v^2} = 39.15\frac{N \times 75 \times 60}{2\pi\,.\,n v^3} = 28053\frac{N}{nv}. \quad \left[m = 205600\frac{N}{nv^3}.\right]$$

Now if the engine has $N = 30$ horse-powers, and if its crank radius $r = 0.4$ meter [15.75 ins.] makes $N = 40$ revolutions per minute, we have

$$v = \frac{2\pi \times 0.4 \times 40}{60} = 1.675 \text{ m. [5.5 ft. per sec.]};$$

consequently the weight of the rotating mass reduced to the crank pin is

$$G = mg = 28053\frac{30 \times 9.81}{40 \times 1.675^3} = 73{,}575 \text{ kg. [162,200 lbs.]}.$$

The weight of a fly-wheel rim whose radius $r_1 = 2.4$ meters [94.49 ins.] is

$$G_1 = G\left(\frac{0.4}{2.4}\right)^2 = \frac{73575}{36} = 2044 \text{ kg. [4500 lbs.]}.$$

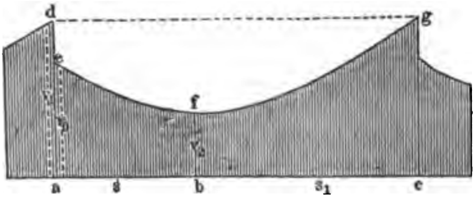
§ 192. The weight of the fly wheel for machines which, like trip hammers and rolling-mill trains, are subjected to sudden variations of velocity can be determined as follows: Let us again suppose all the forces and masses to be reduced to the same point; for example, in a roll-train to the circumference of the rolls, or in a hammer-train to the point at which the hammer is acted upon by the wiper. Let the constant turning force be represented by P, the intermittently moved load, for example the weight of the hammer, by Q, the continuous rotating mass by M and the intermittently moved mass by M_1, and let v_1 and v_2 again represent the maximum and minimum velocities.

Here a period of motion of a machine consists of three parts; at first the load Q is acted upon impact fashion by the machine, and the maximum velocity $v_1 = a\,d$, Fig. 758, of the point of application of the force is suddenly transformed into a smaller velocity $v_0 = a\,c$, which can be determined from the formula

$$v_0 = \frac{M v_1}{M + M_1}$$

given in Vol. I, § 335, for inelastic impact. In the next part of the period, during which the machine moves the load Q and performs actual work, the motion is retarded, the velocity gradually diminishing to its minimum $v_2 = bf$. Then comes the last part of the period, when the machine runs empty, its capacity for work being applied to accelerating the rotating mass M and bringing its velocity up to the maximum value $v_1 = cg = ad$. During impact, or during the first part of the period, the path of the point to which the force, load, and all the masses are reduced is almost zero, but in the second part of the period the force and load act through a certain distance $s = ab$, and in the last part of the period the force acts alone through a certain distance $s_1 = bc$. Consequently for the last two parts of the period we have the known formulas

FIG. 758.



$$Ps = Qs - \tfrac{1}{2}(M + M_1)(v_0^2 - v_2^2)$$

and

$$Ps_1 = \tfrac{1}{2}M(v_1^2 - v_2^2).$$

If we substitute in the last formula, as in § 191,

$$\frac{v_1 + v_2}{2} = v \quad \text{and} \quad v_1 - v_2 = \delta v,$$

where v represents the average velocity and δ the coefficient of fluctuation, we obtain the fundamental formula

$$Ps_1 = \delta M v^2, \quad \text{or} \quad M = \frac{Ps_1}{\delta v^2},$$

from which the rotating mass can be determined for a given coefficient of fluctuation.

If μ is the ratio

$$\frac{s_1}{s + s_1} = \frac{Ps_1}{P(s + s_1)}$$

of the path s_1 traversed by the machine when running empty, to the whole path $s + s_1$ traversed during a period,—in other words, if μ is the ratio of the excess of work during acceleration to the total work performed in a period,—we can also write

$$M = \frac{\mu P(s + s_1)}{\delta v^2} = \frac{60\mu}{\delta} \cdot \frac{L}{n v^2},$$

where n is the number of periods per minute and L the work

$$\frac{n}{60} P(s + s_1)$$

in meterkilograms [foot-pounds] of the machine per second.

As the work $P(s + s_1)$ is expended not only in overcoming the load Q, but also in varying the state of motion of the mass M_1, and as the impact itself, according to Vol. I, § 335, consumes the work

$$\tfrac{1}{2}(v_1 - v_0)^2 \frac{M M_1}{M + M_1},$$

we have

$$P(s + s_1) = Qs + \tfrac{1}{2} M_1 v_2^2 + \frac{1}{2} \frac{M M_1}{M + M_1} (v_1 - v_0)^2,$$

or

$$\frac{n}{60} P(s + s_1) = \frac{n}{60} Qs + \frac{n}{60}\left(\tfrac{1}{2} M_1 v_2^2 + \frac{1}{2} \frac{M M_1}{M + M_1} (v_1 - v_0)^2\right);$$

i.e.,

$$L = L_1 + \frac{n}{60}\left(\tfrac{1}{2} M_1 v_2^2 + \frac{1}{2} \frac{M M_1}{M + M_1} (v_1 - v_0)^2\right),$$

where $L_1 = \frac{n}{60} Qs$ represents the work of the load Q per second. Now

$$v_0 = \frac{M v_1}{M + M_1}$$

and

$$v_1 = \left(1 + \frac{\delta}{2}\right) v;$$

also

$$v_2 = \left(1 - \frac{\delta}{2}\right) v;$$

hence follows

$$L = L_1 + \frac{n}{60} \cdot \frac{v^2}{2} \left[\left(1 - \frac{\delta}{2}\right)^2 M_1 + \left(1 + \frac{\delta}{2}\right)^2 \frac{M M_1^3}{(M + M_1)^3} \right],$$

and the required rotating mass is

$$M = \frac{60\,\mu}{\delta} \cdot \frac{L_1}{n v^2} + \frac{\mu}{2\delta} \left[\left(1 - \frac{\delta}{2}\right)^2 M_1 + \left(1 + \frac{\delta}{2}\right)^2 \frac{M M_1^3}{(M + M_1)^3} \right];$$

or, as δ is very small in comparison with 1, we have

$$M = \frac{60\,\mu}{\delta} \cdot \frac{L_1}{n v^2} + \frac{\mu}{2\delta} \left(M_1 + \frac{M M_1^3}{(M + M_1)^3} \right).$$

If we substitute the weights $G = gM$ and $G_1 = gM_1$, and express L_1 in horse-powers, we shall have for the weight of the fly-wheel rim the formula

$$G = 9.81\, \frac{60\,\mu}{\delta} \cdot \frac{75\, L_1}{n v^2} + \frac{\mu}{2\delta} \left(G_1 + \frac{G G_1^3}{(G + G_1)^3} \right)$$

$$= 44145 \cdot \frac{\mu L_1}{\delta n v^2} + \frac{\mu}{2\delta} \left(G_1 + \frac{G G_1^3}{(G + G_1)^3} \right).$$

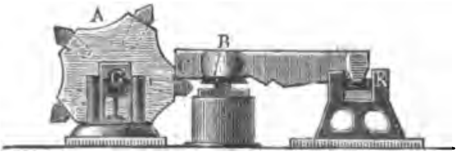
$$\left[G = 1061300\, \frac{\mu L_1}{\delta n v^2} + \frac{\mu}{2\delta} \left(G_1 + \frac{G G_1^3}{(G + G_1)^3} \right). \right]$$

Generally G_1 is so small in comparison with G that the equation can be easily solved, approximately.

REMARK. This subject will be continued in the articles treating of Trip Hammers, Rolling-mill Trains, etc.

EXAMPLE. The tilt-hammer CK, Fig. 759, weighing 11,230 pounds, is lifted 90 times a minute by a wiper wheel turning with a velocity of 0.6 meter [1.97 ft. per sec.], its centre of gravity being raised 0.3 meter [0.984 ft.]: how great should be the weight of the rotating mass if the wheel travels as far during the lifting as when it is running empty, *i.e.*, $\mu = \frac{1}{3}$, and if $\delta = 0.1$ is the required coefficient of fluctuation? The effective work of the machine is

FIG. 759.



$$L_1 = \frac{90}{60}\, 5000 \times 0.3 = 2250 \text{ mkg.} = 30 \text{ horse-powers},$$

for the hammer is lifted 90 times a minute to a height of 0.3 meter [0.984 ft.]. Hence the weight of the rotating mass reduced to the point of application of the wiper becomes, according to the above formula,

$$G = 44145 \times 0.5 \frac{30}{0.1 \times 90 \times 0.36} + \frac{0.5}{2 \times 0.1}\left(G_1 + \frac{G G_1^3}{(G + G_1)^3}\right)$$

$$= 204375 + 2.5\left(G_1 + \frac{G G_1^3}{(G + G_1)^3}\right).$$

$$\left[G = 456700 + 2.5\left(G_1 + \frac{G G_1^3}{(G + G_1)^3}\right).\right]$$

Now if the weight of the hammer and helve reduced to the point of application of the wiper is $G_1 = 3000$ kilograms [6614 lbs.], we have

$$G = 204375 + 2.5\left(3000 + \left(\frac{3000}{G + 3000}\right)^3 G\right),$$

and we can assume with great accuracy

$$G = 204375 + 7500 = 211875 \text{ kg. } [467,100 \text{ lbs.}];$$

for if we substitute in the second term of the parenthesis $G = 211,875$ $[G = 467,100]$, this term becomes only

$$\left(\frac{3000}{214875}\right)^3 211875 = 0.6 \text{ kg. } [1.32 \text{ lbs.}].$$

If the average radius of the fly wheel is 5 times as great as the lever arm GC of the wiper, the weight of the fly-wheel rim will be

$$\frac{G}{5^2} = \frac{211875}{25} = 8475 \text{ kg. } [18,690 \text{ lbs.}].$$

§ 193. Distribution of the Mass of the Fly Wheel.—The preceding formulas determine the mass M reduced to a point having the velocity v and at the distance r from the axis, the inertia of the mass exerting a regulating action on the motion. In practice the rotating mass has the form of a wheel, and this fly wheel must therefore receive such dimensions that its mass reduced to a point with radius r shall equal this value of M. The fly wheel consists of a rim whose weight may be represented by R, a number of arms whose total weight may be represented by A, and the boss. The masses of the separate parts must therefore be reduced to the same radius of rotation r, according to the rules laid down in Vol. I, § 283, and the sum of the reduced masses placed equal to M. The mass of the rim will be considerably larger than any other term of this sum, for its weight is greater and its radius, and consequently its velocity, is also greater. On the other hand, there is only a moderate mass in the arms, while the mass of the nave is so near the axis that it may be neglected in all cases. In the following formulas let r_1 be the external, r_2 the inner, and $r = \frac{r_1 + r_2}{2}$ the average radius of the fly-wheel, and let its radial breadth $r_1 - r_2$ be placed equal to b. Then if R represents the weight of the ring, we shall have, according to Vol. I, § 288,

$$\frac{R}{g}\left(r^2 + \frac{b^2}{4}\right) = W',$$

and the mass of the rim when reduced to the radius r becomes

$$M' = \frac{R}{g r^2}\left(r^2 + \frac{b^2}{4}\right) = \frac{R}{g}\left(1 + \frac{b^2}{4 r^2}\right).$$

If A represents the weight of all the arms, and if we assume that the arms are bars of rectangular cross-section and of the length r_2, we have, according to Vol. I, § 286,

$$W'' = \frac{1}{3}\frac{A}{g} r_2^2 = \frac{A}{3g}\left(r - \frac{b}{2}\right)^2;$$

hence the mass of the arms reduced to the radius r becomes

$$M'' = \frac{A}{3g}\left(1 - \frac{b}{2r}\right)^2.$$

Consequently we have

$$M = M' + M'' = \frac{R}{g}\left(1 + \frac{b^2}{4r^2}\right) + \frac{A}{3g}\left(1 - \frac{b}{2r}\right)^2.$$

Now as b is rarely greater than $0.1\,r$, $\frac{b^2}{4r^2}$ rarely greater than 0.0025, and as A is considerably smaller than R, it will be exact enough for practical purposes to make $Mg = G = R + \frac{1}{3}A$; *i.e.*, the effect of the arms can be assumed equal that of one third their weight concentrated on the average circle of the rim.

Furthermore, let F represent the cross-section of the fly-wheel rim and $F_1 = \nu F$ that of each of the z arms; then if γ represents the specific weight of cast iron, we have $R = 2\pi r F \gamma$, and approximately $A = z \nu F r \gamma$, consequently

$$G = (2\pi + \tfrac{1}{3} z \nu) F r \gamma,$$

from which, when $G = Mg$ is known, we can find the cross-section of the rim,

$$F = \frac{G}{(2\pi + \frac{1}{3} z \nu) r \gamma} = \frac{0.0000221}{1 + 0.053 z \nu} \frac{G}{r} \text{ square meters.}$$

$$\left[F = \frac{0.61}{1 + 0.053 z \nu} \frac{G}{r} \text{ square inches.}\right]$$

The number of arms z varies between 4 and 8, the ratio of cross-sections $\nu = \frac{F_1}{F}$ between 0.25 and 0.5, and the radial breadth of the rim from once to twice its thickness.

EXAMPLE. The weight of the fly wheel calculated in § 192, reduced to the point of application of the wiper, was found to be 211,875 kilograms [467,100 lbs.]. If this point is at the distance 0.6 meter [23.62 ins.] from the axis, while the average circle of the fly-wheel rim has a radius $r = 3$ meters [118.11 ins.], we obtain for the weight reduced to the rim

$$211875 \frac{0.6^2}{3^2} = 8475 \text{ kg. } [18,684 \text{ lbs.}].$$

If we assume that the area of each of the six arms is $F_1 = \frac{1}{2} F$, we get

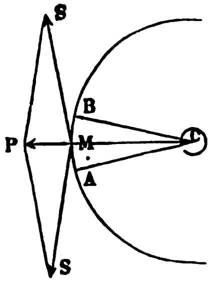
$$F = \frac{0.0000221}{1 + 0.053 z \nu} \frac{8475}{3} = 0.0564 m^2 \; [87.42 \text{ sq. ins.}].$$

If we make the ratio of the breadth b to the thickness d of the rim equal to 1.5, we have, from $1.5 d^2 = 0.0564$ [$1.5 d^2 = 87.42$],

$$d = 0.194 m \; [7.64 \text{ ins.}] \quad \text{and} \quad b = 0.290 m \; [11.42 \text{ ins.}].$$

§ 194. Strength of Fly Wheels.—As the living force of the fly wheel increases as the first power of the weight G, and as the second power of the velocity v or the radius r, it is evident that the weight of a fly wheel can be less the larger the velocity v or radius r is taken. Besides a smaller weight of fly wheel is accompanied by a smaller thrust on the bearings and a consequently smaller friction at the journals. On the other hand, the strength of the rim constitutes a limit to its circumferential velocity. For this reason the circumferential velocity is rarely taken greater than 30 to 35 meters [984 to 1148 ft.] per second. The greatest permissible value of this velocity can be determined as follows:

Fig. 760.



The centrifugal force P of an element AB of the fly-wheel rim, Fig. 760, is given by

$$P = \frac{mv^2}{r} = \frac{F\alpha r\gamma v^2}{rg} = \frac{F\alpha\gamma v^2}{g},$$

when the radius $AC = r$ and the angle at the centre $ACB = \alpha$, and when

$$m = \frac{F\alpha r\gamma}{g}$$

is the mass of the element AB. This force P can be resolved into two pressures S whose directions are tangent at A and B, their value being determined from

$$S = \frac{P}{2\,sin\,\frac{\alpha}{2}},$$

or, since we can place $sin\,\alpha = \alpha$ for a small angle,

$$S = \frac{P}{\alpha} = \frac{F\gamma v^2}{g}.$$

If we substitute in this $S = Fk$, where k is the greatest permissible stress for cast iron, we get $Fk = \frac{F\gamma v^2}{g}$, and from this the stress

$k = \frac{\gamma v^2}{g} = \frac{7200}{9.81} v^2 = 734\,v^2$ kg. per m². [$k = 0.097\,v^2$ lbs. per sq. in. when v is in ft. per second.]

Hence the greatest circumferential velocity of the fly-wheel rim is given by

$$v = \sqrt{\frac{g\,k}{\gamma}} = 0.037\,\sqrt{k}, \quad [v = 3.2\,\sqrt{k},]$$

which expression is independent of the dimensions of the wheel. If for cast iron we assume $k = 3$ kg. per mm² or $k = 3{,}000{,}000$ kg. per m² [$k = 4267$ lbs. per sq. in. or $k = 614{,}450$ lbs. per sq. ft.] and $\gamma = 7{,}200$ kg. [$\gamma = 0.261$ lbs. per cu. in. or $\gamma = 451$ lbs. per cu. ft.], we get

$$v = \sqrt{9.81\,\frac{3000000}{7200}} = 63.5 \text{ m. } [209 \text{ ft.}]$$

as the greatest permissible speed of rim for this stress.

The above formula applies only to a fly-wheel rim cast in one piece; but if it is composed of several segments, the strength of the connecting pieces must also be examined, for they are likewise subject to the stress $S = \frac{F\,\gamma\,v^2}{g}$. Therefore if F_1 represents the cross-section of a connecting piece and k_1 the permissible stress in the material, we shall have

$$F_1\,k_1 = \frac{F\,\gamma\,v^2}{g} \quad \text{or} \quad F_1 = \frac{v^2\gamma}{g\,k_1}\,F.$$

In order that the connecting pieces may not be of very great size they are always made of wrought iron, and if for this material we assume $k_1 = 10$ kilograms [14,223 lbs. per sq. in.], we get

$$F_1 = \frac{7200\,F\,v^2}{9.81 \times 10000000} = 0.0000734\,F\,v^2. \quad [F_1 = 0.00000685\,F\,v^2.]$$

For a maximum velocity, say, of $v = 30$ meters [98.4 ft.] per sec. we get

$$F_1 = 0.066\,F.$$

The same force must of course be resisted by the keys and bolts of the connecting pieces.

The foregoing investigation takes no account of the influence of the arms, but simply considers the rim as acted upon by the centrifugal force; the arms have, however, considerable influence on the stress to which the material is subjected. A more exact investigation of the stresses called forth by the centrifugal forces of the fly-wheel rim can be found in *Grashof's* "Festigkeitslehre." Here the segment included between two arms is treated as a beam whose ends, where they connect with the arms, are regarded as fixed, the centrifugal forces of the parts of the mass being uniformly distributed along the beam and subjecting it to bending stress. This investigation shows, as in the analogous case of a uniformly loaded beam fixed at both ends, Vol. I, § 246, that the greatest bending moment occurs at the ends; that is, in the case of the fly wheel, in the plane of the arms. For a six-armed fly wheel in which the rim and arms are of cast iron, the ratio of the arm section to the rim section $\frac{F_1}{F} = \frac{1}{3}$, and the radial breadth $b = \frac{1}{4} r$, we have for the maximum stress at the inner circumference of the rim in the plane of an arm,

$$k = 0.1088\, v^2, \quad [k = 0.144\, v^2,]$$

where v is the circumferential velocity in meters [feet] per second and k the stress in kilograms per square centimeter [lbs. per sq. in.]. In the above investigation, where the arms were not considered, the stress in the material was found to be $k = 734\, v^2$ kilograms per square meter, that is, only

$$\frac{0.0734}{0.1088} = 0.675$$

of the stress when the arms are taken into account. (See article by *O. Krueger*, Zeitschr. deutsch. Ingen. 1872, p. 97.)

As the living force with which the fly wheel regulates the motion of the machine is transmitted by the arms to the shaft and other machine parts, the arms will be subjected to certain stresses by this transmission. These stresses vary constantly in magnitude and direction, for, during each period, the fly wheel alternately drives and is driven by the shaft and so the arms are alternately bent to one side and the other. Let α be the angle through which the fly wheel turns while its maximum velocity v_1 is changing to the minimum velocity v_2, r the average radius of the fly-wheel rim, and M the mass of the rim reduced to this radius. Then the work transmitted by the arms from the fly wheel during this rotation α is given by

$$L = M\frac{v_1^2 - v_2^2}{2} = M\delta v^2.$$

As this work was performed while the fly-wheel rim traversed the distance $s = r\alpha$, the average force P exerted by the rim on the arms is

$$P = \frac{L}{s} = \frac{M\delta v^2}{r\alpha}.$$

Now if z is the number of arms, h the thickness and $b = mh$ the axial breadth of the arm, then if the arm be regarded as a beam of the length r fixed to the boss and bent by a force at the rim, we shall have

$$\frac{P}{z}r = \frac{bh^2}{6}k = \frac{mh^3}{6}k.$$

Now according to the above equation

$$Pr = \frac{M\delta v^2}{\alpha};$$

hence

$$\frac{M\delta v^2}{z\alpha} = \frac{mh^3}{6}k,$$

or

$$h = \sqrt[3]{\frac{6M\delta v^2}{zm\alpha k}}.$$

If we express the performance of the machine in horse powers by N and the revolutions per minute by n and introduce them and the ratio μ given above, we have

$$M\delta v^2 = \mu\frac{N 60 \times 75}{n}, \quad \left[M\delta v^2 = \mu\frac{N 60 \times 550}{n}\text{ ft. lbs.,}\right]$$

and get for the thickness of the arm

$$h = \sqrt[3]{\frac{6 \times 60 \times 75 \mu N}{z m a n k}} = 30 \sqrt[3]{\frac{\mu N}{z m \alpha n k}} \text{ meters.}$$

$$\left[h = 133.4 \sqrt[3]{\frac{\mu N}{z m \alpha n k}} \text{ ins.}\right]$$

The fly-wheel shaft is subjected to twisting stress by the moment

$$P r = \frac{M \delta v^2}{\alpha} = 60 \times 75 \frac{\mu N}{n \alpha}; \quad \left[P r = 396000 \frac{\mu N}{n \alpha} \text{ in. lbs;}\right]$$

and if we make use of the formula

$$d = 1.02 \sqrt[3]{P r} \quad [d = 0.0907 \sqrt[3]{P r}]$$

given in § 14 for the diameter of a wrought-iron shaft, we shall have

$$d = 1.02 \sqrt[3]{4500000 \frac{\mu N}{n \alpha}} = 168.4 \sqrt[3]{\frac{\mu N}{n \alpha}} \text{ millimeters.}$$

$$\left[d = 6.66 \sqrt[3]{\frac{\mu N}{n \alpha}} \text{ ins.}\right]$$

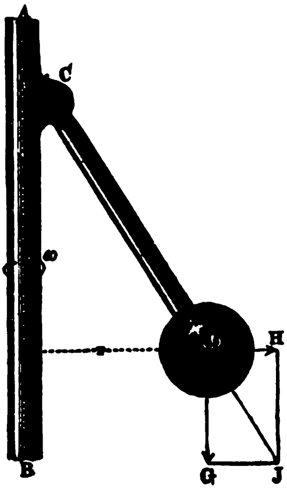
REMARK. When the fly wheels are subject to sudden shocks, as in hammer trains, the dimensions of the arms must be so determined that they can absorb a sufficient quantity of the work transmitted to them when the shaft is suddenly held fast, the arms acting somewhat like flat springs (see Vol. I, § 287, Ger. Ed.) An exact investigation of this case may be found in Grashof's "Festigkeitslehre," p. 285.

§ **195. Governors.**—The object of the devices known as governors is to produce as uniform running of the prime movers as possible, by regulating the supply of the driving motor, steam or water. Almost all the apparatus constructed for this purpose is of such a character that the regulator takes part in the motion of the prime mover, varying the supply of the motor only when the velocity of the machine has deviated a certain amount from the normal velocity. The action then exerted is to diminish the driving force when the machine runs more quickly than the normal speed, and to increase the driving force when the machine runs more slowly. Such a mode of action is theoretically imperfect, for these regulators cannot prevent variations of the velocity, but can only confine them within certain limits when they occur. Hence perfectly uniform motion can never be attained with such a governor; there will always remain certain variations of motion, and, other things being equal, that governor must be regarded as best which permits the smallest fluctuations of velocity. To avoid these imperfections, arrangements have been devised which are brought into action, not by the velocity variation itself, but by its cause, *i.e.*, by the variation of the resistance; but such arrangements, of which Poncelet's governor, to be described hereafter, is an example, have not come into general use.

Centrifugal governors are those most commonly used with steam engines and water wheels; they are so called because they depend upon the action of the centrifugal pendulum first used by *Watt* for regulating the velocity of his steam engines.

A centrifugal pendulum consists of a pendulum CD connected with a rotating, usually vertical, shaft AB, Fig. 761.

FIG. 761.



By its rotation about the shaft AB, a horizontal centrifugal force DH is developed in the weight G of the pendulum, this force and the weight G combined producing the resultant force DJ. For a condition of equilibrium this resultant force must be equal and opposite to the reaction exerted by the point of suspension C on the pendulum, *i.e.*, the direction of the resultant force R must pass through the point of suspension C. That we may state this condition analytically let G be the weight of the ball, r the distance DF of its centre of gravity from the axis, α the angle FED made by the arm of the pendulum with the axis, and ω the angle of velocity of the shaft; the centrifugal force is then given by

$$P = M\omega^2 r = \frac{G}{g}\omega^2 r.$$

The condition of equilibrium is therefore expressed by

$$tan\ \alpha = \frac{P}{G} = \frac{\omega^2 r}{g},$$

or

$$\omega = \sqrt{\frac{g}{r\ cot\ \alpha}} = \sqrt{\frac{g}{h}},$$

where h is the projection $EF = r\ cot\ \alpha$ of the arm of the pendulum on the axis. If we let t represent the time in seconds needed for one revolution of the shaft, so that $\omega = \frac{2\pi}{t}$, we get

$$\frac{2\pi}{t} = \sqrt{\frac{g}{r\ cot\ \alpha}}$$

and

$$t = 2\pi\sqrt{\frac{r\ cot\ \alpha}{g}} = 2\pi\sqrt{\frac{h}{g}}.$$

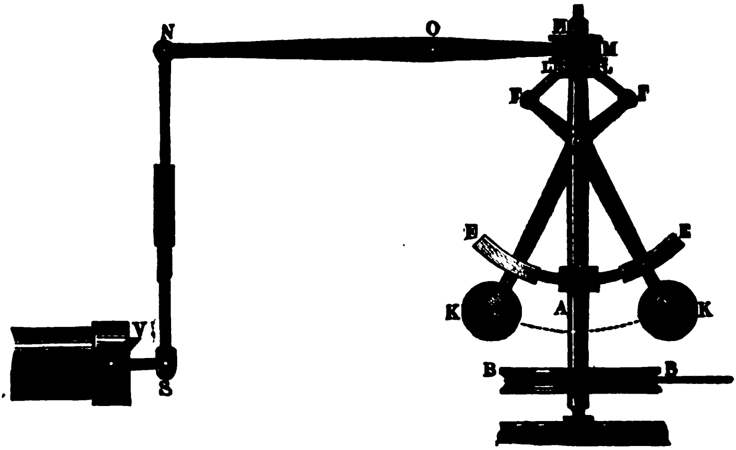
A comparison of this value t with the expression

$$t = 2\pi\sqrt{\frac{r}{g}}$$

given in Vol. I, § 321, for the duration of a complete oscillation of an ordinary circular pendulum of the length r, shows that *the duration of a revolution of the centrifugal pendulum is equal to the duration of a complete oscillation of a simple pendulum whose length is equal to the projection of the centrifugal pendulum on the axis.* Here as there the weight and centrifugal force of the arms CD are neglected. We see from the formula that as the angular velocity ω of the shaft increases, the inclination α of the arm to the axis will increase and the weight will rise, but the arm will never rise into a horizontal position, for this would require an infinitely great angular velocity. Therefore if we suppose the centrifugal pendulum to be so attached to the prime mover that the shaft *A B* takes part directly in the variations of velocity, it is evident that the arm of the pendulum will be in equilibrium at a certain definite position when the machine has a certain definite velocity, while every increase of speed must cause a rise and every decrease a lowering of the pendulum. These motions of the arm caused by the variations of velocity of the prime mover are used to move the valve or gate through which the steam or water is admitted to the prime mover, thus regulating it as desired.

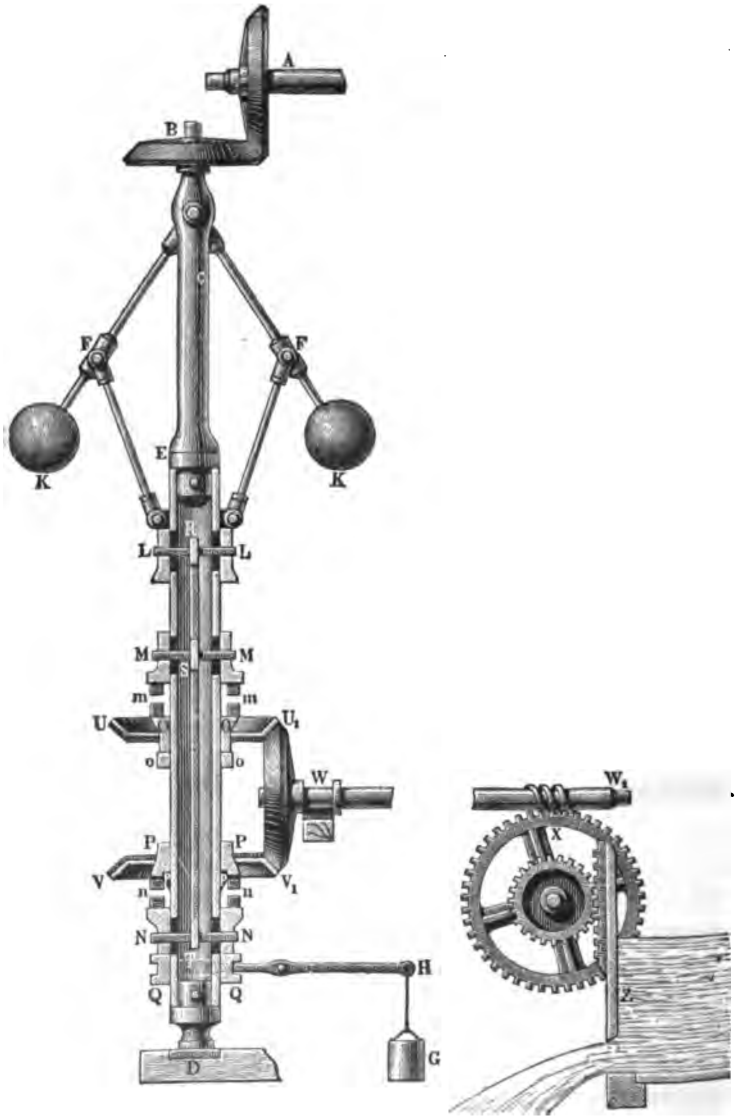
The manner in which this is accomplished is shown in Fig. 762. Here the governor spindle *A* is driven by the cord pulley *B*, which is kept in continual rotation by the prime mover. To avoid lateral pressures the spindle is provided with two equal, diametrically opposite, arms *C K* which are jointed to the links *L F*, and thus compel a sleeve *H* on the governor spindle to slide along the axis a certain amount when the pendulum arm rises or sinks. Therefore if this sleeve *H* has a groove which acts on the fork *M* of the lever *M O N*, a sliding of the sleeve *H* on the governor spindle will cause a corresponding motion of the lever *M N* swinging about *O*. The transmission of this

FIG. 762.



motion to the valve, etc., regulating the supply of the motor, can be accomplished in very different ways; for example, in

Fig. 763.



steam engines the motion of the point N is transmitted by the rod NS to the lever TS, whose motion turns the *throttle valve*

V, thus permitting a more or less free passage to the steam. This arrangement was formerly much used, but is not to be recommended because the resistances of the throttle valve reduce the pressure of the steam and thus partially destroy its capacity for work. Hence it is much more economical and, of late, has become quite common to communicate the motion of the lever *M N* to an *expansion-gear*, the reduction of the driving force being attained, not by contracting the steam passages, but by admitting less steam to the cylinder, *i.e.*, by using a greater ratio of expansion. For further details reference may be made to Vol. II.

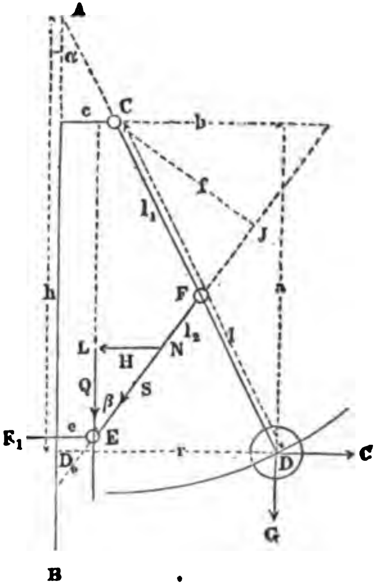
But such a *direct* transmission of the motion of the governor sleeve *H* to a throttle valve or to the expansion-gear of the engine is only possible when the resistances to motion offered by the admission-gear are small. In water wheels the force needed to move the gate is much too great to be exerted directly by the governor sleeve. In such cases indirect transmission is employed, an example of which is given in Fig. 763. The governor spindle *C D* driven by the wheels *A B* is hollow at its lower end and exactly cylindrical on the outside, so that the muffs *L*, *M* and *N* can easily slide on it while they take part in its rotation. The two bevel wheels *U* and *V* turn loosely on the governor spindle, being supported by means of the fixed collars *o* and *p*. The muff *L* is connected with the centrifugal pendulum by the rods *F*, and by means of the rod *R S T* compels the muffs *M* and *N* to take part in the up and down motion. Now as these muffs and the naves *O* and *P* of the bevel wheels are provided with clutch-teeth *m* and *n*, the muff *N* will be coupled to the wheel *P* when the governor sleeve *L* is in its highest position, and the muff *M* will be coupled to the wheel *U* when the governor is in its lowest position, while in the middle position represented in the figure both couplings will be out of gear. Therefore in this middle position of the governor sleeve *L* and balls *K*, the shaft $W W_1$ which moves the gate by the worm gear *X* and rack *C* is not set in motion. But according as the lower or upper coupling, *n* or *m*, is thrown into gear, the wheel $U_1 V_1$ receives motion from the wheel *V* or *U* and turns the shaft *W* in one or the other direction, thus diminishing or increasing the passage for the water flowing to the water wheel. To effect the motion more easily it is customary to balance heavy gates by counterweights; in like manner the weight G is applied to balance the rod $R\,T$ and the muffs L, M, N. By this arrangement the considerable force with which the gate resists motion is not overcome by the living force of the balls K, but by force transmitted from the water wheel through A and B to the governor spindle. Moreover it is unnecessary that the shaft A shall be driven in exactly this manner by the governor spindle, for the wheels U and V can be placed on a special shaft continually turned by the water wheel, and one or the other of the wheels may be thrown into gear by the governor sleeve. Such indirect transmissions can be effected in very different ways. *Indirect* transmission, as already stated, is employed when the resistance in the transmitting gear, *i.e.*, the mechanism for moving the admission valve, is considerable; it is therefore principally employed in water wheels. On the other hand, when there is slight resistance in the transmitting gear, as in some steam-engines, the *direct* transmission is preferred. If the governor sleeve is without weight and if there are no resistances to its motion, then, when the governor occupies the position characterized by

$$\omega = \sqrt{\frac{g}{h}},$$

the slightest variation of the angular velocity ω will cause the sleeve to rise or fall. But as these suppositions are not realized, the sleeve having weight of its own which is often purposely increased, and the muff always offering some resistance to the transmitting gear, the action of the governor will be modified by these influences and may be investigated as follows:

Let G be the weight of a governor ball D, Fig. 764, and let $2\,Q$ be the weight of the sleeve $E\,E_1$ and whatever load may be placed upon it. Moreover let $l = C\,D$ be the length of the arm of the pendulum, let $C\,F$ be represented by l_1, $F\,E$ by l_2, and the distances of the points of suspension C and E from the axis $A\,B$ by c and e. We will at first neglect the weights and

FIG. 764.



centrifugal forces of the rods CD and EF, and also the journal friction at the joints C, E and F. Let us suppose the pendulum to occupy any position of equilibrium CD in which the arm makes the angle $BAD = \alpha$ with the vertical axis, and the sleeve-rod EF the angle $FEL = \beta$ with the vertical. For this position the sum of the moments of all the forces with reference to the fixed point of suspension C must be placed equal to zero. To do this, resolve the weight $Q = LE$ into two components, one,

$$S = NE = \frac{Q}{\cos\beta},$$

acting in the direction of the sleeve-rod, and the other,

$$H = LN = Q \tan\beta,$$

in a horizontal direction. S is the only component which we will consider as affecting the equilibrium, for the horizontal force H is directly sustained by the sleeve and is neutralized by an opposite equal force $-H$ on the other side at E_1. Hence if C represents the centrifugal force of the ball at D, we shall have for the equation of equilibrium

$$Ca = Gb + Sf,$$

or

$$\frac{G}{g}\omega^2 r l \cos\alpha = G l \sin\alpha + \frac{Q}{\cos\beta} l_1 \sin(\alpha + \beta),$$

from which follows

$$\frac{\omega^2 r}{g} = \tan\alpha + \frac{Q}{G}\frac{l_1}{l}(\tan\alpha + \tan\beta).$$

Generally the proportions are so chosen that the arms CF of the pendulum and the sleeve rods FE form equal angles $\alpha = \beta$ with the axis AB by making $l_1 = l_2$ and $c = e$, and the suspension may be called a *rhombic* one. For this case the above equation becomes

$$\frac{\omega^2 r}{g} = tan\ \alpha \left(1 + \frac{2 l_1}{l} \frac{Q}{G}\right),$$

or, when we substitute $r\ cot\ \alpha = A D_0 = h$,

$$\omega = \sqrt{\frac{g}{h}\left(1 + \frac{2 l_1}{l} \frac{Q}{G}\right)}.$$

We see from this that loading the sleeve with $2Q$ necessitates an increase of angular velocity ω. When $Q = 0$ this equation becomes $\omega = \sqrt{\frac{g}{h}}$, like that found for the simple pendulum.

REMARK. In the preceding investigation the weights of the rods were neglected, for their influence is always inconsiderable. But if we wish to take these weights into account, it may be done in the following manner: Let l again be the length of the arm, l_1 the distance of the joint F from the point of suspension C, and l_2 the length EF of the sleeve-rods; also let f_1 represent the area of a pendulum-rod and f_2 that of a sleeve-rod. Then half the weight $f_1 l \gamma$ of an arm CD must be regarded as acting downward at the centre D of the ball, for this weight at the centre of gravity of the arm is at the distance $\frac{l}{2}$ from the point of suspension C and has a turning moment

$$f_1 l \gamma \frac{l}{2} sin\ \alpha$$

with reference to the latter. In like manner half the weight $f_2 l_2 \gamma$ of each sleeve-rod must be regarded as acting at E, increasing the half-load Q on the sleeve, while the other half, $\frac{f_2 l_2 \gamma}{2}$, must be regarded as acting downward at the point F, exerting a moment $\frac{1}{2} f_2 l_2 \gamma l_1\ sin\ \alpha$, which tends to turn the pendulum arm downward. Moreover there are centrifugal forces acting in the rods which tend to lift the pendulum-arms. The centrifugal force developed in such a rod is, according to Vol. I, § 307, expressed by $m \omega^2 r$, where m represents the mass and r the distance of the centre of gravity from the axis. This latter distance for the pendulum-arms is

$$r_1 = c + \frac{l}{2} sin\ \alpha.$$

and for the sleeve-rods

$$r_2 = e + \frac{l_2}{2} \sin \beta;$$

therefore we have the centrifugal forces equal to

$$P_1 = f_1 l \gamma \frac{\omega^2}{g}\left(c + \frac{l}{2} \sin \alpha\right) \text{ for one arm}$$

and

$$P_2 \doteq f_2 l_2 \gamma \frac{\omega^2}{g}\left(e + \frac{l_2}{2} \sin \beta\right) \text{ for one sleeve-rod.}$$

But the points of application of these centrifugal forces do not coincide with the centres of gravity, being determined as follows: Let λ_1 and λ_2 represent the distances of the centres of gravity of the arm and sleeve-rod from the axis, estimated in their directions; then

$$\lambda_1 = \frac{r_1}{\sin \alpha} = \frac{c}{\sin \alpha} + \frac{l}{2}$$

and

$$\lambda_2 = \frac{r_2}{\sin \beta} = \frac{e}{\sin \beta} + \frac{l_2}{2}.$$

The distances of the points of application of the forces from the axis, likewise estimated in the direction of the rods, become, according to Vol. I, § 307,

$$s_1 = \lambda_1 + \frac{l^2}{12\,\lambda_1}$$

for the centrifugal force P_1 of the arm, and

$$s_2 = \lambda_2 + \frac{l_2^2}{12\,\lambda_2}$$

for the centrifugal force P_2 of the sleeve-rod.

In order to obtain the moment $P_1 p_1$ with which the centrifugal force of the arm tends to turn the pendulum we must multiply P_1 by its vertical distance from the point of suspension C, *i.e.*, by $p_1 = s_1 \cos \alpha - c \cot \alpha$. But the centrifugal force P_2 of the sleeve-rod must be resolved into two components acting at F and E. As the distance of the point of application of the centrifugal force from the point E is given by

$$s_2 - \frac{e}{\sin \beta},$$

and from the point F by

$$l_2 + \frac{e}{\sin \beta} - s_2,$$

we obtain for the component at F of the centrifugal force of the sleeve-rod

$$P_2 \frac{s_2 - \dfrac{e}{sin\ \beta}}{l_2 + \dfrac{e}{sin\ \beta} - s_2} = \frac{s_2\ sin\ \beta - e}{(l_2 - s_2)\ sin\ \beta + e} P_2,$$

which force acts on the pendulum-arm with a lever-arm $l_1\ cos\ \alpha$. The other component of the centrifugal force, acting at E, is neutralized by an equal and opposite force exerted by the sleeve-rod on the other side of the sleeve.

§ **196. Sensitiveness of Governors.**—If we suppose a particular position for the governor for which h and α have certain values, then for an angular velocity ω satisfying the above equation the governor will be in equilibrium, the balls having no tendency to rise or fall, for the moment of the centrifugal force of G is exactly equal to the weights G and Q. The pendulum has no tendency to rise or fall till the velocity ω of the shaft changes. Then an actual motion of the balls will not immediately take place, because the friction in the governor and transmitting gear and the resistance offered by the admission gear must be overcome. Therefore if a rising of the governor balls is to take place, the angular velocity must increase to a certain amount ω' which is greater than the normal amount ω, and for the same reason a lowering of the balls cannot occur till the velocity has been reduced to an amount ω'' which is less than ω. The amount of these variations from ω, *i.e.*, the differences $\omega' - \omega$ and $\omega - \omega''$, depends on the magnitude of the aforesaid hurtful resistances in the transmitting gear and in the governor itself. Let all these resistances be reduced to the sleeve $E\,E_1$, their resistance at this place being represented by $2\ W$; then when the governor balls rise the total resistance at the sleeve will be given by $2\ Q + 2\ W$, and when the balls sink, by $2\ Q - 2\ W$, for this resistance $2\ W$, like all hurtful resistances, is always opposed to the motion. The velocities ω' and ω'' can be found from the above equation by substituting $Q + W$ and $Q - W$ in place of Q, for the resistance $2\ W$ like the weight $2\ Q$ of the sleeve must be regarded as divided equally between the sleeve-rods. Consequently for a rhombic suspension we have

$$\omega'^2 = \frac{g}{h}\left(1 + \frac{2l_1}{l}\frac{Q+W}{G}\right)$$

and

$$\omega''^2 = \frac{g}{h}\left(1 + \frac{2l_1}{l}\frac{Q-W}{G}\right).$$

Therefore in any position of the governor for which the normal velocity is ω, the angular velocity of the shaft can increase by the amount $\omega' - \omega$ or decrease by the amount $\omega - \omega''$ without varying the position of the governor; moreover these two velocities ω' and ω'' do not differ by exactly equal amounts from the normal velocity ω. This can be easily shown as follows. According to what has preceded, we have

$$\omega'^2 - \omega^2 = \frac{g}{h}\left(1 + 2\frac{l_1}{l}\frac{Q+W}{G}\right) - \frac{g}{h}\left(1 + 2\frac{l_1}{l}\frac{Q}{G}\right) = \frac{g}{h}2\frac{l_1}{l}\frac{W}{G}$$

and

$$\omega^2 - \omega''^2 = \frac{g}{h}\left(1 + 2\frac{l_1}{l}\frac{Q}{G}\right) - \frac{g}{h}\left(1 + 2\frac{l_1}{l}\frac{Q-W}{G}\right) = \frac{g}{h}2\frac{l_1}{l}\frac{W}{G};$$

hence, under the supposition that W is the same for the raising or lowering of the balls, we have

$$\omega'^2 - \omega^2 = \omega^2 - \omega''^2,$$

or

$$2\omega^2 = \omega'^2 + \omega''^2.$$

Now if we make

$$\omega' = \omega + \Delta_{\prime}, \quad \text{and} \quad \omega'' = \omega - \Delta_{\prime\prime},$$

we get

$$\omega^2 + 2\omega\Delta_{\prime} + \Delta_{\prime}^2 - \omega^2 = \omega^2 - \omega^2 + 2\omega\Delta_{\prime\prime} - \Delta_{\prime\prime}^2,$$

or

$$\Delta_{\prime}^2 + \Delta_{\prime\prime}^2 = 2\omega(\Delta_{\prime\prime} - \Delta_{\prime}).$$

As the left member of this equation is always positive, it follows that

$$\Delta_{\prime\prime} > \Delta_{\prime},$$

i.e., the velocity ω'' of the governor corresponding to the sinking of the balls deviates more from the normal angular velocity ω than the velocity ω' corresponding to the rising of the balls. But as Δ is always a small fraction of ω (say 0.02), it will be exact enough to make

$$\Delta_{,} = \Delta_{,,}, \quad \text{hence} \quad \omega = \frac{\omega' + \omega''}{2};$$

i.e., the *average* velocity for each position of the governor is assumed equal to its *normal* velocity, *i.e.*, to the velocity which corresponds to the position of equilibrium when hurtful resistances are neglected.

Within these two velocities ω' and ω'' the governor is *insensible* to variations of velocity, and the ratio $\frac{\omega' - \omega''}{\omega} = \epsilon$, as the measure of this relative insensibility, may be called the *coefficient of insensibility.**

To obtain an expression for this value of ϵ, assume

$$\frac{\omega' - \omega}{\omega} = \frac{1}{2}\epsilon \quad \text{and} \quad \omega' + \omega = 2\,\omega,$$

and we then have the product

$$\frac{\omega'^2 - \omega^2}{\omega^2} = \epsilon.$$

If we introduce in the left member of the equation the values found above for the numerator and denominator, we obtain

$$\epsilon = \frac{\frac{g}{h}\left(1 + 2\frac{l_1}{l}\frac{Q + W}{G}\right) - \frac{g}{h}\left(1 + 2\frac{l_1}{l}\frac{Q}{G}\right)}{\frac{g}{h}\left(1 + 2\frac{l_1}{l}\frac{Q}{G}\right)} = \frac{W}{\frac{l}{2l_1}G + Q}.$$

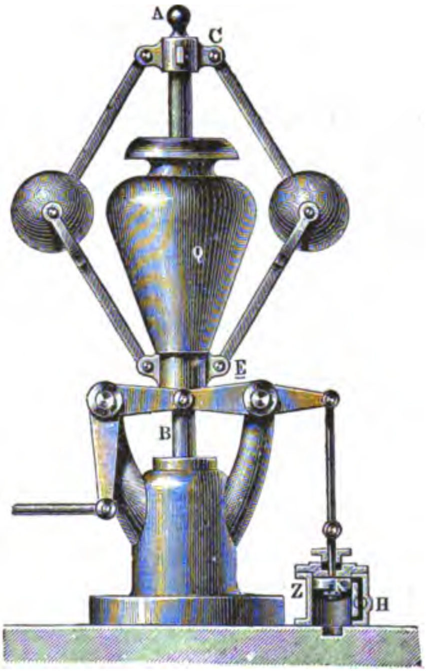
---

* Occasionally the value $\frac{\omega' - \omega}{\omega}$ is called the *coefficient of insensibility;* see *Prœll*, Civil-Ingenieur, vol. 18.

The insensibility of the governor is therefore directly proportional to W; for example, a screwing up of the stuffing box through which the valve rod passes increases the insensibility. The formula also shows the influence of the load $2\,Q$ of the sleeve on the sensitiveness of the governor. The application of heavy loads to the sleeve first occurred with the *Porter* governor, Fig. 765, and, of late, has become very popular. In the *Porter* governor the balls are at the joints uniting the pendulum rods with the sleeve rods, hence $l = l_1$ and

$$\epsilon = \frac{W}{\frac{1}{2}\,G + Q}.$$

FIG. 765.



For example, if we assume the load on the sleeve to be $2\,Q = 3\,G$, we get

$$\epsilon = \frac{W}{2\,G},$$

while without any load on the sleeve the coefficient of insensibility becomes $\epsilon = \frac{2\,W}{G}$, in other words, four times as great. It was remarked above that the velocity of the governor is greater with a load $2\,Q$ than without this load; for instance, when $2\,Q = 3\,G$ we have

$$\omega = \sqrt{\frac{g}{h}\left(1 + \frac{2\,l_1}{l}\,\frac{Q}{G}\right)} = \sqrt{\frac{g}{h}\left(1 + 2\tfrac{2}{3}\right)} = \sqrt{2\frac{g}{h}},$$

i.e., exactly twice as great as when there is no load on the sleeve. From the formula

$$\epsilon = \frac{W}{\frac{l}{2\,l_1} G + Q}$$

found above, it follows that the coefficient of insensibility of the governor is directly proportional to the resistance 2 W of the transmitting gear. The smaller this load 2 W the more sensitive is the governor. We also speak of the ***energy*** of a governor, and by this term mean that force E which is exerted by the governor on the sleeve at the instant when the velocity of the spindle has reached the value ω' or has fallen to the value ω'' in consequence of a variation of velocity $\omega' - \omega = \omega - \omega''$. The value of this energy $E = 2\,W$ can be found directly from the above equation, being equal to

$$E = 2W = \epsilon \left(\frac{l}{l_1} G + 2\,Q\right).$$

The preceding formulas for ϵ and E were developed for the *Watt* and *Porter* governors under the supposition of rhombic suspension, *i.e.*, $\alpha = \beta$, and it was shown that in this particular case the insensibility and the energy are independent of the angle α, *i.e.*, of the position of the governor. But if a different mode of suspension is chosen in which α and β are unequal, we must return to the general equation

$$\omega^2 \frac{r}{g} = \tan\alpha + \frac{Q}{G}\frac{l_1}{l}(\tan\alpha + \tan\beta),$$

and then by a perfectly similar development find

$$\epsilon = \frac{\omega'^2 - \omega^2}{\omega^2} = \frac{\frac{W}{G}\frac{l_1}{l}(\tan\alpha + \tan\beta)}{\tan\alpha + \frac{Q}{G}\frac{l_1}{l}(\tan\alpha + \tan\beta)}$$

$$= \frac{W}{G\frac{l}{l_1}\frac{\tan\alpha}{\tan\alpha + \tan\beta} + Q}.$$

We see from this that the insensibility ϵ and the energy $2\ W$ depend on the position of the governor, differing for different angles of inclination α. To determine these values for any angle of inclination α, the value of β must be found from the quantities l_1, l_2, c, and e using for this purpose the equation

$$c + l_1 \sin \alpha = e + l_2 \sin \beta$$

deduced from Fig. 764. The determination and substitution of the value $\tan \beta$ leads to a very complex formula, and therefore in practice it is more convenient to draw the governor in different positions, *i.e.*, with different angles of inclination α, and take the corresponding angles β from the drawing. The smallest angle of inclination α is not usually taken less than 15° to 20° nor greater than 40° to 45°. The length of the pendulum-arms and sleeve-rods must be so chosen that with the assumed swing of the arms the motion of the sleeve will suffice to completely close the admission-gear. From the length of the arms we can then determine the angular velocity by means of the formula for ω, and can thus determine the velocity ratio for the train of gears connecting the governor spindle with engine shaft. The number of revolutions n of the governor spindle may be found from ω by the relation

$$\omega = \frac{2 \pi n}{60}.$$

The sensitiveness of a governor for a steam engine must *not be too great*, i.e., the coefficient of insensibility must not sink below a certain value. It is easy to see that this smallest value of ϵ is prescribed by the value δ of the coefficient of fluctuation of the engine. For if the governor could be influenced by the variations of velocity inseparably connected with the crank motion and fly wheel used in the engine, it would be set in motion during every revolution, and the result would be an unsteady pulsating motion. Moreover, it is evident that in engines working with expansion, the action of the governor on the throttle valve is of influence only while the steam is being admitted to the cylinder.

§ **197. Static and Astatic Governors.**—From the formula for the velocity of a centrifugal governor

$$\omega = \sqrt{\frac{g}{r}}\sqrt{tan\ \alpha + \frac{Q}{G}\frac{l_1}{l}(tan\ \alpha + tan\ \beta)}$$

found in the preceding article, it is evident that the normal velocity ω, for which the pendulum is in equilibrium, hurtful resistances being neglected, depends on the distance r of the balls from the axis and on the angles α and β, *i.e.*, on the inclination of the arms. If α_1 represents the greatest and α_2 the least angle of inclination, the highest position of equilibrium will correspond to a certain angular velocity ω_1 which differs from the velocity ω_2 belonging to the lowest position. In any position between the limits α_1 and α_2 for which the angle of inclination may be represented by α, there will be needed for equilibrium an angular velocity ω which generally lies between the limiting velocities ω_1 and ω_2. Therefore if the governor spindle assumes any velocity ω lying between ω_1 and ω_2 this velocity will correspond to a position of equilibrium given by a perfectly determinate position of the pendulum arms. A governor which possesses this property of having a position of equilibrium for every velocity lying between ω_1 and ω_2, we may call a *static* governor. On the other hand, if the arrangement of the governor is such that the angular velocities have a constant value ω for the different angles of inclination α, the governor in all its positions will be in equilibrium with this velocity and this velocity only. Consequently when there is a change of velocity there will be no position in which the governor can remain stationary. Such a governor may be called an *astatic* or *isochronous* governor. To illustrate what has been said let us take the simplest case of a simple centrifugal pendulum with the arm CD suspended in the axis AB, Fig. 766, for which case $\omega = \sqrt{\frac{g}{h}}$ was found in § 195. The value $h = CE$, or the projection of the pendulum arm $CD = l$ on the axis, is $h = l$ for an angle of inclination $\alpha = 0$ and diminishes to zero as α increases to 90°. For this last position an infinite velocity is needed, but for $\omega = 0$ the pendulum hangs vertically. The ratio of the velocity ω_1 in the position CD_1 to ω_2 in CD_2 is expressed by

$$\omega_1 : \omega_2 = \sqrt{\frac{1}{h_1}} : \sqrt{\frac{1}{h_2}}.$$

We see from this that the centrifugal pendulum is a static governor. Such a one will lift the sleeve and move the transmitting mechanism when the velocity increases, but when it occupies the higher position CD_1 it will not be able to restore to the engine the same velocity which it possessed when the pendulum was in its lowest position, CD_2. Consequently the static property of a governor is accompanied by a certain variation of velocity in the engine, which variation is greater the more the governor differs from an astatic one, *i.e.*, the greater the difference between the velocities ω_1 and ω_2 at the limiting positions. We may also speak of the coefficient of fluctuation of a gov-

FIG. 766.

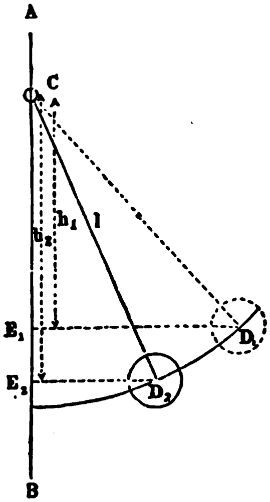
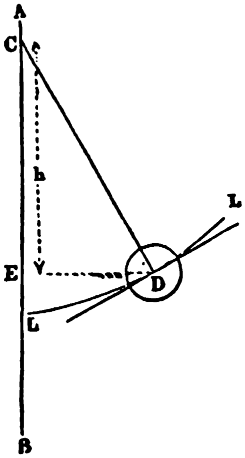


FIG. 767.



ernor, by which is meant, as in crank trains and fly wheels, the small fraction

$$\delta = \frac{\omega_1 - \omega_2}{\omega}.$$

To make this pendulum astatic, the centre of the ball must be so guided that h will be constant for all positions of the ball. Here h must be understood to mean the projection on the axis of that portion of the pendulum arm which lies between the centre of the ball and the spindle $A\,B$. Now if we imagine the ball to be guided along a certain curve $L\,L$, Fig. 767, instead of being suspended from the fixed point C, the action of this point C will be replaced by the normal reaction $D\,C$ of the curve at D. Therefore the formulas developed in § 195 for the centrifugal pendulum are applicable here also, and we have the angular velocity $\omega = \sqrt{\frac{g}{h}}$ when $h = C\,E$ represents the sub-normal of the guide curve L. If ω is to be constant for all positions of the ball the sub-normal $C\,E$ must also be constant, from which follows that the guide curve L must be a parabola whose axis is $A\,B$ and its parameter $\frac{2\,g}{\omega^2}$, since the parameter is equal to twice the sub-normal. Inversely, for a parabolic curve whose parameter is p the necessary angular velocity of the spindle is $\omega = \sqrt{\frac{2\,g}{p}}$.

The parabola is the astatic curve for the *Watt* and *Porter* governors only so long as the angle β formed by the sleeve rods with the axis is equal to zero, because for this case the general equation

$$\frac{\omega^2 r}{g} = \tan\alpha + \frac{Q}{G}\frac{l_1}{l}(\tan\alpha + \tan\beta)$$

of the preceding article becomes

$$\omega = \sqrt{\frac{g}{r}\tan\alpha\left(1 + \frac{Q}{G}\frac{l_1}{l}\right)} = c\sqrt{\frac{g}{h}}$$

when h again represents the sub-normal and

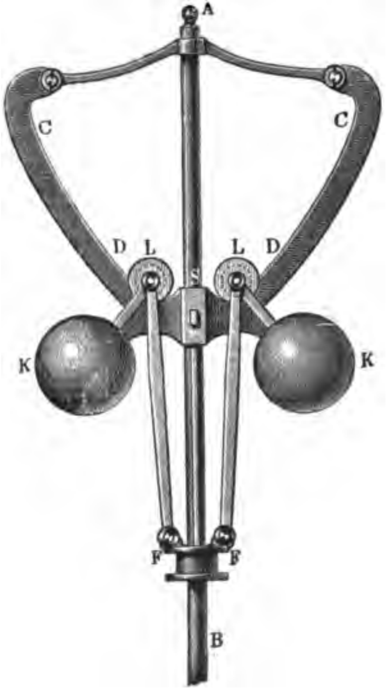
$$c = \sqrt{1 + Q\frac{l_1}{l}}$$

a constant. The parameter of the parabola is now

$$p = \frac{2g}{\omega^2}\left(1 + \frac{Q\,l_1}{l\,G}\right).$$

We accordingly find astatic governors constructed so that the balls are compelled to move along parabolic curves. *Franke's* parabolic governor, Fig. 768, is an example, in which the balls hang from rollers L which move along guide bars $C\,D$ rigidly attached to the spindle $A\,B$. These guides are *equidistants* to the parabolas in which the centres of gravity of the balls must move. The sleeve rods $L\,F$ are suspended from the journals of the rollers L, and as the rods are not parallel to the axis, it follows that the governor is not perfectly astatic. Moreover, the eccentric action of the centrifugal force of the balls increases the hurtful resistances at the rollers L. A better construction is that of *Garnett*, Fig. 769, in which the sleeve E has a horizontal bar H on each side, in whose slots the journals K of the ball are guided. The guides $C\,D$ are here also equidistants to the parabola. But these governors have been little used, principally on account of the defects inherent in all astatic governors.

Fig. 768.



If we wish to determine the general astatic curve, *i.e.*, the

Fig. 769

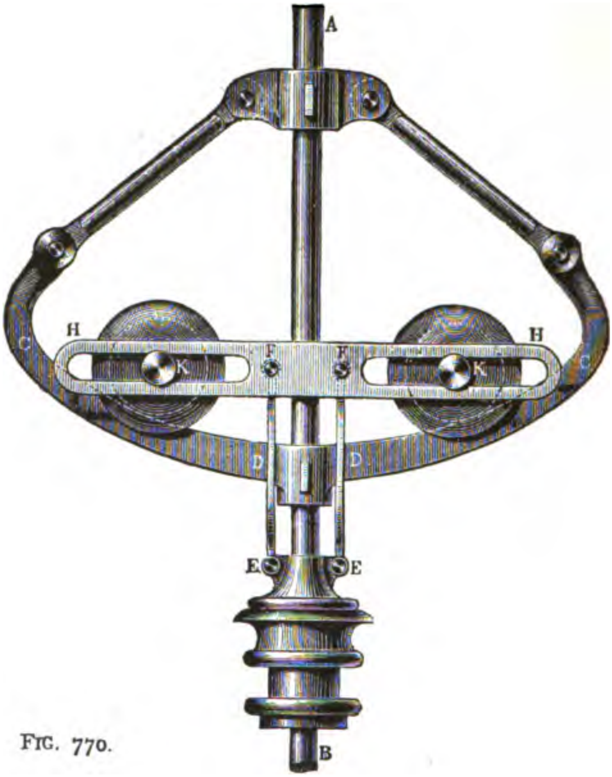
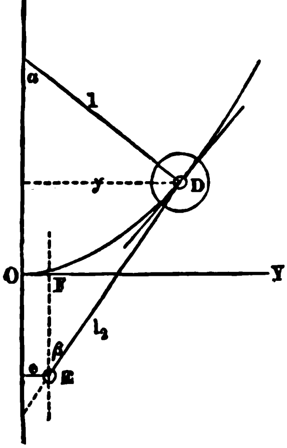


Fig. 770.



curve along which the centres of the balls must be led when the sleeve rods DE, Fig. 770, make a variable angle $DEF = \beta$, we must start with the general equation of § 195:

$$\frac{\omega^2 r}{g} = \tan \alpha + \frac{Q}{G}\frac{l_1}{l}(\tan \alpha + \tan \beta).$$

If in this we substitute $l_1 = l$ for the case in which the sleeve-rods are jointed to the centres of the balls, Fig. 770, and if we assume the axis of the governor as the axis of X, and the lowest point O of the curve as the origin of co-ordinates, we have from the figure

$$r = y, \quad \tan\alpha = \frac{\delta x}{\delta y}$$

and

$$\sin\beta = \frac{y - e}{l_2};$$

hence

$$\tan\beta = \frac{y - e}{l_2\sqrt{1 - \frac{(y - e)^2}{l_2^2}}} = \frac{y - e}{\sqrt{l_2^2 - (y - e)^2}}.$$

Introducing these values in the above equation, we get

$$\frac{\omega^2}{g} y = \left(1 + \frac{Q}{G}\right)\frac{\delta x}{\delta y} + \frac{Q}{G}\frac{y - e}{\sqrt{l_2^2 - (y - e)^2}}$$

or

$$\frac{\omega^2}{G} y\,\delta y - \frac{Q}{G}\frac{(y - e)\,\delta y}{\sqrt{l_2^2 - (y - e)^2}} = \left(1 + \frac{Q}{G}\right)\delta x,$$

from which by integration we get

$$\frac{\omega_2}{g}\frac{y^2}{2} + \frac{Q}{G}\sqrt{l_2^2 - (y - e)^2} = \left(1 + \frac{Q}{G}\right)x + \text{Constant.}$$

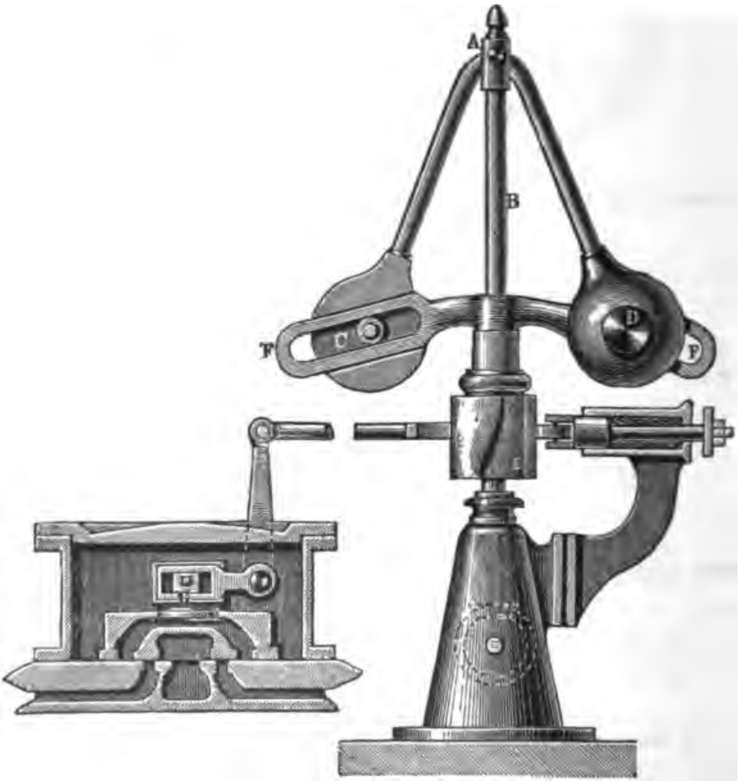
The constant is determined by making $x = 0$ and $y = 0$ when Constant $= \frac{Q}{G}\sqrt{{}_2^2 - e^2}$.

§ 198. In the arrangements hitherto discussed the governor is rendered astatic by replacing the pendulum suspension of the ball by a curved guide so formed that the direction of its reaction against the ball makes with the axis an angle α that satisfies the equation

$$\frac{\omega^2 r}{g} = \tan \alpha + \frac{Q}{G}\frac{l_1}{l}(\tan \alpha + \tan \beta)$$

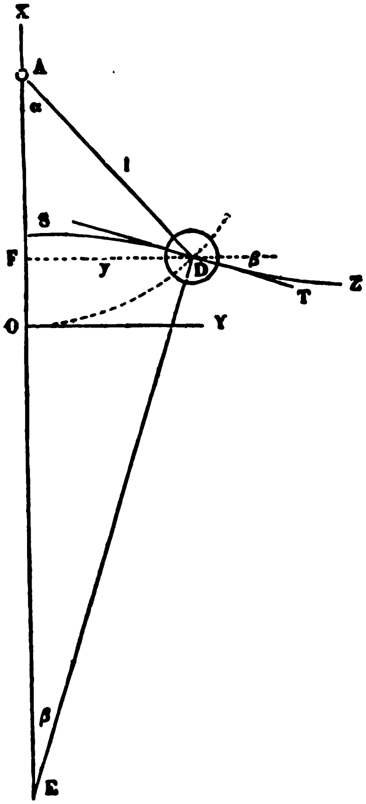
when the angular velocity ω is constant. Now this equation shows that this condition can be fulfilled when the balls are suspended as in *Watt's* governor, provided the angle β, formed by the pulling force of the sleeve with the axis, has in every position a value equal to that given by the equation. The

FIG. 771.



sleeve cannot then be connected with the balls by links, but a curved slot of suitable form must be employed, whose up and down motion is communicated to the sleeve. Such a governor has been constructed by *Werner** and is represented in Fig. 771. Here the pendulum arms *AD* are suspended from a point on the axis *AB* and the balls have slits through which the slotted bar *FF* passes, each ball being provided with a roller *C* which moves in a slot. The sleeve *E* is directly connected with the slotted bar *F* which slides along the axis. The sleeve is a conoidal body whose height regulates the time during which the admission port is kept open by the expansion valve *G*, thus controlling the cut-off of the steam engine (see Vol. II, Steam Engines).

FIG. 772.



To determine the form of this curve, let the governor axis, Fig. 772, be taken as the axis of *X*, and the lowest point *O* of the ball as the origin of co-ordinates. Then in the fundamental equation

$$\frac{\omega^2 r}{g} = \tan\alpha + \frac{Q}{G}(\tan\alpha + \tan\beta),$$

we have

$$r = y \quad \text{and} \quad \tan\alpha = \frac{y}{\sqrt{l^2 - y^2}}.$$

Moreover, if *DE* is the normal at *D* to the curve *S* of the slot in which the ball moves, then if friction is neglected this normal will represent the direction of the pulling force exerted by the pendulum on the sleeve. Hence $OED = \beta$, and as this angle β is also included between the axis of V and the tangent DT to the curve, we have

---

* See Zeitschr. deutsch. Ingen., 1868, p. 489.

$$\tan \beta = \frac{\delta x}{\delta y}.$$

Substituting these values, the fundamental equation becomes

$$\frac{\omega^2 y}{g} = \frac{y}{\sqrt{l^2 - y^2}}\left(1 + \frac{Q}{G}\right) + \frac{Q}{G}\frac{\delta x}{\delta y},$$

or

$$\frac{\omega^2}{g} y \delta y - \left(1 + \frac{Q}{G}\right)\frac{y \delta y}{\sqrt{l^2 - y^2}} = \frac{Q}{G}\delta x,$$

from which we get, by integration,

$$\frac{\omega^2}{2g}y^2 + \left(1 + \frac{Q}{G}\right)\sqrt{l^2 - y^2} = \frac{Q}{G}x + \text{Constant}.$$

It is easy to see that x becomes a maximum in the governor axis and a minimum at some other point Z, and that between these places there is a point of inflection.

As regards the applicability of astatic governors, *Reuleaux** was the first to call attention to the fact that they were only suited for *direct* transmission, *i.e.*, for an arrangement of transmitting gear in which the sliding of the sleeve is directly employed to adjust the admission valve. For if an astatic governor is connected with an indirect transmitting gear so that the up and down motion of the sleeve throws into action an adjusting mechanism and thus causes an independent shaft to close or open the admission valve, a very unsteady motion of the machine must result. Thus if the prime mover is running with its normal velocity ω and the governor is at its middle position and the adjusting mechanism thrown out of action, as soon as the velocity has reached the value ω' prescribed by the frictional resistances, the governor will rise and throw the lower wheel into action, with the result that the admission valve will be partially closed, thus modifying the velocity of the prime mover. But now the governor keeps the adjusting mechanism in gear even when the velocity has been reduced to its normal amount ω, for the astatic character of the governor allows it with this velocity to remain in equilibrium in every position. Consequently a still further closing of the admission valve will take place, which will continue till the velocity has been reduced to ω''. At this instant the governor will begin to sink and will continue sinking till the upper wheel is thrown into gear, whereupon the same operation will be repeated in inverse order. Hence the governor is kept in constant vibration even when there is no fluctuation in the useful resistance or load on the prime mover. But if the governor used with indirect transmission is a static one it will, to be sure, rise when the velocity ω' is reached and throw the adjusting mechanism into action, but as it can only remain in the higher position when the velocity is greater than ω, it tends to leave this position as soon as the velocity again approaches its normal value ω, and a throwing into gear of the upper wheel will not generally occur until the increase of load causes the velocity to fall below the value ω''.

---

* Zeitschr. deutsch. Ingen., 1869, p. 165.

It follows from this that *astatic* governors are only suitable for direct transmission. Even then they often cause unsteady motion in the prime mover. The cause of this is to be sought principally in the action of the masses of the parts constituting the governor, for when the normal velocity ω of the prime mover varies, the sleeve is set in motion as soon as the velocity has reached the amount ω' or ω'' required by the hurtful resistances. But when this motion of the sleeve takes place, the equilibrium of the forces acting on the governor is not restored, principally because the frictional resistances that correspond to a state of rest and determine ω' and ω'' are greater than the frictional resistances that accompany a state of motion. As a consequence the sleeve and the parts connected with it are, during their up and down motion, subjected to a certain acceleration which will not allow them to come to rest when the normal velocity is again established, but compels them to continue their motion by virtue of the living force they have thus received. This causes a further variation in the same sense of the velocity of the prime mover. After this living force has been entirely consumed the sleeve will at once begin its return motion, and thus vibrations will arise in the governor, which must be regarded as the cause of the unsteady motion of the prime mover, so frequently observed. This behavior is also observed in static governors though in less degree, there being less of this action the more static they are.

Efforts have been made to remove this defect of governors by providing them with cataracts or dash-pots, *i.e.*, with an artificial resistance which opposes the acceleration of the sleeve. This dash-pot consists of a closed cylinder *Z*, Fig. 773, in which slides a piston *K* which directly or indirectly takes part in the motion of the sleeve. This motion of the piston causes the fluid or air contained in the cylinder to escape through a passage connecting the spaces on opposite sides of the piston, the cock *H* regulating the resistance experienced by the fluid in its passage. In this way a restraining action is exerted, preventing the governor from flying out too quickly, and this is accomplished without any great resistance to a slow change of position of the governor. The resisting fluid employed in dash-pots is water, oil, or glycerine, but air is still better. The counterpoise *Q* connected with the sleeve is sometimes constructed in the form of a cylinder, the piston being fastened to the governor spindle.*

To avoid the hurtful piston friction of the ordinary dash-pot, which diminishes the sensitiveness of the governor, *Buss* † has invented a peculiar cataract in which air causes the weight of a certain quantity of water to resist the tendency of the moving parts to pass beyond their proper position. This apparatus consists of an annular drum *T* which turns about its shaft *A* and by means of a lever *A B* and rod *Z* is so connected with the sleeve of the governor that an up or down motion of the sleeve causes an oscillation of the drum in one direction or the other. The drum *T* is half filled with a liquid, and the air space above is divided by a partition *CD* in which is a small opening *E* connecting the air in the two spaces. It is evident from this that with a slow motion of the drum the level of the liquid in the two chambers will differ but little, for the air can pass with sufficient rapidity through the opening *E* from one chamber to the other. But with a rapid motion of the drum the level of the liquid in one chamber *F* will be lower than that in the other chamber *G*, and therefore the weight of a mass of fluid corresponding to the difference of level *H* will oppose the motion of the sleeve.

---

* See Zeitschr. deutsch. Ingen., 1874, p. 138.

† See Civil-Ingenieur, 1872, p. 25.

FIG. 773.

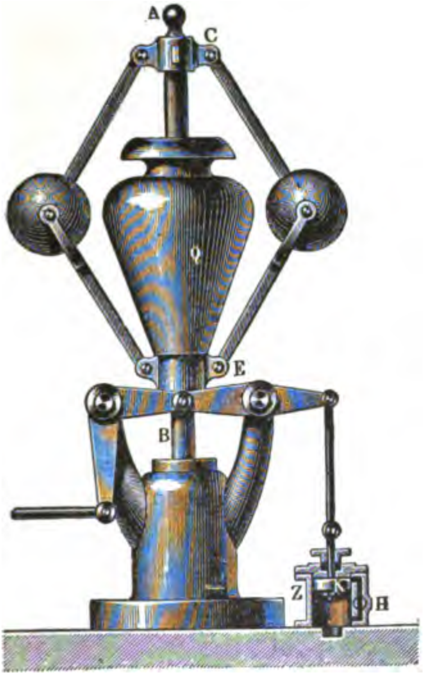
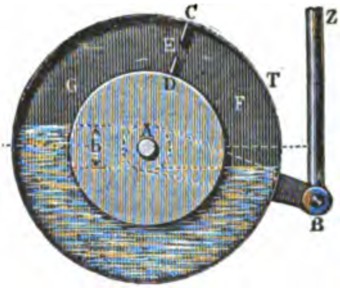


FIG. 774.



There is an interesting investigation by *Wischnegradski* (Civil-Ingenieur, 1877) concerning the action of dash-pots on governors, in which the conclusion is reached that an *astatic* governor cannot be freed from the aforesaid defects by any dash-pot, and that consequently the astatic governors, as such, must be regarded as useless; that, on the other hand, it is possible to find for every given *static* governor a suitable dash-pot, which will partially if not wholly remove the hurtful kind of motion above mentioned, and thus render it more useful.

§ 199. Pseudo-astatic Governors.—The defects of the perfectly astatic governors mentioned in the preceding articles have caused them to be little used in practice. On the contrary, efforts have been made to construct governors which should be approximately astatic and yet possess a certain stability, *i.e.*, possess to a certain extent the property of the static governors of being in equilibrium in different sleeve positions with different angular velocities. It has already been mentioned that, by virtue of this property, static governors are less subject to the so-called *jumping* which is due to the inertia of the masses and causes a very irregular running of the machine. On the other hand, the original *Watt* and *Porter* governors would not answer, for the velocities corresponding to the different ball positions differ from each other more than is compatible with the good working of most machinery. Consequently, of late, efforts have been principally directed to making both the *Watt* and the *Porter* governors as astatic as possible, while leaving them as much stability as is consistent with the work of the machinery driven. Such approximately astatic governors may be called *pseudo-astatic* or *approximately isochronous governors.*

The difference in the angular velocities for the extreme positions can be found from the general equation given in § 195,

$$\frac{\omega^2 r}{g} = tan\ \alpha + \frac{Q}{G}\frac{l_1}{l}(tan\ \alpha + tan\ \beta),$$

for both the *Watt* and *Porter* governors.

For rhombic suspension, *i.e.*, for $\alpha = \beta$, this equation becomes

$$\frac{\omega^2 r}{g} = tan\ \alpha\left(1 + \frac{Q}{G}\frac{2l_1}{l}\right),$$

or

$$\omega = C\sqrt{\frac{g}{r \cot \alpha}} = C\sqrt{\frac{g}{h}},$$

when for a given construction the constant quantity

$$\sqrt{1 + \frac{Q}{G}\frac{2l_1}{l}}$$

is represented by C and the projection on the axis of the portion of the pendulum arm between axis and centre of ball is again represented by h. This projection AE, Fig. 775, is expressed by

$$h = l \cos \alpha + c \cot \alpha,$$

where α again represents the inclination of the arms to the axis, $CD = l$ its length, and $c = CC_0$ the eccentricity of the point of suspension. Therefore the ratio of the angular velocities ω_1 and ω_2 corresponding to the angles of inclination α_1 and α_2 at the extreme positions is given by

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{l \cos \alpha_2 + c \cot \alpha_2}{l \cos \alpha_1 + c \cot \alpha_1}}.$$

FIG. 775.

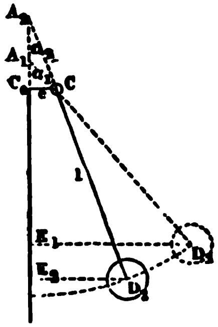


FIG. 776.

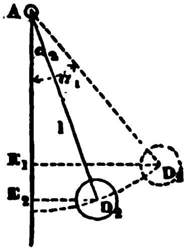
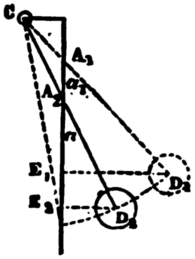


FIG. 777.



For example, if we assume in a *Watt* governor $\alpha_1 = 40^\circ$, $\alpha_2 = 20^\circ$, and $c = 0.1\, l$, we obtain the velocity ratio

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{\cos 20^\circ + 0.1 \cot 20^\circ}{\cos 40^\circ + 0.1 \cot 40^\circ}} = 1.17,$$

which ratio is larger than is desirable for most machinery.

We see from the figure that when the pendulum arm moves, the projection AE increases simultaneously at both ends A and E or decreases simultaneously at those points. The variation at one end, A, can be removed by making $c = 0$, *i.e.*, by suspending the pendulum from a point in the axis, Fig. 776, but the variation at the lower end still remains and then the ratio of the limiting velocities is

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{l \cos \alpha_2}{l \cos \alpha_1}} = \sqrt{\frac{\cos 20^\circ}{\cos 40^\circ}} = 1.11,$$

which is still too large.

If we assume c still smaller, *i.e.*, place the point of suspension on the opposite side of the axis, Fig. 777, we have the height

$$h = l \cos \alpha - c \cot \alpha,$$

and we see from this that h becomes equal to zero when $\alpha = 90^\circ$, and also when $l \cos \alpha = c \cot \alpha$ or when $\sin \alpha = \frac{c}{l}$, *i.e.*, when the centre D of the ball lies on the axis $A\,B$. Therefore somewhere between this position and the horizontal one of the pendulum arm h must have a maximum value. The angle α_0 belonging to this maximum h_0 is easily found from

$$\frac{\delta h}{\delta \alpha} = - l \sin \alpha + \frac{c}{\sin^2 \alpha} = 0 \quad \text{or} \quad \sin \alpha_0 = \sqrt[3]{\frac{c}{l}}.$$

If we suppose the pendulum to have this angle of inclination, it will be in equilibrium when the angular velocity is

$$\omega_0 = C\sqrt{\frac{g}{h}}.$$

Now as h is smaller on each side of this position, every other position will require greater velocity of the spindle for the maintenance of equilibrium. Therefore if the angle of inclination α_0 is increased, the pendulum will behave like those shown in Fig. 775 and Fig. 776, *i.e.*, in every higher position the pendulum can only be in equilibrium with a correspondingly greater velocity. Hence if the velocity increases from the minimum value ω_0 to a value ω, there will always be a certain angle α corresponding to this greater velocity ω for which there is equilibrium, this angle α being greater than α_0.

But the pendulum behaves quite differently when the velocity ω sinks below the value ω_0; when this occurs, the pendulum arm sinks on account of the diminished centrifugal force, and there is *no position* in which, with the diminished velocity, the pendulum can be in equilibrium, for to every lower position there belongs a smaller value h smaller than h_0, and consequently a greater velocity than ω_0. Therefore when the ball is in the position α_0 the slightest decrease of velocity will cause it to fall to the axis. Therefore in every position between the axis and that angle of inclination α_0 which corresponds to the maximum value of h, the pendulum will be in a condition of *unstable equilibrium*, because for every such position the slightest diminution of the velocity ω corresponding to that position will cause the pendulum to sink quickly to the axis, while, on the other hand, the slightest increase of ω will cause the pendulum to rise and pass beyond the angle α_0 till it reaches another position having the same smaller height h corresponding to the greater velocity. Hence the pendulum positions whose angles of inclination are greater than α_0 correspond to the condition of *stable equilibrium*.

It is evident from the foregoing discussion that if we wish to make use of this form of governor suspension, with *crossed* rods (*i.e.*, $c < 0$), we must reject all positions belonging to the region of unstable equilibrium, and consequently the aforesaid angle α_0 belonging to the maximum of h must be regarded as the *smallest permissible angle of inclination* α_2 of the governor. The pseudo-astatic governors of *Kley*, *Wiebe*, and *Werner* have been designed in this manner. The rules for this form of suspension readily follow from what has been said. For example, if the smallest angle α_2 of the pendulum is fixed by constructive considerations, we first determine the ratio $\frac{c}{l}$ from the equation

$$sin^3 \alpha_2 = \frac{c}{l},$$

which corresponds to the maximum of h, and then find l and consequently c by means of the general condition of equilibrium of the rhombic governor,

$$\omega_2 = \sqrt{1 + \frac{Q}{G}\frac{2l_1}{l}}\sqrt{\frac{g}{h}} =$$

$$\sqrt{1 + \frac{Q}{G}\frac{2l_1}{l}}\sqrt{\frac{g}{l\left(cos\ \alpha_2 - \frac{c}{l}\ cot\ \alpha_2\right)}},$$

in which ω_2 is the smallest angular velocity of the governor spindle. Now if we also assume the greatest angle of inclination α_1 in accordance with the required stroke of the sleeve, we find the greatest velocity corresponding to the highest position α_1 to be

$$\omega_1 = \sqrt{1 + \frac{Q}{G}\frac{2l_1}{l}}\sqrt{\frac{g}{l\left(cos\ \alpha_1 - \frac{c}{l}\ cot\ \alpha_1\right)}};$$

consequently the ratio of the extreme velocities is

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{\cos \alpha_2 - \frac{c}{l} \cot \alpha_2}{\cos \alpha_1 - \frac{c}{l} \cot \alpha_1}}.$$

For example, if, with *Kley*, we assume the extreme angles to be

$$\alpha_2 = 25^\circ \quad \text{and} \quad \alpha_1 = 45^\circ,$$

we get

$$\frac{c}{l} = \sin^3 25^\circ = 0.075$$

and

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{\cos 25^\circ - 0.075 \cot 25^\circ}{\cos 45^\circ - 0.075 \cot 45^\circ}} = \sqrt{\frac{0.745}{0.632}} = 1.085,$$

so that the greatest velocity exceeds the smallest by 8.5 per cent, which is sufficiently small for most cases. For intermediate positions the velocities of course lie between ω_1 and ω_2. If the governor in its lowest position is to make 100 revolutions per minute, the angular velocity becomes

$$\omega_2 = \tfrac{100}{60} 2\pi = 10.472 \text{ linear units,}$$

and hence if for a *Porter* governor we assume $Q = \frac{3}{2} G$ and $l_1 = l$, we can obtain the value of l from

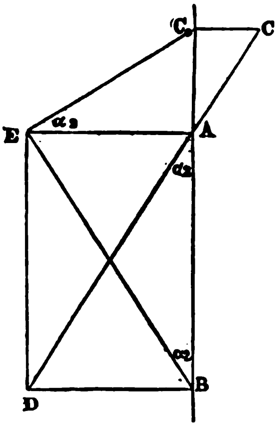
$$\omega_2 = 10.472 \text{ meters} = \sqrt{(1 + \tfrac{3}{2} 2) \frac{9.81}{l(\cos 25^\circ - 0.075 \cot 25^\circ)}}$$

$$= \sqrt{\frac{52.67}{l}}; \quad \left[\omega_2 = 10.472 \text{ inches} = \sqrt{\frac{2073.62}{l}} \text{ inches};\right]$$

namely, $l = 0.480$ meter [18.9 ins.] and $c = 0.075$, $l = 0.036$ meter [1.42 ins.]. In this case the greatest angular velocity is $\omega_1 = 1.085 \times 10.472 = 11.362$ meters [corresponding to $\omega_1 = 11.362$ ins.], which corresponds to $n_1 = 108.5$ revolutions.

REMARK. *Werner* gives the following graphical method of determining the length l of the pendulum and the eccentricity c of the point of suspension (see Zeitschr. deutsch. Ingen., 1865, p. 277). Lay off on the axis the distance AB, Fig. 778, equal to that value of h which belongs to the lowest position of the governor and corresponds to the angular velocity ω_2 given by the formula

FIG. 778.



$$\omega = C\sqrt{\frac{g}{h}},$$

and draw AD so as to make the smallest angle of inclination α_2 with the axis. Complete the rectangle $ABDE$, and at E erect a perpendicular to the diagonal BE; this perpendicular will cut the axis in the point C_0 which, when projected perpendicularly to the axis on to the pendulum arm AD, gives in CD the length l of the arm, and in C_0C the eccentricity c. This is proved as follows: We have

$$AB = h = l \cos \alpha - c \cot \alpha,$$

and also

$$h = BE \cos \alpha_2 = BC_0 \cos^2 \alpha_2 = CD \cos^3 \alpha_2 = l \cos^3 \alpha_2;$$

consequently by equating we get

$$l \cos \alpha - c \cot \alpha = l \cos^3 \alpha,$$

or

$$l(1 - \cos^2 \alpha) = \frac{c}{\sin \alpha}, \quad \text{or} \quad \sin^3 \alpha = \frac{c}{l},$$

as above.

The formula occasionally found in text-books,

$$c = l\frac{\cos \alpha_2 - \cos \alpha_1}{\cot \alpha_2 - \cot \alpha_1},$$

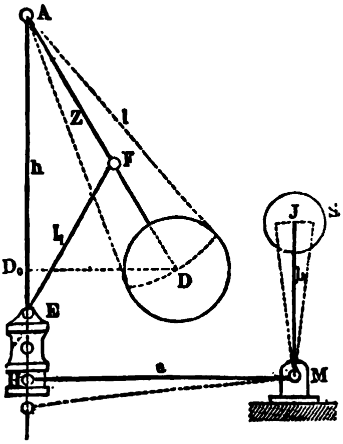
i.e.,

$$l \cos \alpha_2 - c \cot \alpha_2 = l \cos \alpha_1 - c \cot \alpha_1,$$

is based on the condition that in the extreme positions the projections h of the pendulum arms shall be equal, and therefore the velocities ω_1 and ω_2 shall also be equal. But such an arrangement cannot give us any governor that can be used, for between these extreme positions having the equal projections h there lies the pendulum position for which h is a maximum. As a consequence, that portion of the governor movement which lies between the last-mentioned and the lowest position of the pendulum belongs to the region of *unstable equilibrium*. Experience has also shown that governors constructed according to this formula have been particularly subject to *jumping*. Theoretically, these governors possess this defect in a more marked degree than the perfectly astatic (isochronous) governors.

But, without crossing the arms, there is another way of making the *Watt* governor nearly astatic, namely, by varying the load $2\,Q$ of the sleeve with the angle of inclination of the pendulum. Such an arrangement has been designed by *Grossmann*. Suppose an ordinary *Watt* governor suspended from a point in the axis to be in equilibrium under the influence of the normal velocity ω when it occupies its middle position AD, Fig. 779, in which $AD = h$; then when it occupies a higher position, the velocity must be greater than ω in order that the centrifugal force may keep the weights of the balls and of the sleeve in equilibrium. Now instead of maintaining this equilibrium by means of centrifugal force, the same object may be attained by diminishing the load Q. In like manner when the governor sinks, equilibrium may be maintained without diminishing the velocity, simply by increasing the load Q on the sleeve. To effect this *Grossmann* subjects the sleeve E to the action of a bell-crank lever MHJ, which, turning about the fulcrum M, carries a weight S at J and, by means of a fork at H, acts on the sleeve E. In the middle position of the governor the lever arm MH is horizontal and MJ is vertical, so that in this position there is no action of the weight S on the sleeve E, while for a motion of the sleeve above or below this position the weight S exerts an upward or downward pressure on the sleeve. If e represents the travel of the sleeve E from its middle position, the lever MH will be inclined to the horizon an amount which can be expressed by $\phi = \frac{e}{a}$ when the length of the lever arm $MH = a$ is made sufficiently great. The lever arm $MJ = b$ has the same inclination ϕ to the vertical, so that the moment of the weight S is given by

FIG. 779.



$$S b \phi = S \frac{b}{a} e.$$

Now if $2K$ represents the action of the weight S on the sleeve, we get from

$$2 K a = S \frac{b}{a} e$$

for K the value

$$K = S \frac{b}{2a^2} e.$$

By lifting or lowering the sleeve through the distance e, the point F, by reason of the rhombic suspension, moves through the distance $\frac{e}{2}$; consequently the centre of the ball is lifted or lowered in a vertical direction through $\frac{l}{l_1} \frac{e}{2}$, and the projection on the axis will be diminished or increased by this amount. For the equality of the angular velocities in the middle and in this new position we have the general equation

$$\frac{\omega^2}{g} = \frac{1}{h}\left(1 + \frac{Q}{G} \frac{2l_1}{l}\right) = \frac{1}{h \pm \frac{l}{2l_1} e}\left(1 + \frac{Q \pm K}{G} \frac{2l_1}{l}\right).$$

From this follows

$$\frac{l}{2l_1} e\left(1 + \frac{Q}{G} \frac{2l_1}{l}\right) = h \frac{K}{G} \frac{2l_1}{l} = h \frac{S}{G} \frac{b}{2a^2} e \frac{2l_1}{l}.$$

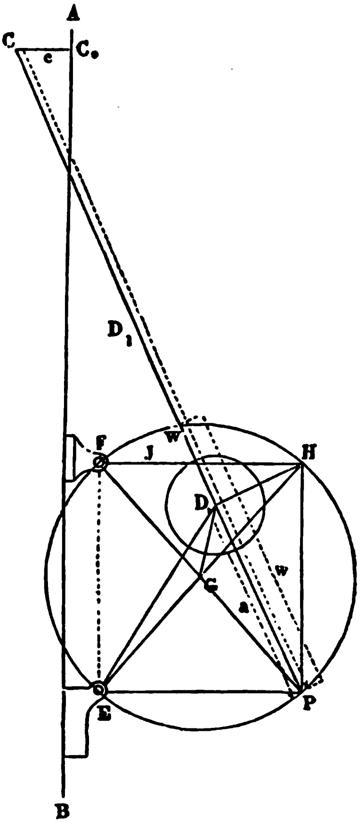
Consequently we have the equation

$$S \frac{b}{2a^2} = \frac{G}{h}\left(\frac{l}{2l_1}\right)^2 \left(1 + \frac{Q}{G} \frac{2l_1}{l}\right) = \left(\frac{l}{2l_1}\right)^2 \frac{G\omega^2}{g}.$$

The values for the weight S and the lever arms a and b must be chosen to satisfy this equation if the governor is to be approximately astatic. The approximation is a very close one, and becomes closer the more nearly the supposition $e = a\,\phi$ is realized, *i.e.*, the longer the lever $MH = a$ is in comparison with the half stroke e of the sleeve; in other words, the less the path of the end H deviates from the vertical path of the sleeve. By suitably choosing the length of the lever-arm MH the governor may be made as nearly astatic as desired.

§ **200. Proell's Governor.**—The pseudo-astatic governor with crossed pendulum-arms, Fig. 777, as designed by *Kley*, works very satisfactorily in practice, particularly when its energy is increased by loading the sleeve, as in the *Porter* governor. But the crossing of the arms gives the construction a great height which is inconvenient in practice. To meet this defect efforts have been made to arrange the governor so that its height will be considerably reduced without essentially altering its efficiency. These efforts have led to the invention of the *Proell* governor, and in describing it we will follow the method of development given by the inventor.* Let Fig. 780 represent a pseudo-astatic governor with crossed pendulum-arms *CD*, the balls being placed at the joints *D* connecting the arms *CD* with the sleeve rods *ED*. The motion of a sleeve rod *ED* at any instant can be regarded as an infinitely small rotation about the instantaneous centre. For a given position, say the middle one *CDE*, the instantaneous centre is found at the intersection *P* of the direction of the arm *CD* with the horizontal through *E*, for these lines are perpendicular to the elements of the paths described by the points *D* and *E*. The sleeve rod *DE* and the ball rigidly connected with it have a constrained motion due to the guiding of the point *D* in a circular arc about *C* and of the point *E* along the vertical. On account of this constrained motion every other point rigidly connected with *ED* must describe a

FIG. 780.



---

* See Civil-Ingenieur, 1872, p. 185. and Zeitschr. deutsch. Ingen., 1873, p. 65.

perfectly determinate path. Now as the motion of a rigid system moving in a plane is completely determined by the paths of *any two* of its points, it is evident that the motion of the sleeve rod and its ball *D* can be obtained if, in place of the point *D*, some other point *G* of the sleeve rod is guided in the path which it describes when connected with the original governor *CDE*, of which *Proell s* is to be a modification. When this is done we can dispense with the guiding of the point *D*, *i.e.*, we can omit the pendulum-arms *CD*. Now to guide any point, for example *G*, so that it will move as if it were connected with the original governor, we must know the path of this point. The path of the point *G* is known to be perpendicular to the instantaneous radius *PG*, and therefore an infinitely small motion in a direction perpendicular to *PG* at *G* can be replaced by a small rotation about a centre lying on the instantaneous radius *PG*. The path described during this rotation will agree most nearly with the original path of *G* when the centre of curvature of the element of this path of *G* is taken as a centre for the aforesaid rotation. Therefore if the centre of curvature of the path of *G* at the instant in question, *i.e.*, for the middle position of the governor, is supposed to be at *F*, the point *G* of the sleeve rod will, for this position, receive the desired motion by means of a radius bar *FG* jointed to the spindle *AB* at *F* and to the sleeve rod at *G*. This takes the place of the pendulum-arm *CD*. To be sure, the agreement of the circular motion of the point *G* about the point of suspension *F* with the original motion of *G* when the suspension was at *C* obtains for only an instant; but as the osculatory circle of a curve differs but little from the curve for a short distance on each side of the point of contact, we may, when we consider the small angular variations of the governor to each side of its middle position, regard the construction here given as a sufficiently close approximation. The figure at once shows that, by suitably choosing the guided point *G* or centre of curvature *F*, the height of the governor can be greatly reduced.

It is therefore only a question of finding some simple exact method of determining the centre of curvature *F* of any point *G*. The *inflexion circle* discussed in § 16 of the Introduction furnishes us with such a simple exact method. If we suppose this circle, which is known to pass through the instantaneous centre P, to be represented by PWF, we know from § 17 of the Introduction that any instantaneous radius PD drawn to a point of the moving system is cut at W by the inflexion circle at a distance $PW = w$ from the instantaneous centre, and we have

$$\frac{1}{w} = \frac{1}{a} \pm \frac{1}{a_0},$$

where a is the distance PD of the instantaneous centre from the point of the system, and $a_0 = PC$ the distance of the instantaneous centre from the centre of curvature of the path D. Now as the centre of curvature of the path of the point D is given by the point of suspension C, we known that $a_0 = PC$ and $a = PD$, and we can find from the equation

$$\frac{1}{PW} = \frac{1}{PD} - \frac{1}{PC}$$

the distance PW cut from the instantaneous radius PC by the inflexion circle. According to § 17 of the Introduction, we may also accomplish this by laying off $PD = DD_1$; the required point W will form an *harmonic series* with P, D_1 and C, and will be conjugate to C. Moreover, as the centre of curvature of the path of the point E is infinitely distant, we have

$$\frac{1}{a_0} = \frac{1}{\infty} = 0, \quad a = PE, \text{ therefore } \frac{1}{w} = \frac{1}{PE},$$

i.e., the inflexion circle passes through the point E, which agrees with the known property of the inflexion circle according to which all of its points describe the points of inflexion of their paths. When, therefore, the point W has been determined in the present case, the circle passed through this point W, the instantaneous centre P and the point E will be the inflexion circle. With the help of this circle it is now easy to find the centre of curvature of any other point G rigidly attached to the rod ED, or inversely it is easy to determine the point G of a system belonging to any assumed centre of curvature F. In *Proell's* governor the centre of curvature is a point F on the inflexion circle, whose distance from the axis

$A\,B$ is equal to that of the point E, so that $E\,F \parallel A\,B$. The corresponding point of the system then evidently lies at the middle G of the instantaneous radius $P\,F$, *i.e.*, at the centre of the inflexion circle, for in this case

$$a_0 = d, \quad a = \frac{d}{2} \quad \text{and} \quad w = d,$$

consequently the equation

$$\frac{1}{w} = \frac{1}{a} - \frac{1}{a_0} \quad \text{or} \quad \frac{1}{d} = \frac{1}{\frac{d}{2}} - \frac{1}{d}$$

is satisfied. Accordingly in the governor in question a radius rod $F\,G$ is jointed to the governor spindle at F, and at G to a rod jointed to the sleeve at E, the latter rod having a bent end $G\,D$ for receiving the ball. This governor is represented in

FIG. 781.

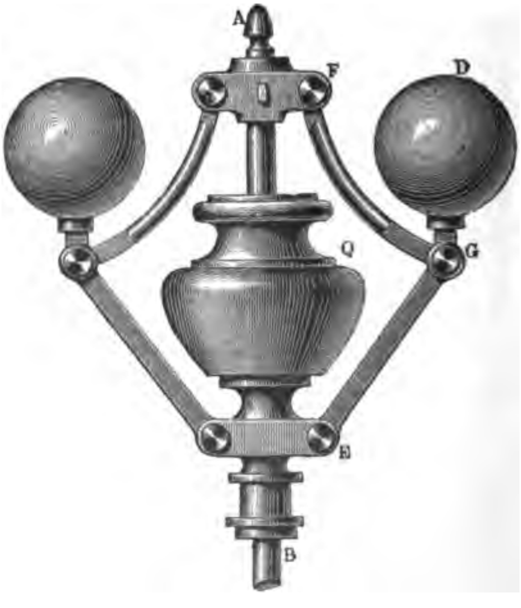
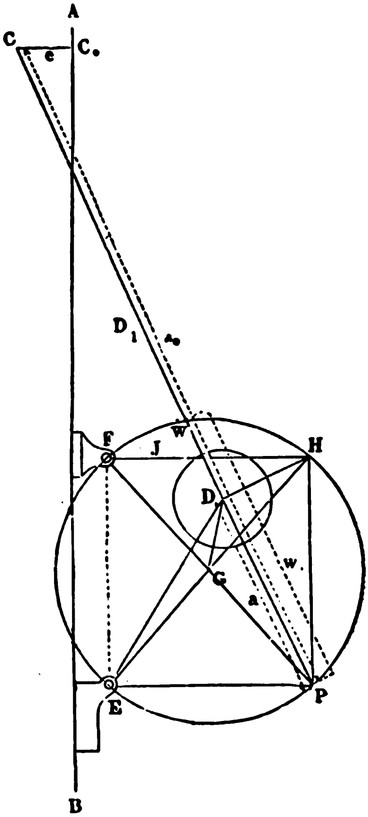


Fig. 781. This construction reduces the height of a spindle to about half that of *Kley's* governor; moreover the crossing of the pendulum-arm and the twisting of the joints by the inertia of the balls disappear. In addition to this the construction

permits the employment of heavy balls on the free ends of the arms and the use of a heavy counterpoise *Q*, thus increasing the *energy* of the governor and diminishing its *insensibility*.

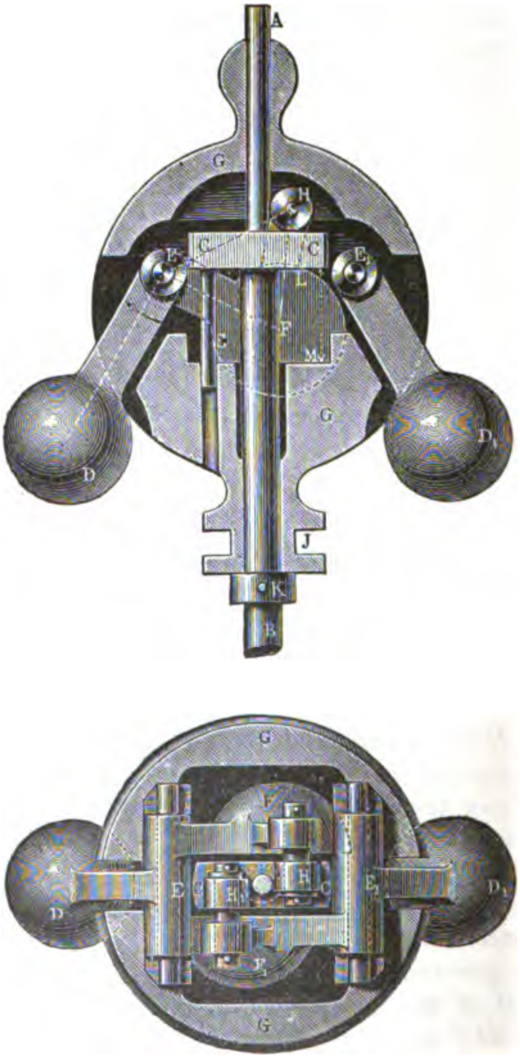
§ **201. The Cosine-Governor.**—Instead of replacing the *Kley* governor, Fig. 777, by the *Proell* governor, in which the point *G* is guided in a circular arc by the radius bar *F G*, we can replace it by an arrangement in which a point *H* on the inflexion circle *F W P*, Fig. 782, is constrained by external means to move in the path belonging to it. All points of the inflexion circle describe the points of inflexion of their paths, *i.e.*, the radii of curvature of the instantaneous elements of the paths are infinitely large. The elements of the paths of these points can therefore be regarded as rectilinear at the instant under consideration. For example, if we regard the point *H* of the inflexion circle as rigidly attached to the sleeve rod *E D*, its instantaneous motion will be in the direction *H J* perpendicular to the instantaneous radius *P H*. We can therefore suppose the pendulum-rod *C D* to be replaced by an arrangement in which a rectilinear guide *H J*, connected with the spindle *A B* of the governor, compels the point *H* of the rod *E D H* to describe a straight line. This guide *H J* is perpendicular to the governor axis *A B* when the point *H* is assumed vertically over the instantaneous centre *P*. The motion of the rod *E D H* is then evidently like that discussed in § 11 of the Introduction, and every point of the rod will accordingly describe an ellipse.

FIG. 782.



On this is based the *cosine-governor*, Fig. 783, invented by *Gruson*.*) In this governor the spindle A B is provided with

Fig. 783.



a collar C C whose upper horizontal surface serves to support a guide roller H on each side of the spindle, the roller being attached to the arm E F H of a bell-crank lever D E F H. This lever is loaded at *D* and *F* by pendulum-weights, and has its fulcrum at *E* on the sliding sleeve *G*, which in this case is of a spherical shape almost completely enclosing the pendulums. The sleeve is compelled to rotate with the spindle by a cylindrical pin *S* rigidly attached to the collar *C*, and enters a suitable hole in the lower part of the sleeve. This permits the sleeve to slide vertically along the spindle while it is rotating. If, while the whole system rotates, we suppose the pendulum *D F E* to swing about *E* in the direction of the arrow in consequence of the centrifugal force of the masses *D* and *F*, then, as the roller *H* finds a fixed support on the surface *C C*, the fulcrum *E*, and therefore the sleeve *G*, will rise, which rising or sliding motion is communicated in the ordinary manner to the transmitting mechanism by means of a fork in the groove *J*. The balls *D* of both pendulums lie in the same axial plane, but the weights *F* have a compressed form, and are placed near the spindle. Consequently the arms of each bell-crank lever *D E F* are not in the same plane, but are set off from each other as shown in the plan of the accompanying figure. The stroke *L M* of the sleeve is limited below by the collar *K* and above by the collar *C C*.

---

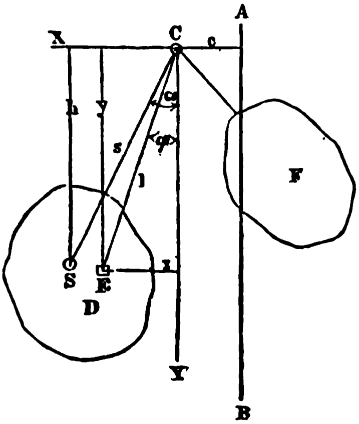
* Zeitschr. deutsch. Ingen., 1877, p. 97.

The designation *cosine-governor* is chosen for this device because the pendulum used has the property that for a given velocity its centrifugal moment with reference to the fulcrum is proportional to the cosine of the angle of inclination, as will be shown in what follows. If *C*, Fig. 784, is the point of suspension of a pendulum *D* of any form whatever, which rotates about the axis *A D*, its centrifugal moment with reference to the point of suspension *C* will be given by

$$M = \frac{\omega^2}{g} \Sigma q(c + x)y,$$

when the angular velocity ω is given, and when q represents the weight of an element of mass *E*, x and y its co-ordinates

FIG. 784.



referred to rectangular axes with origin C and vertical axis of Y, and c the distance of the point of suspension from the axis of rotation. If, besides, $l = CE$ represents the distance of the element of mass from the point of suspension, and ϕ the angle made by this distance with the axis of Y, we can also write

$$M = \frac{\omega^2}{g} \Sigma q (c + x) y = \frac{\omega^2}{g} \Sigma q (c + l \sin \phi) l \cos \phi$$
$$= \frac{\omega^2}{g} \left[c \Sigma (q l \cos \phi) + \Sigma \left(q \frac{l^2}{2} \sin 2 \phi \right) \right].$$

If a pendulum were constructed so that for every position, *i.e.*, for every angle of inclination α formed by the distance $CS = s$ (of the centre of gravity) with the axis of rotation, the relation

$$\Sigma \left(\frac{q l^2}{2} \sin 2 \phi \right) = 0$$

obtains, then in every position the centrifugal moment would be

$$M = \frac{\omega^2 c}{g} \Sigma q l \cos \phi = \frac{\omega^2 c}{g} G h = \frac{\omega^2 c}{g} G s \cos \alpha,$$

where G is the weight of the pendulum and h the vertical distance of its centre of gravity S below the point of suspension C. This value of M will then be proportional to the cosine for every value of the angle α. Now it is easy to see that the condition

$$\Sigma \frac{q l^2}{2} \sin 2 \phi = 0$$

will be fulfilled for *every* value of α, when for any *single* position we have

$$\Sigma (q y^2) - \Sigma (q x^2) = 0$$

and

$$\Sigma (q x y) = 0.$$

For if both these equations are satisfied by any angle of inclination α, and if we write them as follows,

$$\tfrac{1}{2}\Sigma q\, l^2 \cos^2\phi - \tfrac{1}{2}\Sigma q\, l^2 \sin^2\phi = \Sigma \frac{q\, l^2}{2} \cos 2\phi = 0$$

and

$$\Sigma q\, l^2 \sin\phi \cos\phi = \Sigma \frac{q\, l^2}{2} \sin 2\phi = 0,$$

then, if δ represents any angular deviation of the pendulum from the particular position for which the last two equations hold good, the following equation

$$\Sigma \frac{q\, l^2}{2} \sin(2\phi + 2\delta) =$$
$$\Sigma \frac{q\, l^2}{2} \sin 2\phi \cos 2\delta + \Sigma \frac{q\, l^2}{2} \cos 2\phi \sin 2\delta = 0$$

will also be satisfied, for each of the two terms in the last expression is separately equal to zero. Now as δ represents any angle, it follows that the equation

$$\Sigma \frac{q\, l^2}{2} \sin 2\phi = 0$$

is satisfied for *every* angle of inclination α, provided the equations

$$\Sigma(q\, y^2) - \Sigma(q\, x^2) = 0 \quad \text{and} \quad \Sigma(q\, x\, y) = 0$$

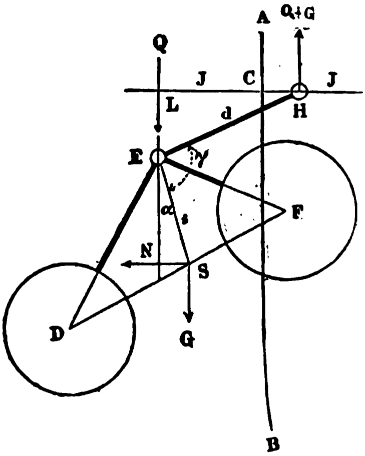
are true for some one position. Now if we suppose that these equations of condition are not satisfied by a pendulum D, but that we have instead

$$\Sigma(q\, y^2) - \Sigma(q\, x^2) = +A \quad \text{and} \quad \Sigma(q\, x\, y) = +B,$$

then we may suppose another weight F attached to the pendulum, for which weight

$$\Sigma(q\, y^2) - \Sigma(q\, x^2) = -A \quad \text{and} \quad \Sigma(q\, x\, y) = -B,$$

Fig. 785.



so that the condition stated above will be satisfied by combining D and F. For this reason each pendulum of the cosine-governor is loaded with two weights D and F. Now let DEF, Fig. 785, be such a cosine-pendulum turning about E, G its weight acting at the centre of gravity S, α the angle of inclination made by ES with the vertical, and γ the angle SEH, where H is the centre of the horizontally guided roller already mentioned. Then, when the spindle has the angular velocity ω, the moment of the centrifugal force with reference to E becomes

$$M = \frac{\omega^2 c}{g} G s \cos \alpha.$$

This action of the centrifugal force is opposed by the pendulum-weight G acting at the centre of gravity S, and by the weight Q (of half the sleeve) acting at the fulcrum E of the pendulum. As the point H of the pendulum is supported by the fixed guide JJ, the latter will react at H with a vertical, upward, force $G + Q$. Consequently we have for the equilibrium of the forces the following equation of moments with reference to E:

$$\frac{\omega^2 c}{g} G s \cos \alpha = G . SN + (G + Q) . HL;$$

or if the length EH is represented by d,

$$\frac{\omega^2 c}{g} G s \cos \alpha = G s \sin \alpha + (G + Q) d \sin (\gamma - \alpha).$$

From which follows:

$$\frac{\omega^2 c}{g} G s \cos\alpha$$
$$= G s \sin\alpha + (G + Q) d \sin\gamma \cos\alpha - (G + Q) d \cos\gamma \sin\alpha,$$

or

$$\frac{\omega^2 c}{g} G s \cos\alpha$$
$$= (G + Q) d \sin\gamma \cos\alpha + [G s - (G + Q) d \cos\gamma] \sin\alpha.$$

When the angle γ is determined by

$$G s = (G + Q) d \cos\gamma,$$

i.e., when

$$\cos\gamma = \frac{G}{G + Q} \frac{s}{d},$$

we have

$$\omega^2 = \frac{g}{c} \frac{G + Q}{G} \frac{d \sin\gamma}{s}.$$

As this equation does not contain the angle of inclination α, it follows that when

$$\cos\gamma = \frac{G}{G + Q} \frac{s}{d},$$

the cosine-pendulum is in equilibrium in every position with the same angular velocity

$$\omega = \sqrt{\frac{g}{c} \frac{G + Q}{G} \frac{d \sin\gamma}{s}},$$

and consequently in this case the governor is perfectly astatic (isochronous). If the governor is to possess a certain degree of stability, we must give to the angle γ a value differing somewhat from that corresponding to the perfectly astatic condition. To render this possible the cosine-governor is so arranged that the distance HL of the roller H from the axis of rotation can be varied within certain narrow limits. In *Gruson's* governor, for example, $Q = 3\,G$ and $d = 1.5\,s$: accordingly for the perfectly astatic condition we find the angle γ from

$$\cos \gamma = \frac{G}{G + Q}\frac{s}{d} = \frac{1}{6} \quad \text{or} \quad \gamma = 80^\circ\ 24'.$$

The governor is *stable* for a greater value of γ. For example, if we assume $\gamma = 90^\circ$, we obtain from

$$\frac{\omega^2 c}{g} G s \cos \alpha = (G + Q)\, d \cos \alpha + G s \sin \alpha,$$

$$\omega = \sqrt{\frac{g}{c}} \sqrt{\frac{G + Q}{G}\frac{d}{s} + \tan \alpha}.$$

This value of ω increases when α increases from zero on, and diminishes when α is negative. As the centre of gravity of the pendulum is directly below the point of suspension E, it follows that the pendulum is stable in the two lower quadrants. Assuming the ratios $Q = 3\,G$ and $d = 1.5\,s$ given above, we get

$$\omega = \sqrt{\frac{g}{c}} \sqrt{6 + \tan \alpha}.$$

This expression gives for

$\alpha =$	-20°	-10°	0°	$+10^\circ$	$+20^\circ$
$\sqrt{6 + \tan \alpha} =$	2.3741	2.4131	2.4495	2.4852	2.5226;

hence the ratio of the corresponding angular velocities ω will be expressed by

$$96.9 \qquad 98.5 \qquad 100 \qquad 101.5 \qquad 103.0.$$

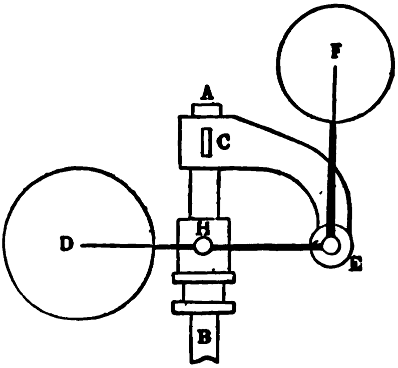
Accordingly for a swing of 20° to the right and left of the vertical position the angular velocity ω varies about 3 per cent; the coefficient of fluctuation is therefore

$$\eta = \frac{103.0 - 96.9}{100} = 0.061.$$

If the angle γ is taken between 80° 24′ and 90° the stability of the governor is less. The energy of this governor is comparatively great, or the insensibility is small, for the weight of the sleeve as well as of the pendulum, *i.e.*, the principal parts of the governor, act to make it energetic (see § 196).

The *Buss* governor has also worked well in practice and of late has become quite popular.* Its spindle carries two pendulums each of which is also loaded by two weights D and F, but these pendulums have the positions shown in Fig. 786, and their points of suspension E are rigidly attached to the spindle AB. To a certain extent this governor can be regarded as an *inversion* of the cosine-governor, which inversion can be obtained by imparting to all the parts of the latter an additional motion at every instant equal and opposite to the vertical sliding of the sleeve. The point of suspension E then becomes rigidly attached to the spindle, and the point H, possessing the horizontal sliding motion, receives in addition a vertical sliding motion which it communicates to the sleeve.

Fig. 786.

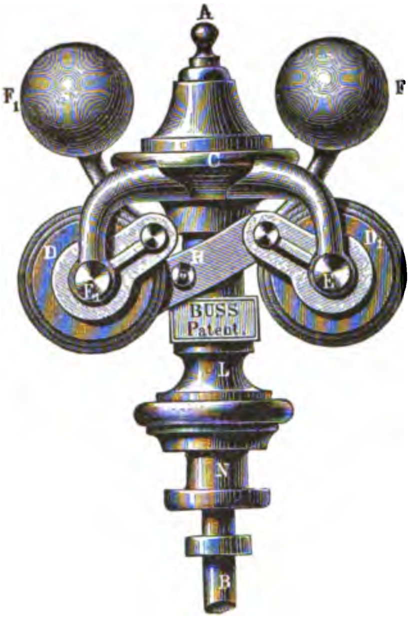


These remarks will render the arrangement and mode of action of the *Buss* governor, Fig. 787, intelligible. We will only add that the four-pronged pendulum-carrier CE, which is attached to the spindle AB, carries a pendulum at each of the two joints E and E_1, the pendulums being loaded by the weights D, F and D_1, F_1 respectively. Each of the horizontal arms ED, of which only the front one is visible in the figure, acts on the sliding sleeve L by means of the pin H in such a way that this pin is also capable of a slight horizontal sliding relatively to the sleeve. The motion of the latter is communicated to the transmitting gear in the ordinary manner by means of the groove N and a forked lever.

---

* Civil-Ingenieur, 1872, p. 1.

FIG. 787.

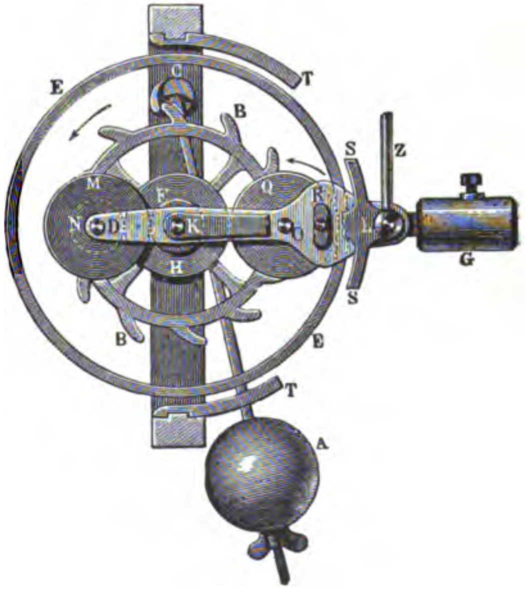


§ **202. Differential Governors.**—These governors, like the centrifugal ones, only come into action after a variation of the velocity has taken place. Generally these governors contain two shafts, one of which is turned directly by the prime mover to be regulated, while the other receives as uniform a rotation as possible by some means, say clock-work. A differential train introduced between these two shafts is connected with the transmitting gear and does not move when the two shafts have the same velocity. But when the prime mover, and consequently the shaft driven by it, moves faster or slower than the uniformly rotating shaft, the differential mechanism will assume a motion corresponding to the difference between these two velocities, and this motion communicated to the adjusting gear regulates the flow of the motor to the prime mover. A few examples will render this action clear.

In the apparatus of *Laukner* Brothers, employed for regulating the gate of a water-wheel, the axle of the wheel is prolonged so as to form a screw whose nut is the hub of a special, small, water-wheel. This is supplied with as constant a supply of water as possible, and as it runs empty the work of this water is consumed in overcoming the hurtful resistances. So long as the little wheel makes the same number of revolutions as the larger wheel the nut will not shift its position relatively to the screw. But such a shifting will take place when the velocity of the principal water-wheel varies from the normal velocity and the sliding of the nut due to this difference of speed will be communicated by the adjusting mechanism to the gate, opening or closing the latter according as the speed of the water-wheel shaft is less or greater than the normal speed.

Hick's Fan-governor and Wiede's Pendulum-governor are based on the same principle; in the former it is the fan-wheel, in the latter the pendulum, which regulates the motion of the nut.

FIG. 788.



There is also an interesting pendulum-governor by *Cohen, David,* and *Siama* of Paris. An ordinary pendulum *C A*, Fig. 788, is connected by a cylinder escapement *C* (see § 173) with an escape-wheel *BB*, so that the latter turns uniformly. This rotation in the direction of the arrow is shared by the annular, toothed, wheel *E E* (the teeth are omitted in this and in the other wheels of the figure), for the escape-wheel *B* is connected with the wheel E by a spiral spring. The two wheels F and H and the two levers $K\,L$ and $D\,O$ turn loosely on the shaft K of the escape-wheel. The lever $K\,L$ is jointed to the rod Z which is connected with the admission valve. On the other hand, the lever $D\,O$ carries on an arbor D the toothed wheels M and N which gear with F and H respectively, and on another arbor O the wheel Q which gears with both H and the annular wheel E. This combination of wheels forms an *epicyclic train.* Now when the wheel F is turned by the prime mover, the wheels M, N, H and Q likewise rotate about their axes. If the prime mover is running at its normal speed the wheel Q will have the same circumferential velocity as its partner, the internally toothed wheel E, and then the lever $D\,O$ will not rotate about the shaft K. But when the speed of the prime mover becomes greater or less than the normal speed, the wheel Q will rise or fall within the annular wheel E and thus communicate motion to the lever $D\,O$, which imparts it to the adjusting lever $K\,L$ by means of the pin R. Here G is a counterweight and T are stops against which the projections S of the adjusting lever strike and thus limit the stroke of the latter.

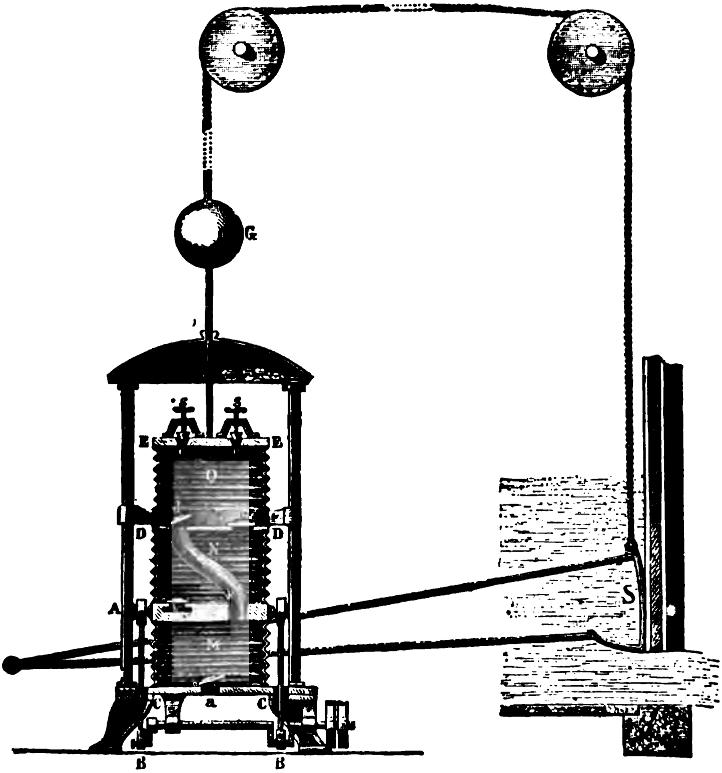
§ **203. Hydraulic and Pneumatic Governors.**—The hydraulic governor consists principally of a small pump which, driven by the prime mover to be regulated, forces water into a reservoir from which the water again flows into the pump. When the prime mover is running at its normal speed the level of the water in the reservoir is at a particular height h above the orifice of efflux, the water under this head flowing out of the reservoir as rapidly as it is supplied by the pump. It is therefore evident that with a greater speed, that is, with a greater supply of water from the pump, the level of the reservoir will rise to a height h_1, and that with a slower speed the level will fall to a height h_2. A float placed in the reservoir will share these fluctuations between h_1 and h_2 and serve to move the adjusting gear. This governor is essentially a *static* one because for different float positions, *i.e.*, for different positions of the admission gear, the quantity of water supplied by the pump and consequently the velocity of the prime mover is different. If the orifice of efflux is kept constant, the discharge will be proportional to the velocity of efflux and consequently to the square root of the head. Hence the ratio of the extreme velocities ω_1 and ω_2 of the prime mover will vary with the extreme heads h_1 and h_2 according to the proportion

$$\omega_1 : \omega_2 = \sqrt{h_1} : \sqrt{h_2},$$

and the corresponding travel of the float will be $h_1 - h_2$.

In *Schiele's* governor the pump is a centrifugal one, and in place of the float a loaded piston is used which works in a

FIG. 789.



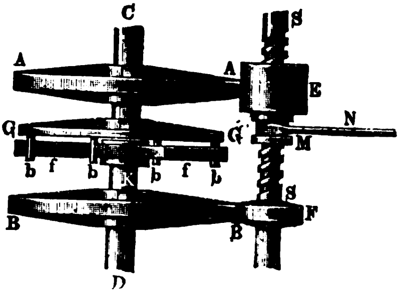
cylinder of slightly larger diameter, thus leaving a certain annular space between piston and cylinder. The water forced under this piston by the pump escapes through the annular space, and consequently the pressure under the piston increases with the quantity delivered by the pump. This governor and the next, pneumatic, one to be described are to be regarded as *astatic* governors, for the pressure exerted by the water on the piston is independent of the piston position.

Of late *Molinié's pneumatic governor* has been more frequently used for regulating prime movers, particularly water-wheels. This governor is essentially a double-acting bellows, Fig. 789, and is arranged as follows:

A is a piston moved up and down by the prime mover by means of the double crank *B* and connecting rods *A B*; it separates the two air spaces *M* and *N* which are further bounded by the fixed ends *C* and *D* and by the corrugated leather cylinder. Above the fixed cover *D* there is a third air chamber which possesses a movable cover *E* to which is fastened a rod connected with the valve gear regulating the motor supply. The two lower spaces *M* and *N* communicate with the atmosphere by the suction valves *a* and *b*, and with the upper air chamber *O* by the delivery valves *c* and *d*. When the piston *A* rises the outer air enters the gradually enlarging chamber *M* through the valve *a*, and the air in the gradually contracting chamber *N* passes through *c* into the third reservoir *O*; when the piston sinks, the outer air enters the gradually expanding space *N* through the valve *b*, and the air from the gradually contracting space *M* flows through *d* into the upper reservoir, the valves *a* and *c* being kept shut by the internal pressure. In order that the outer air may easily pass through *b* into the chamber *N* the entrance to the channel admitting the air from without is placed on the cylindrical surface of the piston *A*, and in order that the air may readily flow from *M* to *N* through *O* these spaces are connected by a piece of hose *F* whose exit is covered by the valve *d*. The air which is thus pressed out of the blast chambers *M* and *N* into the reservoir *O* flows into the open air through the orifices *e e* in the movable cover *E*. This efflux can be regulated as desired by conical valves adjusted by means of screws *s s*. When the prime mover is acting normally the blast chambers *M* and *N* deliver as much air to the reservoir as flows from the latter through the orifices of efflux *e*; consequently the cover *E*, its rod loaded with the counterweight *G*, and the gate or valve *S* connected with the latter remain stationary. But when the velocity of the prime mover and the blast piston *A* changes, the quantity of air delivered to the reservoir *O* will change and the pressure of air in this reservoir will increase or decrease and cause the cover *E* to rise or fall till the action of the valve *S* restores the normal velocity. As the pressure of the air under the cover depends only on the quantity of air delivered and the orifices of efflux *e*, and not on the position of the cover *E* and the gate, we must class this apparatus with *astatic* governors.

§ **204. Poncelet's Governor.**—This governor, Fig. 790, differs from those hitherto discussed principally in that it acts as soon as the resistance to be overcome varies from its normal amount, instead of depending on a variation of velocity for its action. Here *C* is the shaft driven by the prime mover and *D* the shaft which experiences the resistance of the machinery. These shafts are connected by a flexible coupling *G* so that a disk *G* on the power shaft *C* carries pins *b* which press against steel springs *f* attached to the muff *K* of the driven shaft *D*. When the prime mover is acting normally the springs are bent an amount corresponding to the force transmitted and both shafts turn with the same speed. But when the resistance increases, the deflection of the springs *f* will increase, *i.e.*, the driving shaft *C* will get slightly ahead of the driven shaft *D*; on the other hand, when the resisttance diminishes, the springs straighten and the driven shaft *D* gets slightly ahead of the driving shaft *C*. This relative motion of the two shafts, caused by the variation of the resistance, is utilized to move the adjusting gear, the two shafts *C* and *D* being provided with equal wheels *A* and *B* which gear with two equal pinions *E* and *F*. The driving wheel *F* is fastened to the screw *S* which has

FIG. 790.

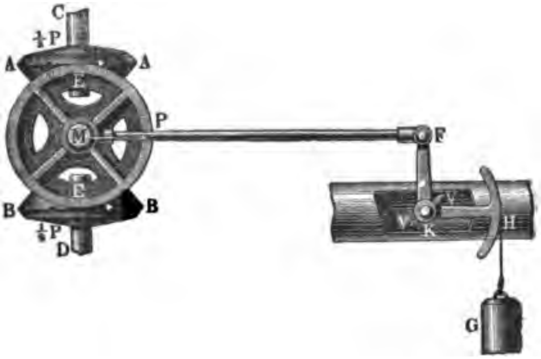


bearings that permit rotation but prevent axial motion. The nut of this screw is the pinion *E*. It is now easy to see how a difference of velocity of *C* and *D*, and consequently of *E* and *F*, must cause a shifting of the nut *E* on the screw *S*, the amount of this shifting depending on the motion of the nut relatively to the spindle, and its direction one way or the other on whether the resistance of the shaft *D* is greater or less than the normal. The figure shows how this shifting of the nut is transferred to the lever *M N* of the adjusting gear.

This ingenious arrangement has not, however, worked well in practice. Overlooking the difficulty of making an elastic coupling sufficiently durable for the transmission of great forces, we find that this governor possesses the defect of exerting a regulating effect *only when there is a variation of resistance and not when there is a variation of driving force.* For suppose the prime mover to be working normally and that from some cause the driving force becomes less intense, then in consequence of the smaller pressure on the springs the latter will bend less and the adjusting gear will act on the admission valve exactly as if the diminished deflection were due to a decrease in the resistance. The motor supply is therefore diminished by the adjusting gear when just the opposite should take place on account of the diminished intensity of the driving force. Consequently a variation of the latter causes the governor to act in a hurtful manner by still further increasing the irregularities caused by this variation.

Siemens' governor must also be classed with those which are brought into action by a variation in the force transmitted. In this governor the shaft *D*, Fig. 791, is driven by the prime mover to be regulated, and by means of its bevel wheel *B* and the intermediate wheel *E* an equally large wheel *A* and its shaft *C* are driven with equal velocity in an opposite direction. On the shaft *C* there is a conical pendulum with very large balls which is to act principally by the inertia of its moving masses. When the prime mover is acting normally, a force is transmitted from *B* through *E* to *A* just sufficient to overcome the frictional resistances of the shaft *C* carrying the conical pendulum. Let this pressure be represented by $\frac{1}{2} P$; then the wheel *E* will be acted upon by two equal forces acting in the same direction, and these combined will give a resultant force *P* which must be sustained by the bearing at *M*. Now the journal *M* does not run in a fixed bearing but in the eye of a rod *M F*, which is connected with a bell-crank lever *F K H* and subjected to a pull *P* by a counterweight *G*. Consequently there is equilibrium when the prime mover is running normally; but if the shaft *D* for any reason, whether an increase of driving force or a decrease of resistance, tends to accelerate its motion, the balls of the pendulum will take part in the acceleration, and a greater force than before must be transmitted through the wheels *B* and *E* and also through *E* and *A*. As a consequence the pressure of the journals at *M* will now exceed the pull *P* of the counterweight, with the result that *G* will rise and the throttle valve *V* restrict the admission of the steam. On the other hand, when the shaft *D* tends to retard its motion the living force of the pendulum balls will cause the wheel *A* to advance relatively to the circumference of *E*, and consequently the journal *M* will shift in the direction of the pulling weight *G* and increase the passage for the motor. This governor, therefore, like that of Poncelet, acts even before a variation of velocity has taken place, namely, as soon as the cause of such a variation appears; in other words, as soon as there is a disturbance of the equilibrium of the acting forces.

FIG. 791.



Remark. The theory of governors is given very fully by *Poncelet* in his Cours de mécanique appliqué. Many articles on ball-governors have also appeared in the various technical periodicals, for instance in the Civil-Ingenieur, Zeitschrift des Vereins deutscher Ingenieure, Bulletin de la Société d'encouragement, etc., Bulletin de la Société de Mulhouse, and Engineering. See also *Radinger's* Die Regulatoren, Bericht ueber die Pariser Weltausstellung 1867, also *Padelletti's* Regolatori a Forza Centrifuga. The pendulum-governor by *Cohen*, etc., is described in the Polytechnisches Centralblatt for 1851 and *Molinié's* pneumatic governor in *Armengaua's* Traité des moteurs hydrauliques et à vapeurs.

Concerning *brakes* see the article "Brems" in *Huelsse's* Allgemeine Maschinenencyklopaedie. There are numerous communications on railway brakes in *Heusinger's* Organ fuer die Fortschritte des Eisenbahnwesens, in the weekly "Die Eisenbahn" and in other technical periodicals. On this matter read also *Ruehlmann*, Allgemeine Maschinenlehre, vol. iii.

APPENDIX.

The Graphical Statics of Mechanisms.—The change of motion effected by any mechanism is always accompanied by a change in the acting forces. If there were no *hurtful resistances* connected with the moving of the separate machine parts on each other, we could easily determine in every case the change of force, according to the principle of virtual velocities. Thus, if we suppose any machine to be acting normally, *i.e.*, if we suppose the motion to be always uniform or to repeat itself in regular periods, we can, for this condition, place the algebraic sum performed by all the acting forces equal to 0. The investigation is therefore one of equilibrium in so far as the normal condition is characterized by equilibrium of motion.

Therefore if we represent by Q the useful resistance to be overcome, and by q the distance through which this resistance is overcome while the driving force P acts through the distance p, we shall have $Qq = Pp$ if we neglect all hurtful resistances, and from this equation the *theoretical* driving force

$$P_0 = Q\frac{q}{p}.$$

But as the motion of any two bodies relatively to each other is accompanied by certain frictional resistances, there will be needed, in addition to the work Qq required to overcome the useful resistance, a certain quantity of work Ww to overcome the hurtful resistances, w representing the distance through which the hurtful resistance W acts. With the hurtful resistance taken into account, the general equation becomes $Qq + Ww = Pp$, and from this we obtain the *actual* force

$$P = \frac{Qq + Ww}{p}.$$

As the hurtful resistances in all cases oppose the motion and therefore act here in the same sense as the load Q, it follows that the actual driving force P is always greater than the theoretical force P_0; in other words, the ratio $\eta = \frac{P_0}{P}$ is always less than unity. This ratio $\eta = \frac{P_0}{P}$ is called the *efficiency* of the mechanism, η representing that part of the driving force P which is engaged in overcoming the useful resistance.

In such machines as hoisting gear where the useful resistance Q is an active force which, left to itself, tends to turn the machine in an opposite direction, we speak of the *efficiency of the reverse motion*, and mean by this the ratio $(\eta) = \frac{(P)}{P_0}$, *i.e.*, the ratio of the actual force (P) exerted by Q at the point of application P to the theoretical force P_0. It is plain that (η) also must always be less than unity; in this connection we may remark that (η) may become *negative*, in which case the load Q assumes the reverse motion only when it is assisted by a force (P) acting in *the same sense* as Q. A negative value of (η) is therefore a characteristic of a mechanism which is *self-locking* for the reverse motion. Such a case has already been discussed in § 127 in connection with screws.

From what has preceded we see how important it is in practice to know the efficiency of a machine; for when the efficiency (η) is as great as possible, the stresses in the individual parts and the consequent expenditure of material are as small as possible and the expenditure of work is economical. How the hurtful resistances of friction, etc., are to be determined has been shown in a general manner in Vol. I, and in special cases in Vol. II and Vol. III, 1, and such determinations will often be made hereafter. In very complex machines the usually long calculations can in many cases be greatly simplified by the application of a general law. For if we suppose a complex machine to be resolved into its separate simple *mechanisms* (see § 30, Introduction), for example a crane into the loose and fixed pulley, the rope drum, the separate pairs of gears and the crank, and designate by η_1, η_2, η_3, . . . the efficiencies of these separate mechanisms, *the efficiency of the whole machine* is given by $\eta = \eta_1 \cdot \eta_2 \cdot \eta_3 \ldots$ That this law is correct is evident when we consider that the *driving force* P_1, applied to the first mechanism to overcome the useful resistance Q, is without any change to be regarded as the useful resistance of the following mechanism.

Nevertheless a calculation in which all the frictional resistances are considered is in most cases so long that the efficiency is often determined approximately or simply estimated. For example, let us suppose the simplest imaginable case, that of a fixed guide pulley in which the ends of the rope are not parallel; then to determine the journal friction we must introduce into the calculation an inconvenient radical,

$$\sqrt{P^2 + Q^2 + 2 P Q \cos \alpha},$$

for the pressure on the journal, and similar remarks apply to the shaft journals of gears, etc.

In determining the forces in systems that are at rest, the methods of graphical statics are found much easier than the analytical methods, and the thought arises that it may be well to determine the efficiency of machines by graphical means. Indeed a closer examination shows that the graphical methods of determining the efficiency are both simple and general in their application, and give a degree of accuracy as great as can be practically attained by any method, and sufficient for application in almost all cases. As there will be frequent opportunity to apply this graphical method in the following sections, we will here give its leading features.*

---

* For further details see *Herrmann's* "Zur graphischen Statik der Maschinengetriebe."

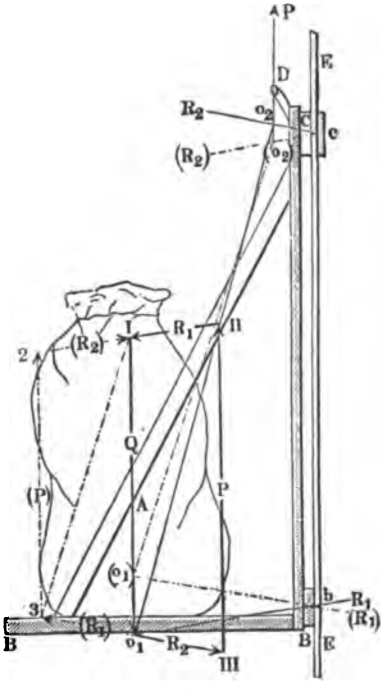
In every mechanism a certain link is held fast (see § 30, Introduction), which acts as a *frame* to support the other links at certain points or surfaces. At every such supporting surface two equal and opposite forces or reactions are called forth, one exerted by the supporting and the other by the supported piece, and we may therefore completely replace the supporting piece by the reactions which it exerts upon the supported links. When this is done the mechanism may be considered as simply a system of forces, namely, these *reactions* and the *external forces*, which system must be in equilibrium. The graphical investigation of this equilibrium may then be easily undertaken according to well-known rules; hence the main problem before us is the correct determination of the aforesaid reactions.

If no frictional resistances accompanied the motion of two surfaces in contact, the reactions between the surfaces called forth when the latter are in equilibrium would necessarily coincide in direction with the common normal to the surfaces in contact, for every direction deviating from this would cause the surfaces to move relatively to each other. But we know from the theory of friction (Vol I, § 172) that in the limiting condition, here always assumed, in which the body is just on the eve of motion, the reaction between the supporting surfaces deviates from the normal to these surfaces by the angle of friction $\rho = arc\ tan\ \phi$. We may therefore conclude that *every supporting surface in a mechanism can be replaced by a reaction whose direction deviates from the normal to the surface by the angle of friction ρ, the reaction acting in the direction of the motion which the supporting piece possesses relatively to the supported body when the motion begins;* in other words, the friction *opposes* the tendency to motion of the supported piece.

This law holds not only for the sliding motion of two flat surfaces upon each other, but, as the following investigations will show, for all kinds of frictional resistances, as journal-, tooth-, rope-, and chain-friction. In what has preceded we have spoken of a *point of contact* of the supporting surfaces and we may retain this term, for although bodies always touch along certain *measurable surfaces*, we can nevertheless regard that point as the point of application of the reaction, in which the contact surface is pierced by the resultant of the reactions of all the elements of the surface of contact.

The mode of applying the above proposition may be best shown by a few examples. Let *B C*, Fig. 792, represent an elevator used in grist-mills, *A* the load, and *D* the rope which lifts them. The elevator is guided by rails *E* against which the slides *B* and *C* press. If we suppose the reactions R_1 and R_2 of the rails to act at the middle points *b* and *c* of the slide plates and to make the angle of friction ρ with the horizontal, the forces *Q*, *P*, R_1 and R_2 must be in equilibrium. This is only possible when the resultant of the two forces *Q* and R_1 is equal and exactly opposite to the resultant of the other two forces *P* and R_2. Hence the resultants must coincide with the line o_1 o_2 connecting the intersection of *Q* and R_1 with that of *P* and R_2. From this we may at once construct the lifting force *P* for a certain load *Q*. Make $I\,o_1 = Q$, draw *I II* parallel to R_1, and through o_1 a parallel to R_2; the latter will cut from the vertical drawn through *II* the portion $III\,II = P$.

FIG. 792.



If we wish to find the force (*P*) exerted by the *sinking load* *Q* on the rope *D*, it is only necessary to draw the reactions (R_1) and (R_2) on the opposite sides of the horizontal and making with the latter the angle of friction ρ; then draw the connecting line (o_1) (o_2), and parallel to it the resultant force *I* 3, and thus obtain in 3 2 the force (*P*) which must be consumed by the brake in order that the elevator may sink uniformly. As we here have $P_0 = Q$, we easily find

$$\eta = \frac{P_0}{P} \quad \text{and} \quad (\eta) = \frac{P}{P_0}.$$

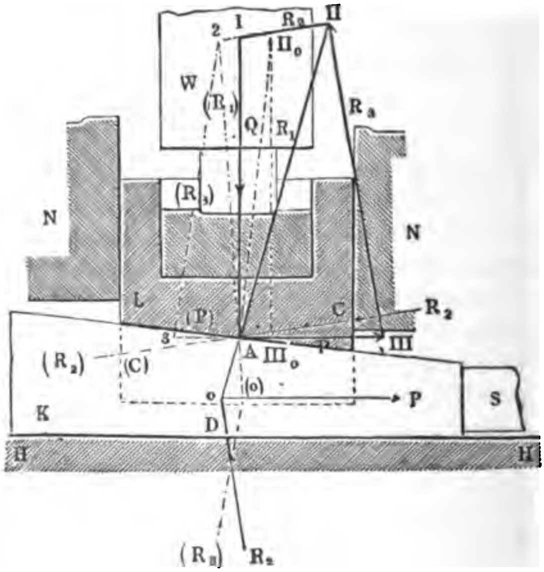
When the coefficient of friction is

$$\phi = tan\ \rho = 0.16,$$

we obtain from the figure $Q = 100$, $P = 108.8$ and $(P) = 89.5$; hence

$$\eta = \frac{P_0}{P} = 0.919 \quad \text{and} \quad (\eta) = \frac{(P)}{P_0} = 0.895.^*$$

FIG. 793.



Another example is the arrangement employed for adjusting the bearing of a pivot by means of a key, Fig. 793. Here the upright shaft W is to be lifted by a key K forced under the pivot cup L by a screw S exerting a horizontal pull P, the key sliding along the base plate H. If we assume that the resultant of all the elementary reactions with which the upper surface of the key K presses against the pivot cup L acts at the same point A through which the vertical load $Q = I\,A$ passes, then the direction $A\,II$ of this reaction R_1 must be so taken that it will make the angle of friction $\rho = II\,A\,II_0$ with the normal $A\,II_0$ to the surface of the key. The two forces Q and R_1 can only be in equilibrium when there is a third force acting on the cup L and passing through the intersection A. Such a third force is given by the reaction R_2 with which the pivot box N acts upon the sliding cup L. Therefore if we draw through A the straight line $A\,C$, making the angle of friction ρ with the horizontal, the $C\,A$ will represent the direction and position of the lateral reaction R_2, and its intensity will be given by the side $II\,I = R_2$ of the force polygon $I\,A\,II$. To find the pulling force P exerted on the key, it is only necessary to remember that the key must be held in equilibrium by the force P, the reaction $R_1 = II\,A$ and the reaction R_3 exerted by the base plate H against the key. The latter must therefore pass through the intersection o of P and R_1 and, of course, must form the angle of friction with the vertical. When this is done we may complete the force polygon by resolving the reaction $R_1 = II\,A$ into the directions of R_3 and P, *i.e.*, drawing $II\,III$ parallel to $o\,D$ and $A\,III$ parallel to P, and then get in $A\,III$ the required force P. It is easy to see that $A\,III_0$ will give P_0 when the reactions R are taken normal to the respective supporting surfaces. The figure also gives the dotted diagram $I\,A\,2\,3$ for the reverse motion, which now needs no separate explanation. Moreover we see from this diagram that the force $(P) = A\,3$, exerted on the key to lower the load, acts on the key in the direction in which the load Q tends to shift the key, and we conclude from this that Q alone is not capable of pushing back the key and that this mechanism belongs to the self-locking class.

---

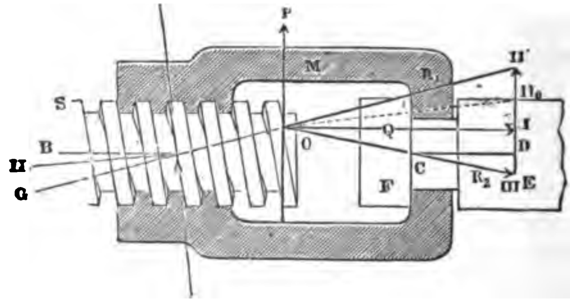
* The numerical values given in connection with these figures have been taken from drawings made to a larger scale.

For a coefficient of friction $\phi = 0.16$ and a taper of the key equal to $\frac{1}{8}$ the figure gives $Q = 100$, $P = 45$, $P_0 = 11.1$, and $(P) = -19.8$; hence

$$\eta = \frac{11.1}{45} = 0.247 \quad \text{and} \quad (\eta) = \frac{19.8}{11.1} = -1.78.$$

In like manner the frictional resistances of screws may be taken into account. Let AB, Fig. 794, be a screw whose nut M turns in the neck-groove C of the fixed stud FD. The problem is to find that turning force P of the nut which, acting with a lever arm equal to the average radius of the screw thread, will cause the screw to exert a pull Q. If we neglect the *circumferential friction* of the nut in the neck groove C, due to the lateral action of P and determined like journal friction, and suppose the elementary reactions of the nut at A and of the ring at C to be concentrated at a distance from the axis equal to the average radius of the threads, then the reaction R_1 of the nut will be found in AG and the reaction R_2 in CE, so that $GAH = ECD = \rho$ when AH is the normal to the middle helix at A. These two reactions intersect at O, and through this point must pass the force P whose intensity $III\,II = P$ can be obtained by making $Q = IO$ and drawing through I a parallel $II\,III$ to the direction of the force P. If there were no friction we should get $P_0 = I\,II_0$ when $O\,II_0$ is drawn paral-

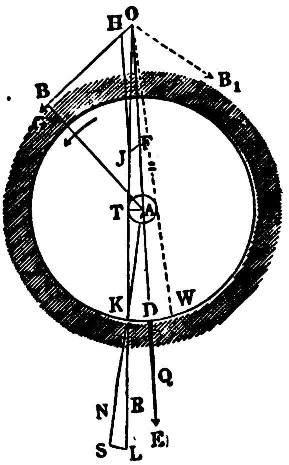
FIG. 794.



lel to the normal HA. For the reverse motion (η) would here also become negative. The construction is essentially the same when the stud FD is likewise provided with threads (left-handed ones), as is the case with the car-coupling, Fig. 515.

Journal friction can also be easily determined by means of the angle of friction. Let A, Fig. 795, be the centre of a cylindrical journal which is pressed with a force $DE = Q$ against its bearings. If there were no friction the bearing would react with a force ED normal to the surface of contact, *i.e.*, the reaction would pass through the centre, but as there is journal friction the reaction must pass to one side of the centre. Let $BG = OH$ represent that turning force P exerted at the end of the lever-arm AB, which is just sufficient to turn the journal; then this force and $Q = DE = OF$ form a resultant $OJ = KL$ which must be resisted by an equal and opposite reaction LK of the bearing. If we resolve LK into the tangential and radial directions LS and SK, we get in $SK = N$ the normal pressure at K, and in LS the friction $F = \phi N = N \tan \rho$, which is to be overcome, *i.e.*, we have $LKS = \rho$. Therefore the direction of the reaction $R = LK$ of the bearing passes by the *centre* A at a distance $AT = KA \sin \rho = \phi r$, when r is the radius of the journal and when we assume $\sin \rho = \tan \rho = \phi$. We easily see that a force acting in the direction OB_1 and thus tending to produce rotation in the opposite direction will call forth a reaction of the bearing in the direction WO, whose distance from the centre of the journal must also be equal to ϕr.

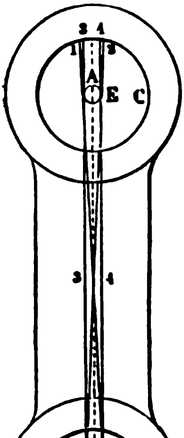
FIG. 795.



If we suppose the force $P = BG$ to remain at the same distance AB from the axis and to turn about the latter, then for every position the reaction of the bearing will pass by the centre A at the distance ϕr, *i.e.*, the reaction of the bearing will in all cases touch a circle concentric to A with a radius ϕr. Let us call this circle the *friction-circle* of the journal, we may then state the proposition that *the reaction between a bearing and its journal can only take place along a tangent to the friction-circle of the journal.* In what *direction* this reaction must be taken can be ascertained with certainty, as in sliding friction, from the general law, according to which *the reaction of either of two pieces on the other must take the direction of the motion which the reacting piece tends to assume relatively to the piece calling forth the reaction.*

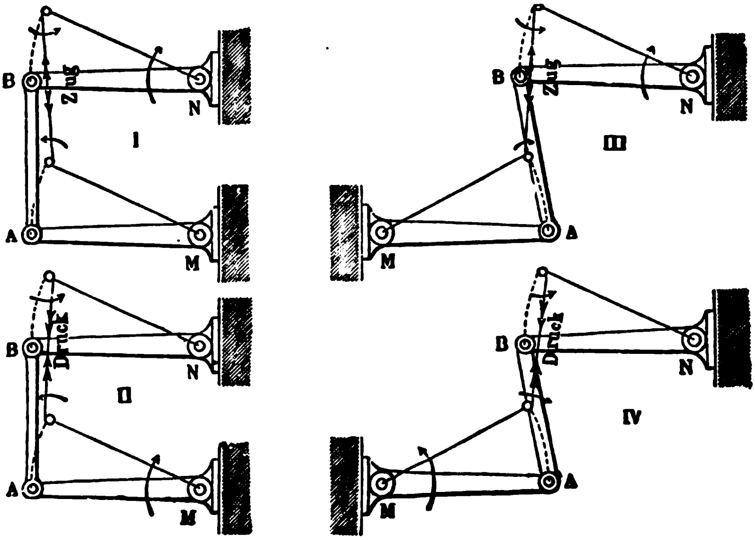
A very frequently occurring example will explain this. Let CD, Fig. 796, be a connecting rod whose bearings enclose the pins A and B of two other pieces, for example of two levers. According to what has been said with reference to journal friction this rod can only react, *i.e.*, transmit forces, in directions tangent to both the friction circles of A and B. There are four such tangents, represented in the figure by 1, 2, 3 and 4. Which of these is to be taken as the direction of reaction can be ascertained from the mechanisms sketched in Fig. 797, where they are designated by the corresponding numbers *I*, *II*, *III* and *IV*. In this figure the larger arrows at M and N indicate the direction of motion and which of the two levers MA and NB is the driver, while the small arrows at A and B indicate the rotation of the connecting rod N relatively to its lever-end. If we at the same time indicate by arrows whether the rod AB is subjected to *tensile* or *compressive forces* there will be no doubt as to the direction of a particular reaction, for the force in the rod must act so that it will tend to produce its rotation relatively to the lever-ends. When the rotation at M or N is in the opposite direction, the direction of the reactions between A and B is not changed; for although

FIG. 796.



in such a case the directions of the relative rotations of *A* and *B* are likewise opposite, the forces in the rod at the same time change from tension to compression or *vice versa.*

FIG. 797.



We will take the bell-crank lever, Fig. 798, as a simple example of the determination of journal friction. If a load *Q* acts in the direction *O A* on the pin *A*, and if this is overcome by a force *P* pulling in the direction *O B* at *B*, then if there were no journal friction we could simply connect the intersection *O* with the centre *C* of the journal and, making $O\,I_0 = Q$, get in the side $O\,III_0$ of the parallelogram $O\,I_0\,II_0\,III_0$ the required force P_0 and in $O\,II_0$ the pressure of the journal at *C*.

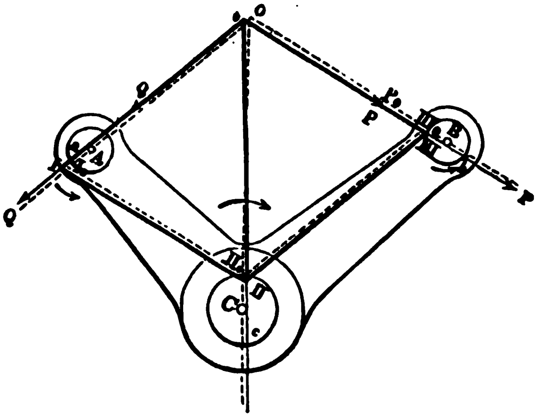
Journal friction, however, requires that the forces shall not pass through the centres of the pins, but shall be tangential to the friction circles, and therefore if we again make $o\,I = Q$ we shall obtain in $o\,III$ the actual driving force *P* required. This construction shows that the influence of the journal friction is to increase the lever-arm of the resistance *Q* and decrease that of the driving force *P*. Assuming a coefficient of friction

$\phi = 0.1$ the figure gives $Q = 100$, $P = 91.3$, and $P_0 = 87.8$; hence the efficiency is

$$\eta = \frac{87.8}{91.3} = 0.962.$$

Another example is given in Fig. 799. Here an eccentric *B* is attached to the shaft *A*, the eccentric rod *B C* driving the piston *C D* of a pump. It is required to determine the turning

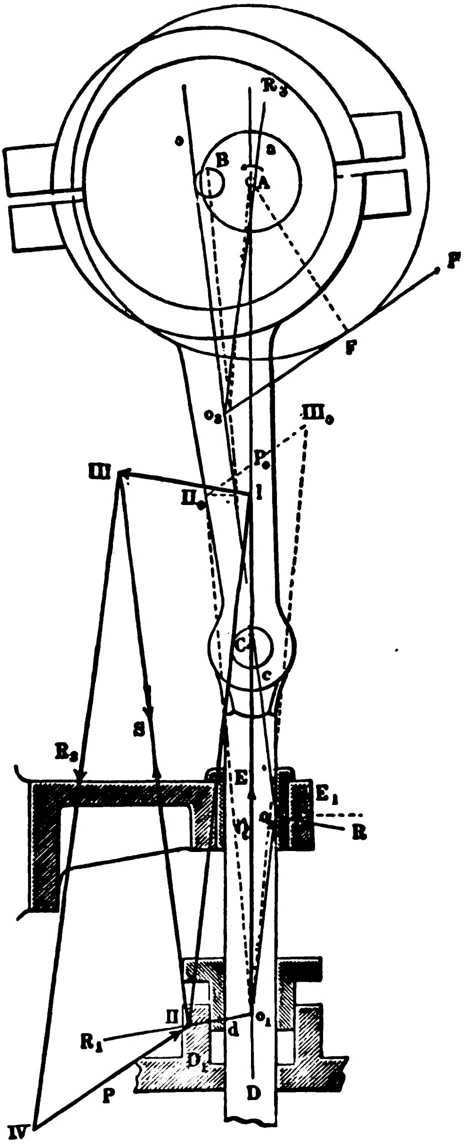
FIG. 798.



force *P* acting with a radius *A F*, say of a wheel, when the resistance $Q = o_1 I$, which the pump piston offers to the motion of the rod *C D*, is given. The resistance *Q* here acts in the direction *D C* of the piston rod, while the eccentric rod *B C* can only transmit forces in the direction of the tangent *bc* to the two friction circles. This force *S* along the rod and the resistance *Q* cannot establish equilibrium without the help of the reaction R_1 of the stuffing box and R_2 of the guide *E*, which reactions we may suppose to be concentrated at *d* and *e* respectively. That these four forces *Q*, *S*, R_1 and R_2 may be in equilibrium it is necessary that the resultant of *Q* and R_1 shall be equal and exactly opposite to that of *S* and R_2. This resultant is therefore given in position and direction by the line

$o_1\,o_2$ connecting the corresponding points of intersection. Therefore if we draw through I a parallel to $o_1\,o_2$, then $II\,I$ is the resultant composed of Q and the reaction $II\,o_1 = R_1$ of the stuffing box. A resolution of the resultant $I\,II$ in the direction of S and R_2 gives in $I\,III$ the reaction R_2 exerted by the guide at e and in $III\,II = S$ the force acting along the eccentric rod. This force S acts in the direction $b\,c$ and intersects the driving force P at o_3; consequently the reaction R_3 exerted by the shaft A must pass through the point of intersection o_3 and take the direction of the tangent $o_3\,a$ to the friction circle of A. If therefore we draw through III a parallel to $a\,o_3$, and through II a parallel to P, we get in $IV\,II$ the required driving force P at F, and in $III\,IV$ the reaction of the bearing against the shaft A. If we assume the angle of friction equal to zero, the reactions R_1 and R_2 be-

FIG. 799.

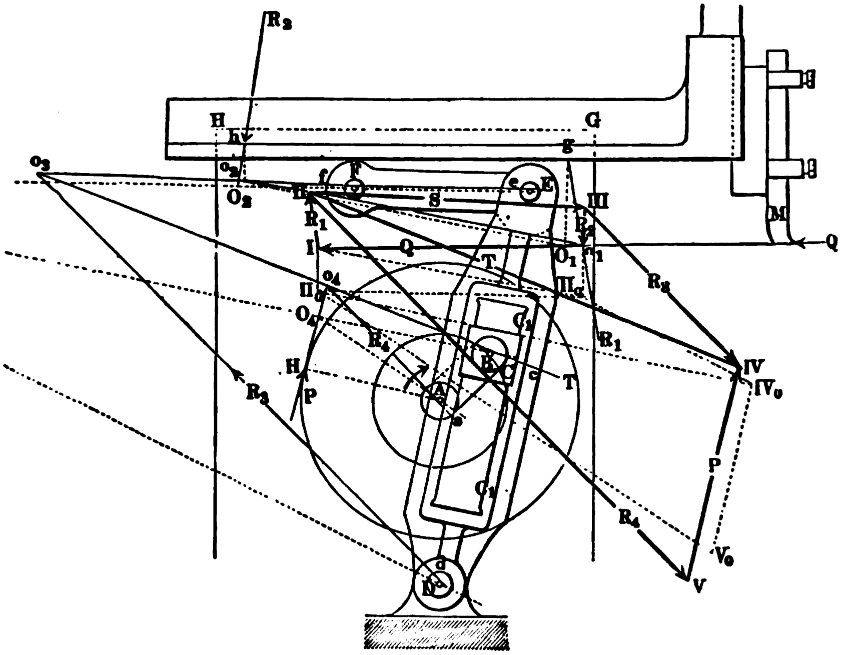


come normal to their guides and the directions of S and R_3 become central and give us the dotted diagram $Io_1\ II_0\ III_0$, in which $II_0\ III_0$ represents the theoretical driving force P_c. When the coefficient of friction at the guides is $\phi = 0.16$ and at the journals $\phi_1 = 0.1$, the figure gives $Q = 100$, $P = 36.2$, and $P_0 = 22.8$; hence the efficiency becomes

$$\eta = \frac{22.8}{36.2} = 0.630.$$

Moreover it is clear that the value of η depends upon the position assumed by the eccentric and will vary with this position.

FIG. 800.



There is still another example given in Fig. 800. The mechanism shown is one of the crank trains already discussed

(§ 140, Fig. 552), and here the slotted link *ED* communicates reciprocating motion by means of the connecting rod *EF* to a slide *HG* whose front end carries the holder for the tool *M*.

If $Q = o_1 I$ is the resistance which the work offers to the tool *M*, this force together with the force *S* transmitted to the slide by the connecting rod and the two reactions R_1 and R_2 of the guides must be in equilibrium. The force *S* along the rod acts in the tangent *fe* to the friction circles of the bearings *F* and *E*; the reactions R_1 and R_2 are drawn so as to make the angle of friction with the normals to the guides, say at *g* and *h*. If, as in the preceding example, we unite the intersection o_1 of *Q* with R_1 and the intersection o_2 of *S* with R_2 and then draw through *I* a parallel *I II* to R_1, through o_1 a parallel to R_2, and through *II* a parallel to *fe*, we get in $II\,III = S$ the force exerted by the rocking link *ED* on the connecting rod *EF*.

The rocking link itself must be in equilibrium under the influence of the force *S* exerted by the rod *EF* in the direction *ef*, the force *T* exerted by the crank pin *B* and the reaction R_3 of the bearing at *D*. The force transmitted from the crank-pin *B* through the block *C* to the slotted link $C_1 C_1$ must be tangential to the friction circle of *B* on account of the turning of the crank pin in the block, and must make the angle of friction ρ with the normal to the slot on account of the sliding of the block *C* in the slot. Consequently the force *T* must be drawn in the direction of the tangent $o_3 c$. If we draw from the intersection o_3 of *S* and *T* the tangent $o_3 d$ to the friction circle of *D*, we obtain the direction of the reaction R_3. Therefore if we resolve the force $S = II\,III$ in the force polygon into the directions *T* and R_3, we get in $II\,IV = T$ the force exerted by the crank pin in the direction $o_3 c$ on the slotted link. As this force *T* intersects at o_4 the force *P* driving the crank shaft *A* at *H*, the tangent $o_4 a$ to the friction circle of *A* gives the direction and position of the reaction R_4 of the crank-shaft bearing. It is therefore only necessary to decompose the force $T = II\,IV$ by the triangle *II IV V* into the directions of *P* and R_4, to get in *V IV* the turning force *P* needed at *tI*.

When frictional resistances are neglected we obtain the dotted force polygon $o_1\ I\ II_0\ III_0\ IV_0\ V_0$, and in $V_0\,IV_0 = P_0$

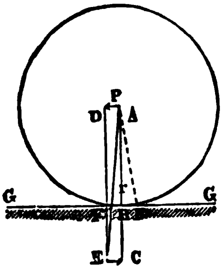
the theoretical driving force. When the coefficient of sliding friction $\phi = 0.16$ and of journal friction $\phi_1 = 0.1$, the figure gives for $Q = 100$, $P = 84.1$, and $P_0 = 66.6$; hence

$$\eta = \frac{66.6}{84.1} = 0.792.$$

In like manner the journal friction may in all cases be determined graphically by means of the friction circle.

If *rolling friction* occurs in a mechanism, it may also be determined graphically. Let A, Fig. 801, be the centre of a roller pressed against the horizontal surface GG by the load $AC = Q$; then, according to Vol. I, § 192, there is needed for the rolling a force

FIG. 801.



$$AD = P = f\frac{Q}{r},$$

where r is the radius of the roller and f an experimental constant which in ordinary cases can be taken equal to 0.0189 for iron and hard wood provided r is expressed in inches. When r is given in millimeters we must make $f = 0.0189 \times 25.4 = 0.5$ millimeter.

AE is the resultant of $Q = AC$ and $P = AD$, and we must therefore conclude that at the limit between equilibrium and motion the fixed path of the roller can react on the roller in the direction EA, and no motion will take place so long as the resultant of all the external forces deviates from the normal AC by less than the angle CAE. To determine this angle γ we have

$$\tan\gamma = \frac{CE}{AC} = \frac{P}{Q} = \frac{f}{r}.$$

It follows from this that, in order to draw the angle γ, it is only necessary to *lay off from the point B, where the path intersects the direction of pressure Q, the distance*

$$BF = f = 0.0189 \text{ in.} = 0.5 \text{ mm.},^*$$

---

* This magnitude must of course be drawn to the same scale as r; the forces, however, can be drawn to any desired scale.

i.e., the line drawn from F to the centre A will give the direction of the reaction of the path of the roller.

For example, let A, Fig. 802, be a roller placed under a load Q, through the centre A draw the normal $A B$ to the path $G G$, make $FB = f = 0.5$ millimeter [0.0189 in.]; then $F A E$ gives the direction of the reaction R of the path G. This direction intersects the force P in the point E; therefore if we make $E C = Q$ and through C draw $C H$ parallel to P, we shall get in $C H$ the driving force P to be applied at $K K$.

FIG. 802.

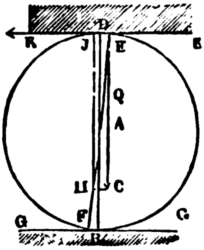
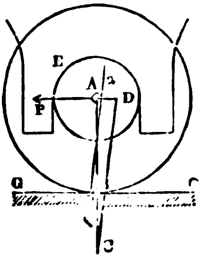


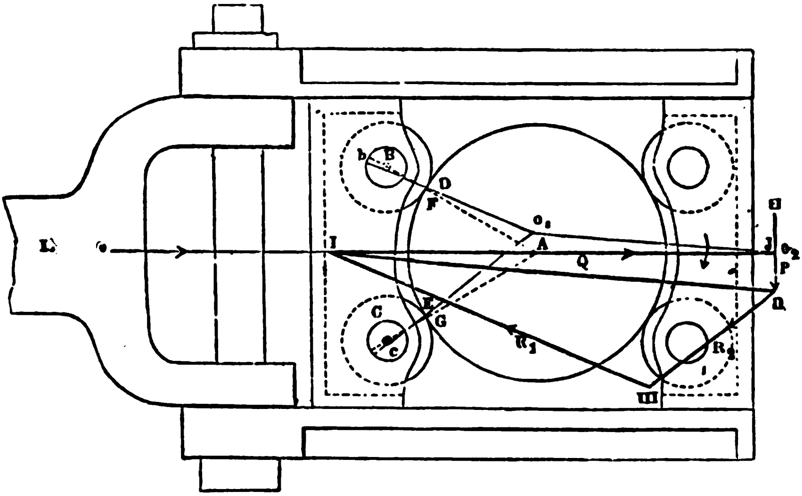
FIG. 803.



If the load Q does not rest on rollers, but on wheels, Fig. 803, we must, in addition to the rolling friction at B, take the journal friction into account. If we again make $B F = f = 0.5$ millimeter [0.0189 in.], the tangent $F a$ drawn from F to the friction circle of the journal A gives the direction of the reaction exerted by the load on the journal through the wheel. Therefore if $A C = Q$ is the load on the wheel and if we draw through C a parallel to the direction of the reaction $F a$, we shall get in $D A$ the pulling force P that is to be applied to the axle.

As an example of the determination of rolling friction, we will take the *roller-bearing*, Fig. 804, which serves to support the crane-jib L and carry it around the fixed crane-post A. If $Q = I o_2$ is the horizontal pressure of the jib L against the foot of the crane-post, the latter will react against the two rollers B and C at the points D and E whose distances from the actual points of contact F and G are equal to $FD = G E = 0.5$ millimeter [0.0189 in.]. If we draw from these points D and E the corresponding tangents Db and Ec to the friction circles of the roller pins, we obtain the direction of the reactions R_1 and R_2 which intersect at o_1. Now if the crane is turned by a force P

FIG. 804.

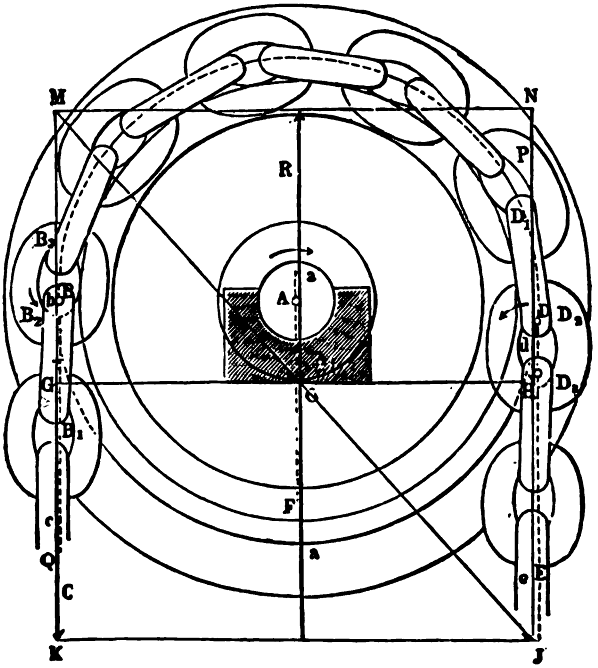


acting in the direction HJ which intersects Q at o_2, equilibrium of the four forces P, Q, R_1 and R_2 requires that the resultant of Q and P and that of R_1 and R_2 coincide in direction with the connecting line $o_1 o_2$. Therefore P is given by $o_2\,II$ when $I o_2 = Q$ and $I\,II$ is drawn through I parallel to $o_1 o_2$. If we resolve the force $II\,I$ into the directions of R_1 and R_2 we obtain in $III\,I = R_1$ and $II\,III = R_2$ the reactions of the post against the rollers. The figure gives for $Q = 100$, $P = 8.7$. We cannot speak of the coefficient of efficiency in this arrangement, for the useful work is here equal to zero, the whole power being absorbed in overcoming the hurtful resistances.

Chain friction, which takes place when chains are wound on and off pulleys and drums, is to be regarded as a sort of journal friction and must be determined in the same way.

In the chain pulley, Fig. 805, BC is the advancing portion of the chain and DE the receding or pulling portion, and it is easy to see that the motion of the links relatively to each other can only take place at a point B at which the chain runs on to the pulley, and at the point D where it is delivered from the pulley. In the straight, stretched, portions of the chain and in the portion BD lying firmly against the pulley there are no relative motions between the links in contact, and consequently no friction in these portions of the chain. At B, however, the bending and at D the straightening of the chain causes the links at these places to rotate relatively to each other, and this rotation is accompanied by journal friction. The diameter of the journal is represented by the diameter d of the iron forming the links of the chain. Therefore if we draw at B and D the

FIG. 805.



corresponding friction circles with the diameter ϕd, it follows that the link BB_1 can transmit a force to the link B_2B_3 only along a tangent to the friction circle of B. The same holds with regard to the links D_2D_3 and DD_1 at the point where the chain runs off the pulley. We see from the small arrows indicating the rotation of BB_1 relatively to B_2B_3, and of D_2D_3 to DD_1, that at B the direction of the reaction is the tangent lying on the outside and at D that lying on the inside of the friction circle. Hence, while the chain friction increases the lever-arm AB of the load by the amount $\phi\frac{d}{2}$, it decreases the lever-arm AD of the pulling force at D by the same amount. If in the pulley in Fig. 805 we take into account the reaction of the bearings acting along the tangent aa of the friction circle of A, we can easily determine the force P needed to overcome Q. For if we make $GK = Q$ and draw GH and KJ normal to GK, and also draw the diagonal JOM, we have

$$GM = GK\frac{GO}{HO} = Q\frac{r+\alpha+\kappa}{r-\alpha-\kappa},$$

where α and κ are the radii of the friction circles of the shaft-journal and the iron of the chain.

In exactly the same manner we must determine the influence of the stiffness of ropes. Here also the stiffness increases the lever-arm of the rope which is being wound on the pulley, by a certain quantity σ, while the lever-arm of the rope which is being unwound from the pulley is diminished by the same quantity σ. It is therefore only a question of determining the value σ by graphical means. For this we may employ the commonly used formula of Eytelwein, Vol. I, § 197, according to which the resistance, due to stiffness, experienced by a rope of diameter d winding itself up on one side and unwinding from the other side of a pulley of radius r is given by

$$S = 18.6\frac{d^2}{r}Q \text{ when } d \text{ and } r \text{ are in meters.}$$

$$\left[S = 0.472\frac{d^2}{r}Q \text{ when } d \text{ and } r \text{ are in inches.}\right]$$

The resistance to simple winding or unwinding is therefore

$$\tfrac{1}{2}S = 9.3\frac{d^2}{r}Q, \qquad \left[\tfrac{1}{2}S = 0.236\frac{d^2}{r}Q,\right]$$

or, when *d* and *r* are expressed in millimeters [inches],

$$\tfrac{1}{2}S = 0.0093\frac{d^2}{r}Q, \quad \left[\tfrac{1}{2}S = 0.236\frac{d^2}{r}Q.\right]$$

Now if σ represents the increase or decrease of the lever-arm r, corresponding to this resistance, we must place

$$\tfrac{1}{2}Sr = Q\sigma,$$

from which follows

$$\sigma = \frac{Sr}{2Q} = 0.0093\,d^2 \text{ mm.} \quad [\sigma = 0.236\,d^2 \text{ in.}]$$

To construct this expression we may write it in the form

$$\sigma = 0.0093 \times 4\left(\frac{d}{2}\right)^2 = 0.0372\left(\frac{d}{2}\right)^2 = \frac{1}{27}\left(\frac{d}{2}\right)^2,$$

$$\left[\sigma = \frac{1}{1.06}\left(\frac{d}{2}\right)^2,\right]$$

which may be constructed as follows: Let BDG, Fig. 806, represent the cross-section of the rope of diameter d, let $AC = 26$ millimeters [1.06 ins.], draw CB and DE parallel to it; then $AE = \sigma$ will be the required variation of the lever-arm r on each side of the pulley. For according to the construction we have

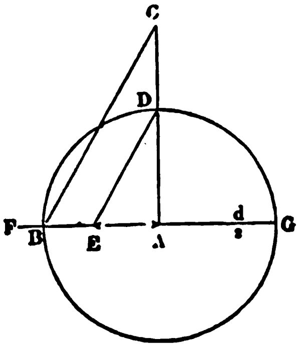
$$AE : AD = AB : AC,$$

or

$$AE = \frac{1}{27}\left(\frac{d}{2}\right)^2 \quad \left[AE = \frac{1}{1.06}\left(\frac{d}{2}\right)^2\right]$$

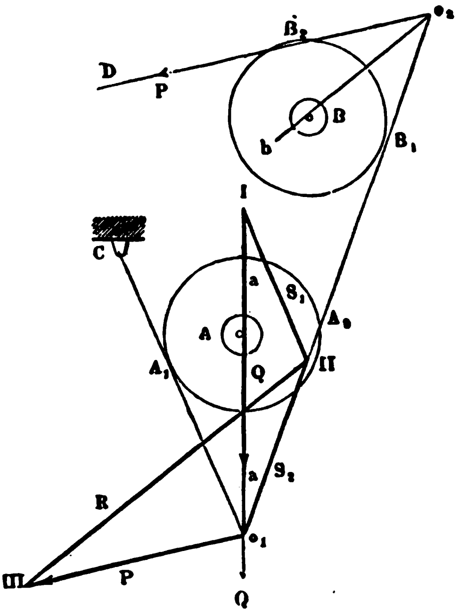
as is required.

FIG. 806.



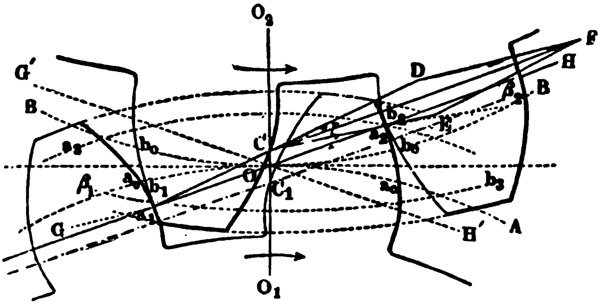
Let the arrangement, Fig. 807, be taken to illustrate the determination of rope- or chain-friction. Here the load $Q = Io_1$ is suspended from a loose pulley A and the lifting is effected by a force acting at the portion D of the rope; it is required to find the intensity of this force. The load Q acts tangentially to the friction circle of A in the direction *a a*. The intersection o_1 of the ropes CA_1 and A_2B_1 also lies in this direction, otherwise equilibrium would be impossible. To determine the directions of the tensions of the rope the radius r must be increased by σ at the points where the rope runs on to the pulleys, as at A_1 and B_1, and the radius r must be diminished by σ at the points where the rope runs off the pulleys, as at A_2 and B_2, where r represents the radius of the pulley (measured to the middle of the rope). Now if we make $Io_1 = Q$ and draw $I\,II$ parallel to CA_1 we shall get in $I\,II$ the tension S_1 of the rope A_1C, and in IIo_1 the tension S_2 in A_2B_1. The latter tension can be resolved into the direction DB_2 of the hauling rope and the direction of the reaction o_2b of the bearing of the journal B. Hence the force polygon will give in o_1III the required pulling force P, and in $II\,III$ the force R with which the journal B of the pulley is pressed against the bearing.

FIG. 807.



The resistance due to *friction of teeth* can also be readily determined by means of the angle of friction. Let A and B, Fig. 808, be the pitch circles of a pair of toothed wheels of which A may be the driver. At any instant let two pairs of teeth be in contact at the points $a_1 b_1$ and $a_2 b_2$; then we know from the theory of gearing that the normals to the tooth surfaces at their points of contact pass through the point of contact O of the pitch circles. Then if the teeth are involutes these two normals will coincide and form an angle of 75° with the line of centres $O_1 O_2$ (§ 73). Now from what was said in § 79 we know that when two teeth are in contact *before* the line of centres, as at $a_1 b_1$, there is a relative sliding of the tooth surface of A on that of B which is directed toward the *interior* of B, while when there is contact *behind* the line of centres, as at $a_2 b_2$, the sliding of the tooth surface of A on that of B is directed toward the *exterior* of the wheel B. This is also shown by the figure, for as the wheels are gradually turned the point of contact passes from the position $a_1 b_1$ to the point O on the line of centres, the two points a_0 and b_0 coming in contact at O, from which follows that the tooth A must have slid on B toward the *interior of B* a distance equal to the difference $b_1 b_0 - a_1 a_0$. Consequently the direction of the reaction existing between the tooth surfaces at A_1 and B_1 must be along the line $a_1 D$, the angle $D a_1 O$ being made equal to the angle of friction ρ. In like manner we can find the direction CE of the reaction of the tooth surfaces when there is contact behind the line of centres at $a_2 b_2$; here CE makes the angle $H a_2 E = \rho$ with the normal $O a_2 H$, and is drawn on the other side of the latter because the relative sliding of the tooth surfaces is toward

FIG. 808.



the *exterior of B*. Therefore the two reactions of the teeth at $a_1 b_1$ and $a_2 b_2$ intersect at a point C and make with each other an angle $DCE = 2\rho$. If we assume, as in § 79, that each of the two pairs of teeth transmits half the force, the two reactions will give a resultant CF which bisects the angle DCE and is therefore parallel to GH. Hence the foregoing investigation shows that we must regard the influence of tooth friction as though the force, theoretically supposed to be acting in the direction of the normal $a_1 O a_2$, were transferred parallel to this direction to the line CF. Then, as in journal-, rope-, and chain-friction, the lever-arm of the driving force of the wheel B is decreased by a certain quantity ζ, while this force, regarded as the *resistance* to be overcome by the wheel A, has its lever arm increased by the same quantity ζ. To determine this quantity ζ, let r_1 and r_2 represent the radii of the pitch circles of the wheels A and B, and γ the angle $G O O_1$ made by the normal with the line of centres. Then the theoretical lever-arms of the force P_0 with reference to the axes of A and B are given by $a = r_1 \sin \gamma$ and $b = r_2 \sin \gamma$. Now if $t = arc\ a_0 O a_0 = arc\ b_0 O b_0$ represents the pitch of the wheels, we can, with sufficient accuracy, place $a_1 a_2 = t \sin \gamma$ and finally get

$$\zeta = a_1 O \tan \rho = \frac{a_1 a_2}{2} \tan \rho = \frac{t}{2} \sin \gamma \tan \rho;$$

or if we assume $\sin \gamma = \sin 75° = 0.966$ as equal to unity, we shall get with sufficient accuracy

$$\zeta = \frac{t}{2} \tan \rho = \phi \frac{t}{2}.$$

This value, which for the sake of simplicity was developed for involute teeth, can be retained for other forms of teeth, for tooth friction is but slightly influenced by the form of the teeth. This value $\zeta = \phi \frac{t}{2}$ can be readily constructed; it can be regarded as *the friction radius of a journal whose diameter is* *equal to the pitch of the teeth.* The force transmitted by the teeth must then be regarded as acting along a tangent to the circle described with this radius about the point of contact of the pitch circles as a centre.

REMARK. It can be shown that this determination of the tooth friction gives the same result as the formulas developed in § 79, according to which the tooth friction is

$$F = Q\phi\pi\left(\frac{1}{z_1}+\frac{1}{z_2}\right),$$

where Q is the resistance to be overcome and z_1 and z_2 are the numbers of teeth of the wheels A and B. Thus, let $b = r_2 \sin\gamma$ be the lever-arm of a force Q acting in the direction GO and just sufficient to drive the wheel B when tooth friction is neglected. Then if a force drives the wheel B by acting along the line CF, that is, with a lever-arm $b - \zeta$, the intensity of the force is given by $Q\frac{b}{b-\zeta}$. If we suppose this resistance, acting along CF, *i.e.*, at a distance $a + \zeta$ from the axis of A, to be replaced by a force acting at the point O of the wheel A in the direction GO, and consequently with a lever-arm $a = r_1 \sin\gamma$, we shall get for its intensity

$$P = \frac{a+\zeta}{a}Q\frac{b}{b-\zeta} = Q\frac{b}{a}\frac{a+\zeta}{b-\zeta}.$$

Now if we substitute

$$r_1 = \frac{z_1 t}{2\pi} \quad\text{and}\quad r_2 = \frac{z_2 t}{2\pi},$$

and

$$a = r_1 \sin\gamma = \frac{z_1 t}{2\pi}\sin\gamma \quad\text{and}\quad b = \frac{z_2 t}{2\pi}\sin\gamma, \quad\text{also}\quad \zeta = \phi\frac{t}{2}\sin\gamma,$$

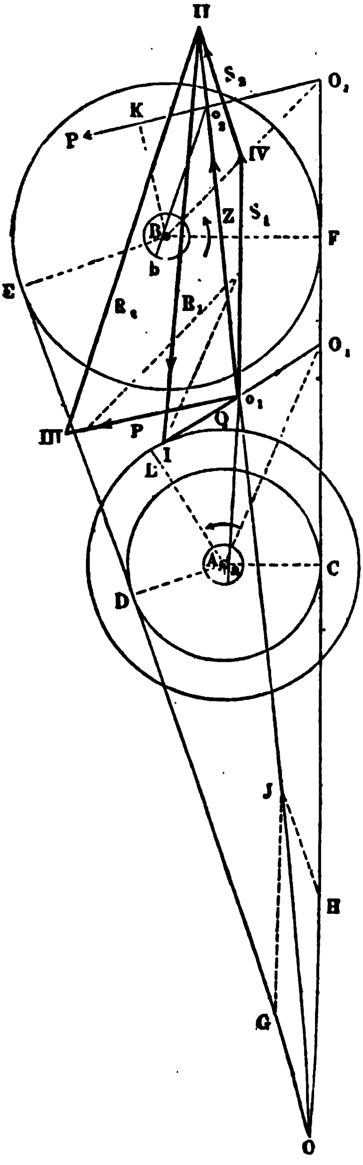
we get

$$P = Q\frac{z_2}{z_1}\frac{\frac{z_1 t}{2\pi}+\phi\frac{t}{2}}{\frac{z_2 t}{2\pi}-\phi\frac{t}{2}} = Q\frac{z_2}{z_2-\phi\pi}\,\frac{z_1+\phi\pi}{z_1} = Q\left[1+\phi\pi\left(\frac{1}{z_1}+\frac{1}{z_2}\right)\right];$$

hence the friction $F = P - Q = Q\phi\pi\left(\frac{1}{z_1}+\frac{1}{z_2}\right)$.

When motion is transmitted between two shafts by a *belt* the distances can be determined graphically as follows: Let Q be the resistance acting with the lever-arm AL, Fig. 809, and let B be the driving shaft, the transmission being effected by

Fig. 809.



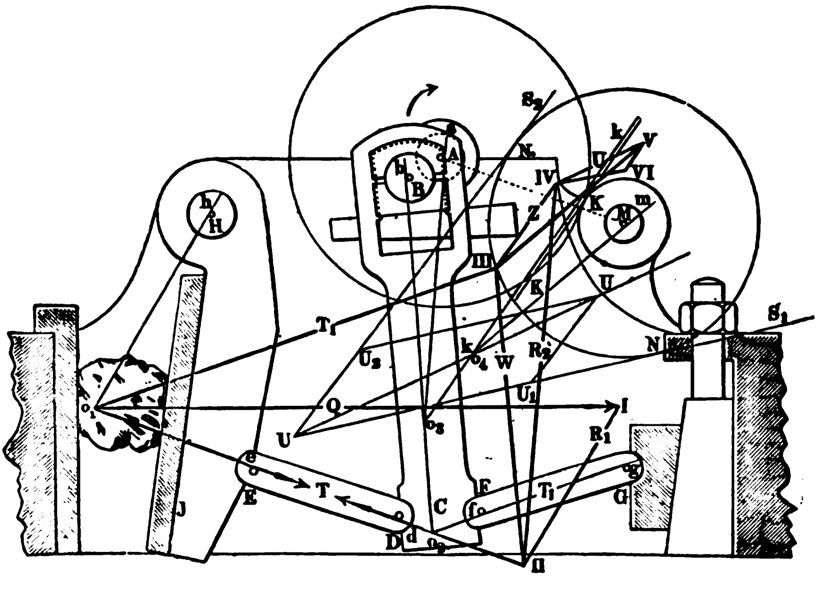
the belt-pulleys *EF* and *DC*; the problem is to determine the driving force *P* acting in the direction *KP*. Let S_1 be the tension in the driving side *CF* of the belt and S_2 the tension in *ED*; then we know from what was proved in a former chapter that for belt-gear the relation

$$S_1 = S_2 e^{\phi\gamma}$$

holds, γ representing the arc of contact *DAC* on the smaller pulley, and e the base of the hyperbolic system of logarithms. If we calculate the value $e^{\phi\gamma}$ or take it from a table, we can lay off from the intersection *O* to any scale along the two belt directions *OC* and *OD* two distances *OH* and *OG* having the ratio $e^{\phi\gamma}$ and then get, in the diagonal *OJ* of the parallelogram, the direction and position of the resultant of the forces exerted by the belt. This direction intersects *Q* at o_1; consequently the tangent $o_1 a$ gives the direction of the reaction R_1 of the bearing at *A*. Therefore if we make $I o_1 = Q$ and draw through *I* a parallel to $a o_1$, we get in $o_1 II$ the intensity of the resultant belt-pull *Z*, which force, resolved into the directions of the belt *CF* and *DE*, gives $o_1 IV = S_1$ and $IV\,II = S_2$ for the two belt tensions. Finally, to determine *P* we must assume the reaction R_2 of the bearing of B as acting along the tangent $o_2 b$ drawn from the intersection o_2 of Z with P to the friction circle of B, and then resolve the belt-pull Z into forces acting in the direction $o_2 b$ and in the direction of P. We then get in $o_1 III$ the force driving the shaft B and acting in the direction KP.

For the further elucidation of what has gone before we will give as a final example an investigation of the *stone-crusher* shown in Fig. 810. Here the material W is crushed by a jaw HJ turning about H, the jaw being moved by the toggle joint $EDFG$, the connecting rod BC and crank AB. If the

FIG. 810.



crank-shaft A is turned by the toothed wheels AK and MK by means of the belt-pulley NN, we can find the pull to be exerted by the belt in overcoming the crushing resistance $Q = o_1 I$, as follows: When the connecting rod moves upward, the two toggles DE and FG can only transmit the forces T and T_1 along tangents de and fg to the respective friction circles. Through the intersection o_1 of the forces Q and T acting on the jaw we must draw the reaction of the bearing at H, its direction being that of the tangent $o_1 h$ to the friction circle of H. If through I we draw a parallel to $o_1 h$, we shall get in $II\,o_1$ the thrust T of the toggle DE. This force cuts T_1 at o_2, consequently the connecting rod CB must exert, in the direction $o_2 b$, a pull $W = II\,III$ which is obtained by resolving $II\,o_1$ into the directions fg and $o_2 b$. If KK is the theoretical direction of pressure of the teeth at K, and if we draw, at the distance $\zeta = \phi \frac{t}{2}$ from this line, the parallel kk, we shall get the *actual* direction of the tooth pressure Z, and the reaction of A will be found in the tangent $o_3 a$ drawn from the intersection o_3 of kk and $o_2 b$ to the friction circle of A. A resolution of $W = II\,III$ into the directions kk and $o_3 a$ gives in $IV\,III$ the pressure Z of the teeth and in $II\,IV = R_2$ the reaction of the bearing on the shaft A. Now if the directions of the belt intersect at U and if we lay off UU_1 and UU_2 in the ratio $e^{\phi\gamma} : 1$, the diagonal UU of the parallelogram will give us the direction of the resultant belt-pull U. The reaction of the shaft M coincides in direction with the tangent $o_4 m$ drawn from the intersection of U and Z to the friction circle of M; hence if we draw $IV\,V$ parallel to U and $III\,V$ parallel to $o_4 m$, we shall get in $IV\,V = U$ the resultant belt-pull. Resolving this force into the directions of the belt, we finally get in $IV\,VI = S_1$ the tension of the driving side and in $VI\,V = S_2$ the tension in the driven side of the belt.

In like manner all such problems may be solved graphically, the sides of the force polygon giving the stresses occurring simultaneously in the individual machine parts, rods, bearings, frames, etc., so that we may determine by this means the required dimensions. The accuracy attainable by this graphical method in all cases satisfies the practical requirements when the scale of the diagram is not taken too small.

INDEX.

PAGE
Accelerating Force 734
Acceleration, Centre of 776
Accumulator 895, 897
Air-brakes 909
Anchor Escapement 883
Anti-parallel Cranks 651
Astatic governors 1006
Automatic Spinning Machine 837, 847
Automatic Stop-motions 878
Balance Spring 882, 890
Band-brakes 900, 912
Beam and Crank 720
Beam, Counterweight 956
Bell Crank 1059
Belts 855
Belts as Disengaging Gear 854
Belt-shifter 854
Block 572
Block-brakes 900
Brake, Cone 920
Brake, Differential 917
Brake, Link 919
Brakes 898
Brakes, Air 909
Brakes, Block 900
Brakes, Eccentric 900, 921
Brake, Steam Pressure as a 911
Brake, Wire-rope 913
Bramah's Lock 867
Brasses 772
Bréguet's Key 834
Buchanan's Steering Wheel 554
Buffer 792
Buss Governor 1040
Cam Disk, Grooved 799
Cam Disks 793
Cam for Sewing Machine 810, 816

PAGE
Cam Rails....826, 829
Cam Surfaces.... 784
Cam Trains.... 784
Cams, Cylindrical.... 815
Cams, Flat 799
Cams, Triangular.... 812
Car Coupling 618
Card-grinder 824
Cataract.... 1016
Catch Gear.... 861
Centre of Acceleration.... 776
Centre of Curvature 1028
Centrifugal Governors.... 991
Chain-friction.... 1067
Chain-locking Devices 568
Chains....535, 560
Chains, Flat-link.... 560
Check Nuts....605, 606
Chronometer Escapement.... 893
Chronometric Apparatus.... 924
Chubb Lock.... 867
Clamping Gear.... 873
Click Gear....860, 861
Clip Pulleys.... 574
Clock, Escapement....832, 882
Clutches.... 833
Coefficient of Fluctuation.... 701, 1007
Coefficient of Insensibility.... 1002
Coefficient of Unsteadiness....701, 1007
Combing Machine.... 803
Compression.... 727
Cone-brake.... 920
Connecting Rod....639. 771
Connecting Rod, Influence of Length of....696, 708
Copping Rail.... 827
Copying Machine.... 829
Cosine-Governor.... 1031
Cotter.... 773
Counter-faller Wire.... 827
Counterweight Beam.... 956
Counterweight, Hydraulic....927. 937
Counterweighting Cranks.... 953
Counterweight, Pneumatic....1042
Counterweights....897, 926, 953, 958
Coupling, Drag-link.... 645
Coupling, Kirchweger's.... 648

PAGE
Coupling, Oldham's 656
Couplings 618, 833
Crank and Beam 720
Crank Constructions 754
Crank Dimensions 763
Crank, Double 702, 757
Crank Motion, Graphics of 742
Cranks, Counterweighting 953
Cranks, Parallel 646
Crank Trains 639
Crank Train Friction 747
Crank Train Motions 657
Crank Train, Oscillating 669
Crank, Triple 715
Curvature, Centre of 1028
Cylinder Escapement 889
Cylindrical Cams 815
Dead-beat Escapement 886, 893
Dead Points 640, 646
Démêloir, Heilmann's 805
Démêloir, Parpaite's 804
Differential Brake 917
Differential Governors 1040
Differential Screws 609, 612
Disengaging Gear 832
Disengaging Gear, Belts as 854
Disengaging Gear, Toothed Wheels as 840
Disk-polishing Machine 655
Double Crank 702, 757
Double-threaded Screw 588
Drag-link 645
Drag-link Coupling 645
Drawbridges 945
Drawing frames 824, 881
Drilling Machines 904
Drill, Ratchet 871
Duplex Escapement 891
Eccentric 757, 805
Eccentric Brakes 900, 921
Efficiency of Mechanisms 1050
Efficiency of the Screw 596
Ejector 910
Elements, Higher Pair of 784
Elevators 860, 903, 1053
Elliptical Wheels 674
Elliptic Chuck 657, 812

PAGE
Embroidering Machine, Heilmann's 843
Endless Screw 590, 622
Energy of a Governor 1004
Engaging Gear 832
Envelopes 785
Epicyclic Train 1042
Equalizing carriage 953
Equidistants 1009
Escapement, Chronometer 893
Escapement, Clock 832, 882
Escapement, Cylinder 889
Escapement, Duplex 891
Escapement, Pin 887
Escapement, Recoil 886
Escape Wheel 883
Expansion 727
Expansion Gear 995
Expansion, Pin 758
Faller-wire 827
Fan Governor 1041
Flat Cams 799
Flat-link Chains 560
Flax-break 648
Flexible Transmitters 535
Fluctuation, Coefficient of 701, 1007
Fly-wheel 965
Fly-wheel, Distribution of Mass of 983
Fly-wheels 962, 977
Fly-wheels, Action of 967
Fly-wheels, Coefficient of Unsteadiness of 702
Fly-wheels, Strength of 985
Force Regenerator 941
Formers 829
Franke's Parabolic Governor 1009
Friction Circle 1058
Friction of Teeth 1070
Friction of the Crank Train 747
Friction, Rolling 1064
Fusee 891
Galloway's Mechanism 651
Gang Saws 833
Gibs 773
Governor, Buss' 1040
Governor, Centrifugal 991
Governor, Cosine 1031
Governor, Differential 1040

PAGE
Governor, Energy of 1004
Governor, Fan 1041
Governor, Franke's Parabolic 1009
Governor, Grossman's 1025
Governor, Kley's 1022
Governor, Parabolic 1009
Governor, Pendulum 1041
Governor, Poncelet's 1046
Governor, Porter's 1003
Governor, Proell's 1027
Governor, Pseudo-astatic 1018
Governor, Sensitiveness of 1000
Governor, Siemens' 1046
Graphical Statics of Mechanisms 1049
Graphics of Crank Motion 742
Grooved Cam Disk 799
Grossman's Governor 1025
Guide Pulleys 570
Gun-locks 863
Hammer 864
Helical Motion 591
Helix 583
Hempen Rope 536
Hoists 860, 903, 1053
Hydraulic Counterweights 927, 937, 1042
Inertia of the Counterweight 958
Inflexion Circle 1029
Insensibility, Coefficient of 1002
Inversions 802, 820, 1039
Key 773
Key, Bréguet's 834
Kirchweger's Coupling 648
Kley's Governor 1022
Left-handed Screw 588, 617
Lever and Shearing Machines 806
Link-brake 919
Lock, Bramah's 867
Lock, Chubb 867
Locking Devices for Nuts 606
Lock, Yale 868
Mangle Mechanism 850
Mangle Wheel 853
Mass of the Fly-wheel, Distribution of 983
Micrometer Screws 835
Milling Tool 624
Motions of the Crank Train 657

PAGE
Mule.................................837, 847
Mule, Self-acting.................................826, 837
Nut-locking Devices.................................606
Nuts, Check.................................605, 606
Oldham's Coupling.................................656
Ordinary Crank Train.................................651
Oscillating Crank Train.................................669
Pallet.................................884
Parabolic Governor.................................1009
Parallel Cranks.................................646
Pawl.................................862
Pendulum.................................832, 882
Pendulum, Governor.................................1041
Permanency, Condition of.................................689, 968
Pile-drivers, Powder.................................865
Pin Escapement.................................887
Pin Expansion.................................758
Pin Rack.................................850
Piston Pressure.................................738
Pneumatic Counterweights.................................927, 1042
Poncelet's Governor.................................1046
Porter's Governor.................................1003
Powder-hammers (pile-drivers).................................865
Power Printing-presses.................................845
Proell's Governor.................................1027
Pseudo-astatic Governors.................................1018
Pulley, Rope.................................575
Pulleys, Clip.................................574
Pulleys, Guide.................................570
Pump Mechanism.................................656
Quadric Crank Train.................................639
Quick-return Gear for Planers.................................858
Quick-return Motion, Whitworth's.................................653, 758
Rack, Pin.................................850
Ratchet Drill.................................871
Ratchet Gear.................................868
Reactions.................................1052
Recoil Escapement.................................886
Regenerator, Force.................................941
Régime.................................689, 968
Regulators.................................882, 895, 898
Reverse Motion, Efficiency of.................................1050
Reversing Gear for Planers.................................858
Right-handed Screw.................................588, 617
Roemer's Mechanism.................................678
Roller-bearing.................................1066

PAGE
Rolling Friction....1064
Rope, Hempen.... 536
Rope, Pulley.... 575
Rope Pulleys for Towing.... 575
Rope Rods.... 577, 647
Ropes and Chains.... 535
Rope Stiffness....1068
Rope, Wire....548, 556
Rotating Mass, Weight of.... 973
Saws, Gang.... 833
Screw, Endless....590, 622
Screws.... 584
Screws, Differential.... 609
Screws for propelling Ships.... 589
Screws, Micrometer.... 835
Screws, Right- and Left-hand....588, 617
Screws, Single-threaded.... 588
Screws used as Fastenings....587, 594, 600
Screw Thread.... 586
Screws with Differential Motion.... 612
Screws with Multiple Threads.... 588
Screws, Wood.... 590
Screw Wheels.... 627
Self-locking Mechanism....597, 1050
Sensitiveness of Governors....1000
Sewing-machine Cam....810, 816
Sheaves....570
Shipper.... 854
Siemens' Governor 1046
Silk-doubling Machines.... 879
Single-threaded Screws.... 588
Slider Crank Train.... 651
Slowing down.... 659
Smoothing Action.... 820
Snap.... 568
Spiral Drum.... 953
Spooling Machine....676, 796, 854
Spring Beam.... 792
Squeezers 806
Stamper.... 817
Standing Rigging.... 556
Static and Astatic Governors....1006
Steam Pressure as a Brake.... 911
Steering Gear.... 619
Stem-Winder.... 835
Stiffness of Ropes.... 565, 1068

PAGE
Stone Crusher 1075
Stop-motions, Automatic 878
Stopping 659
Strands 536, 538
Striking Gear 896
Stud Bolts 608
Thread-break 832
Thread, Screw 586
Throstles, Cam for 796
Throttle-valve Gear 994
Toggle Joint 1075
Toothed Wheels as Disengaging Gear 840
Tooth Friction 1070
Transmitters, Flexible 535
Triangular Cams 812
Trigger 865
Triple Crank 715
Tumbler 863, 866
Unsteadiness, Coefficient of 701, 1007
Vise Motion 592
Wagon Drag 899
Weight of the Rotating Mass 973
Whitworth's Quick Return Motion 653, 758
Wipers 788
Wire Rope 548, 556
Wire-rope Break 913
Wire-rope Machinery 759
Wood Screws 590
Worm 590
Worm Wheel 590, 622
Yale Lock 868

89088908025

b89088908025a

89088908025

B89088908025A